PROBABILISTIC NATURE OF MASS-ENERGY CONVERSION

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Abstract

Special Relativity leads us to draw conclusion on mass-energy conversion but does not tell about the governing limits occupied with the process. This paper has developed a probabilistic approach to show the proper limit of mass-energy conversion in an inertial frame of reference. Here we will see that mass or energy can never be converted fully into each other by itself. An indirect proof & restatement of the 2^{nd} law of thermodynamics has been given from the view-point of special relativity and the conclusion derived from the method. At the end an alternative explanation of radioactive decay & fate of the universe has been discussed.

We will find out whether a definite amount of mass can totally get converted into energy by itself without outer influence in an inertial frame of reference.

Definition of "Outer Influence": The very definition of 'outer influence' is pretty straight forward. It means any change in total mass or energy which is altered by outside.

Thought Experiment 1A : Take a definite amount of mass of magnitude 'm'. In a suitable frame of reference the mass is completely at rest. The total magnitude of mass does not change throughout our experiment. In an instance, if a miniscule amount of mass within the total mass gets totally converted by itself without outer influence into energy then same amount of released energy has to be get converted by itself into that fixed amount of mass to make the total mass fixed. So, the whole procedure contains two stages.

i) Fixed amount of miniscule mass being converted into energy without outer influence.

ii) Fixed amount of energy equivalent to the amount of miniscule converted mass being converted into that amount of miniscule mass.

If we somehow get assured that a definite miniscule amount of mass has been converted into energy by itself then according to mass-energy equivalence principle of special relativity we also get assured about how much energy has to be converted by itself to mass simultaneously to keep the overall mass fixed. Hence, if we are confirmed about the amount of mass in 1^{st} stage of mass to energy conversion, the associated probability (ω_1) for the process will be at maximum because of our confirmation. So, $\omega_1 = 1$. So, according to Boltzmann Entropy law [1,2,3] , associated entropy, $S_1 = -k_1 \ln \omega_1 = 0$. Here '$k_1$' is a fixed constant but not Boltzmann constant. Similarly if we are confirmed about the 1^{st} stage then the second stage will have its full confirmation about the amount of energy to mass conversion and thus the associated probability (ω_2) for the process is maximum, so, $\omega_2 = 1$. So entropy for the process, $S_2 = -k_2 \ln \omega_2 = 0$. (k_1 and k_2 are taken different from each other as we are not sure about the values and related processes entirely) So, total change in entropy, $S_1+S_2 = 0$, which is an obvious outcome as because the two conversion processes are considered isentropic separately within the total mass. From another point of view, total mass was confirmed being fixed, so associated probability (ω) of the mass being constant is 1. So, change in entropy for the total mass, $S= -k \ln \omega =0$, which supports $S= S_1+S_2$. This simple thought experiment now can be applied on the instantaneous miniscule mass itself which is without outer influence on the verge of conversion into energy. If we somehow know the amount of that miniscule mass then according to our previous part of the experiment the change in entropy for that miniscule conversion will be zero. The thought experiment will go on accordingly. By this process it can be shown that a definite amount of mass in a suitable inertial frame of reference at rest no matter how small it is remains in an isentropic condition.

Thought experiment 1B : Take a definite amount of mass of magnitude 'm' which is at rest in a suitable inertial frame of reference. If the mass itself is converted entirely into energy without outer influence then according to the principle of mass-energy conversion of special theory of relativity [4,5,6] the total energy liberated by the conversion process is exactly known. So the associated probability (ω) of the process is at maximum and the value of which will be 1. Hence, the associated entropy, $S = -k \ln \omega = 0$. Again, from 1A the entropy associated to the mass when its magnitude is exactly known is zero if there is no outer influence. So, our thought experiment 1B leads us to draw the conclusion that if a definite amount of mass can be converted fully into energy by itself according to the mass-energy equivalence principle without outer influence then there will be no change in entropy associated with the process- which directly contradicts with the second law of thermodynamics.

Interpretation of the results of 1A & 1B: The interpretation of the two thought experiments is quite simple. "If a definite amount of mass is at rest in an inertial frame of reference then the probability of mass being converted by itself into energy without outer influence is zero"

Thought experiment 1C: Take a definite amount of mass of magnitude 'm' which is at rest in a suitable inertial frame of reference. If a fraction of the whole mass can be converted itself into energy according to mass-energy conversion principle without outer influence then the probability of that fraction of mass having the magnitude 'm_p' is taken 'ω_p' then ω_p is always less than 1 according to the previous thought experiments, because, if we know the exact amount of mass being converted itself into energy without outer influence then it violates the second law of thermodynamics. So, the associated change in entropy in the process, $S = -k \ln \omega_p \neq 0$, which has a definite positive value. Before the initialization of the conversion the entropy associated with that definite amount of mass was zero according to previous experiments. After the conversion process we get a definite change in the value of entropy. So, if a fraction from a fixed amount of mass of definite magnitude starts to get converted into energy by itself without outer influence then the value of entropy changes positively- this is in complete harmony with the second law of thermodynamics.

