

# TOWARDS EQUATIONS FOR BRAIN DYNAMICS AND THE CONCEPT OF EXTENDED CONNECTOME

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The brain is a system at the edge of chaos equipped with nonlinear dynamics and functional energetic landscapes. However, still doubts exist concerning the type of attractors or the trajectories followed by particles in the nervous phase space. Starting from an unusual system governed by differential equations in which a dissipative strange attractor coexists with an invariant conservative torus, we developed a 3D model of brain phase space which has the potential to be operationalized and assessed empirically. We achieved a system displaying both a torus and a strange attractor, depending just on the initial conditions. Further, the system generates a funnel-like attractor equipped with a fractal structure. Changes in three easily detectable brain phase parameters (log frequency, excitatory/inhibitory ratio and fractal slope) lead to modifications in funnel's breadth or in torus/attractor superimposition: it explains a large repertoire of brain functions and activities, such as sensations/perceptions, memory and self-generated thoughts.

Keywords:

Brain dynamics; Strange attractor; Invariant torus

## INTRODUCTION

The brain is a complex, non-linear system, formed by a large number of interacting and inter-dependent components which exhibit emergent properties, spontaneous self-organization (Tognoli; Papo) and sensitivity to initial small changes. It has been proposed that the brain operates at the edge of chaos, near a critical regime, where the maximum information function lies between randomness and regularity (De Arcangelis; Zare). The concept of nonlinear brain needs to be framed into the energy landscape theory, originally built for a statistical description of protein's potential surfaces (Ferreiro; Sutto). Such landscape is characterised not just by low-energy valleys - stationary points where the gradient vanishes -, but also by high-energy peaks and transition states. When nervous activity takes place, the energy tends to converge towards a stable state equipped with a minimum energy level, so that the interplay between neuronal structure and activity at many different spatiotemporal scales gives rise to functional "attractors" (Vuksanovic, Watanabe) that emphasize the importance of random walks (O'Donnell), metastability (Afraimovich) and self-organized criticality (Bak). Although the simplest candidate in which local minima might occur is a simple "fixed-point attractor", which is a funnel located in a functional phase space where trajectories converge as time progresses, other models have also been proposed: for example, resting state networks might emerge as structured noise fluctuations around a stable low firing activity equilibrium state in the presence of latent "ghost" multistable attractors. (Deco). It has been also suggested that brain function does not exhibit erratic brain dynamics nor attractors, rather a stable sequence, called transient heteroclinic channel (Afraimovich). A foremost issue in such a context is the scale-free dynamics – also called  $1/f^\alpha$  behavior, power law, self-similarity, fractal-like distribution (Fraiman; Slomczynski; Milstein; Linkenkaer-Hansen).

Conversely, it has been recently proposed that mental operations follow constrained, topological donut-like trajectories along preferential functional brain railways (Tozzi 2016). Experimental and theoretical clues allow us to conjecture that some brain activities are shaped in guise of a multidimensional torus. The theoretical claims of brain multidimensionality are widespread (Tononi; Papo; Ponce-Alvarez): as an example, models characterized by dimensionality reduction have been used in the study of human central nervous system (Garg). It has been demonstrated that high dimensionality brain spontaneous activity structures occur, consisting of multiple, reproducible temporal sequences (Mittra). It has also been shown that the exceedingly diverse nonlinear selectivity in single-neuron activity in the prefrontal cortex is a signature of high-dimensional neural representations: crucially, this dimensionality

is predictive of animal behaviour as it collapses in error trials (Rigotti). In addition, recent findings suggest that nervous structures process information through topological as well as spatial mechanisms: for example, hippocampal place cells might produce topological templates in order to represent spatial information (Dabaghian). A torus displays a donut-like shape: it means that the trajectory followed by a particle inside it is closed. To make an example, if one walks along one of the curves of a torus, she thinks is crossing a straight trajectory, not recognizing that her environment is embedded in a higher dimension. The torus may be compared with a video game with biplanes in aerial combat: when a biplane flies off one edge of gaming display, it does not crash but rather it comes back from the opposite edge of the screen (Weeks). Mathematically speaking, the display edges have been “glued” together. Our brain exhibits similar behaviour, i.e., the unique ability to connect past, present and future events in a single, coherent picture (Fingelkurts; Nyberg), as if we were allowed to watch the three screens of past-present-future glued together in a mental kaleidoscope. The same occurs during other brain functions, e.g., memory retrieval, recursive imagination and mind wandering (Andrews-Hanna), in which concepts flow from a state to another. The torus is naturally visualized intrinsically, by ignoring any extrinsic properties a surface may have: in the same way, we humans perceive our thoughts intrinsically and naturally adopt “private”, subjective standpoints.