Combined interpretation of 1A, 1B & 1C: The combined interpretation of these thought experiments are pretty serious.

"If a definite amount of mass is at rest in an inertial frame of reference then the probability of mass being converted by itself into energy without outer influence is zero, but the probability of a random portion of that definite mass being converted totally by itself without outer influence into energy is less than unity."

If the probability for total conversion is $(\omega_T)_{m\to e}$ and probability of random portion for conversion is $(\omega_p)_{m\to e}$ then $(\omega_T)_{m\to e} = 0$, and $0 \leq (\omega_p)_{m\to e} < 1$.

Thought experiment for energy: If a definite amount of energy of magnitude 'E' is considered then similar experiments can be constructed. At this point we can take a simple shortcut using mass-energy equivalence principle, otherwise elaborate descriptions of those experiments are necessary. According to special theory of relativity[4, 5, 6] mass & energy are equivalent, hence in 1A,1B,1C the terms mass & energy can be swapped and the governing logics remain exactly same. Only difference in the statement is that now the resting condition for mass is not needed for energy, rather, the term "confined at a place" will be much appropriate. Being confined at place for energy implies that the boundary of that definite amount of energy is not composed of mass. By this, equivalent interpretation for energy turns to be-

"If a definite amount of energy is confined at a place in an inertial frame of reference then the probability of the energy being converted totally by itself without outer influence into mass is zero, but the probability of a random portion of that definite energy being converted totally by itself without outer influence into mass is less than unity."

If the probability for total conversion is $(\omega_T)_{e\to m}$ and probability of random portion for conversion is $(\omega_p)_{e\to m}$ then $(\omega_T)_{e\to m} = 0$, and $0 \le (\omega_p)_{e\to m} < 1$.

In general, "For a definite amount of mass or energy the full conversion between one to another never happens without outer influence in an inertial frame of reference"

Let a definite amount of mass of magnitude 'm' is being considered. Within any given time period 't' some portion of mass 'm_p' is converted into energy by itself without outer influence. If the probability of the mass-fraction converted having magnitude 'x_p' is 'ω' and can be expressed by a function 'f' of 'ω' then $x_p = f(\omega)$. Our goal is to find this particular probability function relating the mass-fraction converted automatically. Considering whole process into stages – for 1st stage: $x_{p1} = f(\omega_1)$ and for 2nd stage: $x_{p2} = f(\omega_2)$.

Now, $x_{p1} = m_{p1}/m$ and $x_{p2} = m_{p2}/(m - m_{p1})$ and $m_{p1} + m_{p2} = m_p$.

Substitution gives

$$x_{p1} + x_{p2} - x_{p1} \times x_{p2} = x_p \quad \text{or,} \quad f(\omega_1) + f(\omega_2) - f(\omega_1)f(\omega_1) = f(\omega).$$

Two processes are consecutive events, hence, $\omega = \omega_1\omega_2$. Substitution gives

$$f(\omega_1) + f(\omega_2) - f(\omega_1)f(\omega_2) = f(\omega_1\omega_2) \qquad (1)$$

Taking partial derivative with respect to ω_1 and ω_2 gives

$f'(\omega_1) - f'(\omega_1)f(\omega_2) = \omega_2 f'(\omega_1\omega_2)$ and $f'(\omega_2) - f'(\omega_2)f(\omega_1) = \omega_1 f'(\omega_1\omega_2)$. Hence,

$$\frac{\omega_1 f'(\omega_1)}{f(\omega_1)-1} = \frac{\omega_2 f'(\omega_2)}{f(\omega_2)-1} \qquad (2)$$

In general, $$\frac{\omega f'(\omega)}{f(\omega)-1} = constant = k_1$$

Indefinite integration on both sides gives

$$f(\omega) = 1 + e^{k_2}\omega^{k_1} \qquad (3)$$

where k_2 is constant of indefinite integral.

So , $$m_p = m\,(1 + e^{k_2}\omega^{k_1}) \qquad (4)$$

Applying $m_p \le m$ leads to $e^{k_2}\omega^{k_1} \le 0$.

If all the masses can be converted into energy by itself then $m_p = m$, hence $e^{k_2}\omega^{k_1} = 0$, which gives ω=0. It is the exact interpretation we have previously made from 1A &1B.

So, "a definite amount of mass at rest in an inertial frame of reference cannot convert itself into energy without outer influence"- this is a universal property of mass and a limiting condition for mass to energy conversion.

Applying $m_p < m$ leads us to the nature of the constants k_1 and k_2 . Two decisions can be made.

i) k_1 and k_2 are both complex numbers. 2) k_1 is real and k_2 is imaginary taking values $i(2n - 1)\pi$ where n ϵ N.

So, the amount of energy released can be found by $\Delta E_p = m_p c^2$.

Hence, $$\Delta E_p = (1 + e^{k_2}\omega^{k_1})mc^2 \qquad (5)$$

Eq. (4) & (5) reveals the probabilistic nature of mass energy conversion. From Eq. (4) we find the value of probability of a certain portion of mass being converted by itself without outer influence. $$\omega = \sqrt[k_1]{\left(\frac{m_p - m}{m}\right)}\, e^{-\frac{k_2}{k_1}} \qquad (6)$$

Rearrangement gives, $$\omega = \sqrt[k_1]{\left(\frac{m - m_p}{m}\right)} \times A \qquad (7)$$

where $A = \sqrt[k_1]{-e^{-k_2}}$

Eq. (7) gives us a clear insight of self-conversion of mass into energy. If the initial mass is large then it is much probable to get a certain amount being automatically converted. If the desired portion of mass to be automatically converted is high then it is a less likely phenomenon to happen.

Setting $k_2 = i(2n - 1)\pi$ leads the equation (4) in the form
$$m_p = m\,(1 - \omega^{k_1}) \qquad (8)$$
where k_1 is real.

And energy released is $$\Delta E_p = (1 - \omega^{k_1})mc^2 \qquad (9)$$

Special case: If this self-generated energy is used to provide the necessary kinetic energy to make the remnant mass $(m - m_p)$ move with speed 'v' relative to its inertial frame of reference then according to the special theory of relativity [4,5,6]
$$\left(\frac{1}{\sqrt{1-\left(\frac{v}{c}\right)^2}} - 1\right)(m - m_p)c^2 = m_p c^2$$

Rearrangement gives $$\omega = \sqrt[2k_1]{1 - \left(\frac{v}{c}\right)^2} \qquad (10)$$

Eq. (10) gives the probability of a definite mass initially at rest to move by its self-gen. energy with velocity 'v' relative to its inertial frame of reference.

An Indirect Proof & Restatement of the Second Law of Thermodynamics: The probability function in Eq. (4) proves that there is no way a definite amount of mass can be fully converted into energy by itself. This same interpretation was taken from the initial thought experiments by applying the second law of thermodynamics by considering it as a universal law. The mathematics here developed hence indirectly proves the Second Law of Thermodynamics. So we can set a new statement describing the Second Law of Thermodynamics:

"Without outer influence a definite amount of mass cannot be fully converted into energy by itself & vice-versa."

Significance of the results:

1) Explanation of radioactive decay from the view point of probabilistic nature of mass-

energy conversion: Atoms of higher nucleon numbers tend to be more radioactive than that of having lower nucleon numbers. Apart from the classic explanation, fundamental cause of radioactive phenomenon can be explained by our probabilistic method developed here. Eq. (8) gives

$$\omega = \sqrt[k_1]{\frac{m-m_p}{m}} \tag{11}$$

Considering a single atom of atomic mass M_a Eq. (11) gives the probability of a definite amount 'm_p' from the atom being converted into energy by itself.

$$\omega = \sqrt[k_1]{\frac{M_a-m_p}{M_a}} \tag{12}$$

With atomic mass M_a being increasingly higher gives the probability of the process a higher value. Thus, scopes of the magnitude of that definite amount 'm_p' being large increases for atoms of higher atomic mass- so does the energy released in Eq. (9). If the available released energy exceeds some portion of the binding energy of the nucleus in any region then the nucleus breaks apart.

2) Fate of the Universe: There are a few noble questions like- "will the entire universe be totally turned into energy or vice-versa?" Our entire discussion here can give a deeper insight in this topic. The mathematics developed here is equally applicable for a definite amount of energy 'E' which is now being analyzed on about whether it can be converted into mass all by itself without outer influence. Eq.6 gives us the probability ω_E in similar form.

$$\omega_E = \sqrt[k_3]{\left(\frac{E_p-E}{E}\right)}\, e^{-\frac{k_4}{k_3}}$$, constants k_3 & k_4 are taken different from k_1 & k_2

Rearrangement gives,

$$\omega_E = \sqrt[k_3]{\left(\frac{E-E_p}{E}\right)} \times A' \tag{13}$$

Depending on the values of constants, initial amount and desired portion- the both probabilities are varied- but never come to zero. Hence we can conclude that the Universe will contain both mass & energy in their distinctive form forever. In far future, a Universe containing only mass or only energy is impossible.

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