In this multifaceted framework, a recently proposed three-dimensional, continuous, time-reversible system - governed by a set of first-order ordinary differential equations (ODEs) - comes into play. This system has an unusual property: it exhibits conservative behavior for some initial conditions, and dissipative for others (Sprott). The conservative regime has quasi-periodic orbits whose amplitude depend on the initial conditions, while the dissipative regime is chaotic. It means that, in the state space, a strange attractor coexists with an invariant torus. Our aim was to build a neural model, starting from the Sprott’s ODEs, which might explain brain dynamics during different mental activities.

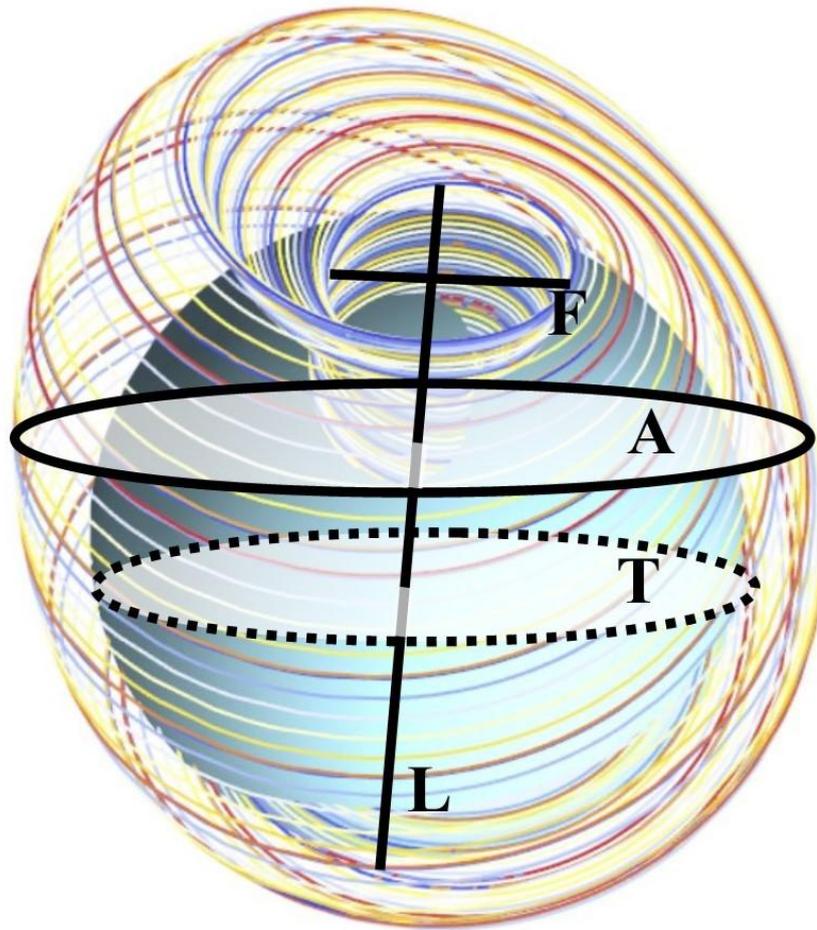
## MATERIALS AND METHODS

**The Sprott’s equations.** We built a 3-dimensional phase space based on the unusual Sprott’s system, equipped with nonequilibrium points (stable or unstable) and only bounded orbits for all initial conditions. The system is described by the following ODEs:

$$\begin{aligned}\dot{x} &= y + 2xy + xz, \\ \dot{y} &= 1 - 2x^2 + yz, \\ \dot{z} &= x - x^2 - y^2.\end{aligned}\quad (1)$$

The only possible solutions are (quasi)-periodic or chaotic, depending just on the starting point of the initial conditions. Notice that Sprott’s images stand for the 4D case (x,y,z,t), with time as a (hidden) parameter on the derivatives.

We achieved the evaluable and measurable features described in **Figure 1**.



**Figure 1** depicts four measurable system's features. In this case, as an example, we displayed the system originating from the initial conditions 0.5002, -0.5, -0.0791. The solid circle A stands for the disk embedded into the strange attractor's maximum transversal diameter, while the solid line F for the diameter of the funnel lying into the strange attractor. The solid line L stands for the length of the funnel lying into the strange attractor, while the dotted circle T for the disk embedded into the torus's circumference.

**A model of brain phase space: the extended connectome.** The next step was to evaluate whether it is possible to assess brain dynamics through our Sprott's system implementation. The natural candidate to the role of nervous phase space is the human connectome. According to the widely accepted description of the connectome's networks, the cortical and subcortical structures form a hierarchical network, equipped with preferential pathways for fast communication and winner-takes-all mechanisms (Van den Heuvel; Reese). However, recent papers unveil the role of a connectome that is more extended than previously believed, investigating network connections in different species: the *Caenorhabditis elegans* globally integrative rich club of neurons (Towlson), the mouse microscale cortico-thalamic pathway (Oh), the rodent peripheral nervous system connectivity and cortical midline structures (Schmitt), the human subcortical connectome (Looi), thalamus (Lenglet) and basal ganglia - including striatum and putamen (Schmidt) -. Taken together, such observations suggest that the nodes and the hubs of the connectome are not confined to the "higher" and phylogenetically recent structures, but are also diffused along the entire neural pathway; including the central and peripheral nervous system. It is possible that, in the years to come, the concept of connectome will be also extended to the peripheral receptors. Indeed, most peripheral structures play a crucial role in information processing: receptors perform complex neural computations that were thought to be carried out by sensory cortex and have a prominent role in sensory processing. As an example, the brain has outsourced some aspects of touch processing to the

nerve endings in the fingertips (Pruszynski). In sum, we can state that the electric oscillatory activity of the extended connectome – shaped in guise of a phase space - gives rise to a free-energy landscape.

Our hypothesis is that external (or internal) inputs activate just a point on a 3D brain phase space (the connectome), and the activation of such a point leads to a trajectory displaying the shape of either a torus, or a strange attractor. The particle trajectories depend just on its initial location in the phase space. We placed, on each one of the three Sprott's axes, three easily detectable parameters of brain activity, obtaining a nervous phase space characterized by the following features:

x = the natural logarithm of the spike frequency.

y = the  $\alpha$  fractal slope.

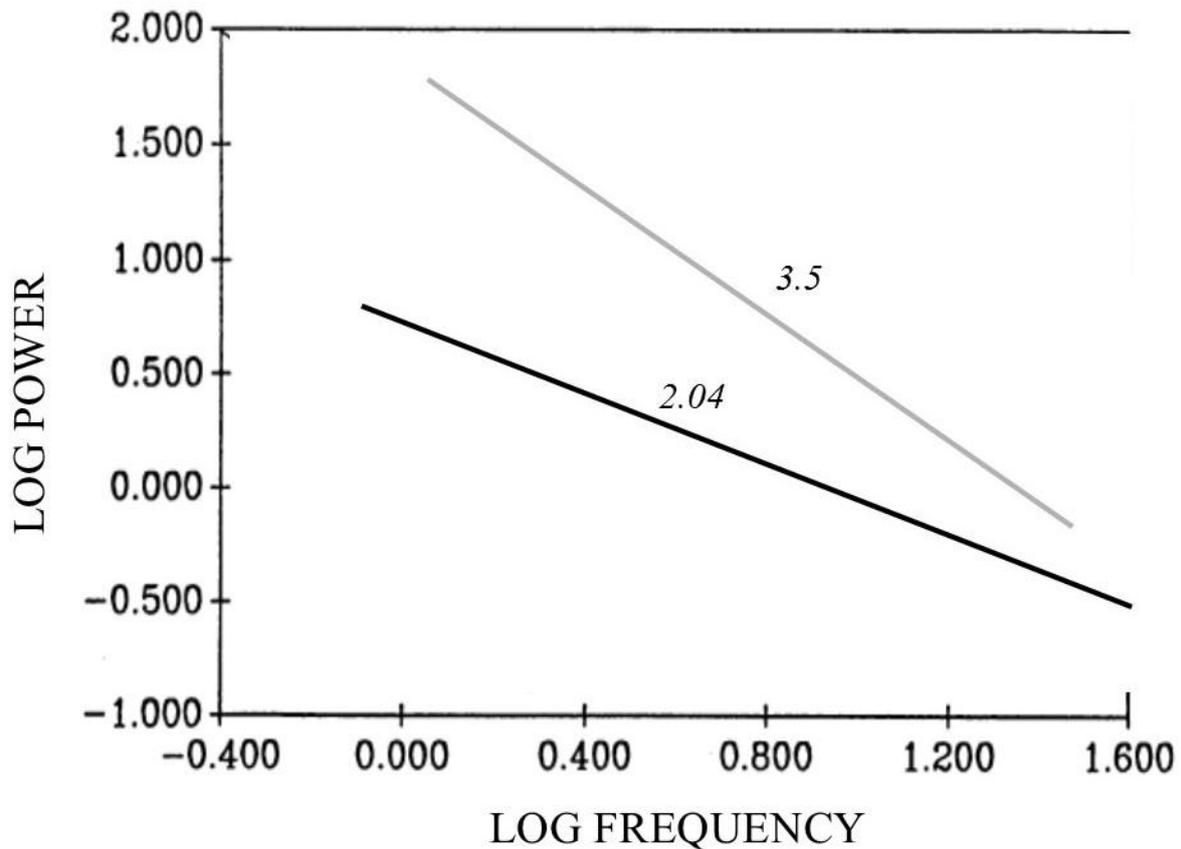
z = the neuronal excitatory/inhibitory ratio.

Note that the spike frequency (our x parameter) is proportional to the brain free-energy and ATP consumption. Indeed, local increases in spike frequency cause a transitory boost of energy consumption and free-energy production, with a metabolic cost of  $6.5\mu\text{mol/ATP/gr/min}$  for each spike (Attwell). Thus, the increase in thermodynamic free-energy during brain activity is mostly due to spiking.

Concerning the y parameter, it must be kept into account that the frequency spectrum displays a scale-invariant behaviour  $S(f) = 1/f^\alpha$ , where  $S(f)$  is the power spectrum and  $f$  is the frequency. Further,  $\alpha$  stands for an exponent, the so-called “dimension” of the fractal, which equals the negative slope of the line in a log power versus log frequency plot (Pritchard, Van de Ville; Newman) (**Figure 2**). Such a general scheme stands both for (spatial) fractals and (temporal) power laws.

We used the neuronal excitatory/inhibitory ratio (E/I ratio) as our z parameter. Such a kind of homeostatic mechanism has been already revealed in the spatial distribution of synaptic strengths, by which neurons balance excitation (E) and inhibition (I), not only in time, but also in space (Xue). Specifically, cortical circuits appear to maintain a constant E/I ratio between the total amount of excitatory and inhibitory stimulation (Sengupta 2013a, Sengupta 2013b). E/I balance is seen both in vitro and in the spontaneously active cortex, not only for individual pyramidal neurons at a given time [Haider], but also across multiple cortical neurons and during neural avalanches (Lombardi). An optimal E/I ratio across neurons is maintained, despite fluctuating cortical activity levels, through the appropriate strengthening or weakening of inhibitory synapses [Xue]. It has been demonstrated that the relationship between the two opposing forces affects many cortical functions, such as feature selectivity and gain (Sengupta 2013b), or memory of past activity [Lombardi], or rapid transitions between relatively stable network states, which allow the modulation of neuronal responsiveness in a behaviourally relevant manner (Haider). The E/I ratio in each area can be calculated, provided that total must give zero.

We evaluated the system behavior during increases or decreases of every one of the three brain parameters, in order to detect which type of trajectory is achieved when a single change in x, y or z occurs. The values of some of the three coordinates were extracted from real-data plots of studies based on EEGs and fMRI neurotechniques (Pritchard, He, Milstein; Miller)



**Figure 2.** Log amplitude versus log frequency scatter plot of brain spikes detected by EEG techniques (modified from Pritchard). The Figure displays on the x axis the frequency (in Hz) and on the y axis the power (in  $\mu V^2$ ) of the electric spikes. Note that the scale is logarithmic: it means that on the x axis, for example,  $-0.400 = 0,39$  Hz,  $0 = 1$  Hz,  $0.4 = 2.52$  Hz,  $1.2 = 16$  Hz and so on. In turn, on the y axis,  $-1.000 = 0,1 \mu V^2$ ,  $-0.5 = 0,32 \mu V^2$ ,  $0.000 = 1 \mu V^2$ ,  $1.000 = 10 \mu V^2$ , and so on. The figure also depicts two possible fractal dimensions, equipped with different slopes ( $\alpha = 2.04$  and  $3.5$ , respectively).

## RESULTS

We achieved a system displaying both a torus and a strange attractor, depending on the starting conditions, i.e., the initial points where the trajectories started their movements. **Figure 3** shows the results of our implementation of Sprott's attractors with different initial conditions  $(x,y,z)$ . A noteworthy result is that the strange attractors display a funnel-shape morphology, with the appearance of a twisting funnel that dips in and out of the sphere: it means that the funnel is embedded in a fractal structure. The funnel is more or less narrow, depending on the initial conditions: to make an example, when  $x$  increases, the funnel is much narrower, while, when  $y$  increases, the funnel is larger. Depending from initial conditions, the strange attractor and the torus are more or less superimposed: i.e., when  $y$  changes, the strange attractor and the torus are closely intertwined, making difficult to evaluate whether a starting point leads to a fractal or non-fractal trajectory; vice versa, when  $z$  increases, the strange attractor surrounds the torus in guise of a cloud; furthermore, when  $x$  increases, the two structures are neatly separated.

In particular, we achieved the following results:

**When the initial conditions are 0,0,0, the following system is displayed:**

L and T are partially superimposed and intertwined.

T diameter = A diameter

F = high values (a large funnel)

**When the initial conditions are 1,0,0:**

L and T are sharply separated

$T < A$

F = the minimum values (very narrow funnel's diameter))

**When the initial conditions are 0,1,0:**

T (almost completely) includes and surrounds L.

$T > A$

A is partially embedded into T

F = the highest values (the widest funnel)

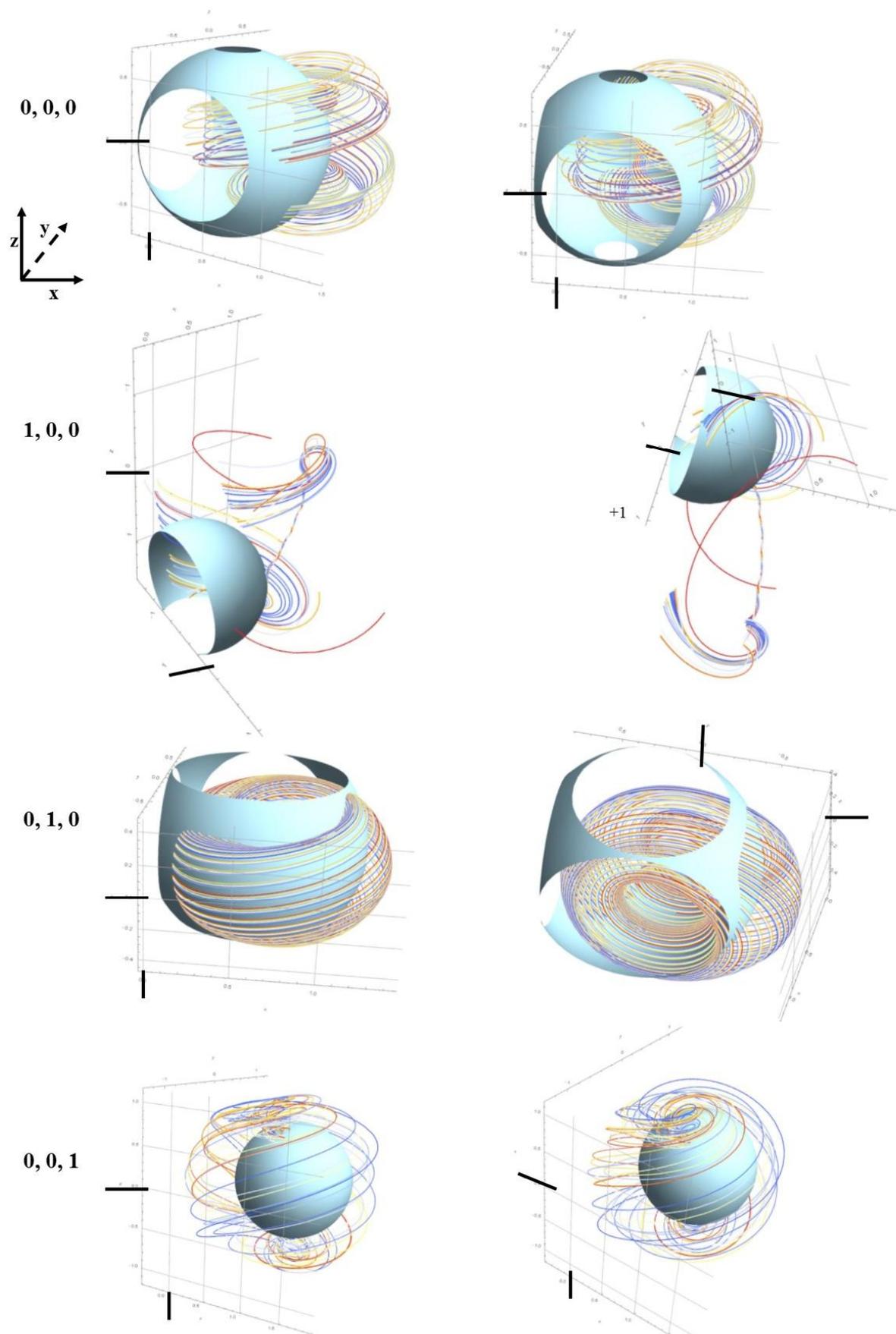
**When the initial conditions are 0,0,1:**

A includes and surrounds T

$A > T$

T is embedded into A

F = low values (narrow funnel)



**Figure 3.** implementation of Spratt's attractors for four different values of  $x$ ,  $y$  and  $z$ . The black lines show the points zero on the respective axis. See the main text for further details.

## CONCLUSIONS

Starting from the unusual Sprott's system of ODEs, we built a system equipped with both a conservative torus and a dissipative strange attractor. When a moving particle starts its trajectory from a given position  $x, y, z$  in the 3D nervous phase space, we may predict whether it will fall in the torus or into the strange attractor. The funnel shape is fractal, and not just a simple fixed-point attractor. A narrower funnel means that the trajectory is constrained towards a small zone of the phase space. When the two structures are closely superimposed, we might hypothesize a state of phase transition at the edge of the chaos, equipped with high symmetry, in which it is difficult to evaluate every single initial position: a slightly change in the starting point could indeed lead to completely different outcomes. When the torus and the strange attractor are clearly splitted, a single starting point gives rise to a sharp outcome. It means that in the latter case, the two conformations are neatly separated, as if the system went out of phase transition and a symmetry breaking occurred.

Looking for possible psychological correlates of our system, we hypothesize that the proposed scheme might stand for a large repertoire of brain functions and activities, such as sensations/perceptions, emotions, mood-state/affect, memory, abstraction, sequencing/planning, choice, judgement, creativity, self-generated thoughts (Andrews-Hanna). For example, when the particle falls into the conservative torus conformation (where two antipodal particles cannot never meet or become closer, due to their recurring rotations) the correlated psychical activities could be either repetitive patterns of movements or thoughts, or preserved memories, or mind wandering, or unsure perceptions, in which different ideas are not properly melted together. In turn, when the particle falls into the dissipative strange attractor conformation (equipped with steep and narrow funnels), the correlated psychical activities could be: the vanishing of memories, or sharp sensations of "known" objects, or non-repetitive motor patterns, or sure perceptions due to the confluence of different sensations, or disparate, unsure thoughts which converge towards a single, fixed idea. Further, the idea of the partial superimposition of the two conformations is in touch with the Kullback-Leibler divergence described by the Friston's free-energy model (Friston).

We might hypothesize that, due to external or internal inputs, changes take place in one or more of the three parameters: spike frequency, fractal slope and E/I ratio. Such changes activate a single point of the 3D functional brain lattice, where a "particle 1" starts its trajectory into one of the two described dynamical conformations. When another input occurs, the corresponding variation in the parameters' value puts in motion a "particle 2" from a different starting point in the phase space. We might speculate that: a) either the particle 2' trajectory prevails and cancel the particle 1's; 2) or the two particles move at the same time, following each one its own orbit; 3) or the particle 1, following the activation of the starting point of the trajectory 2, switches from its trajectory to the trajectory 2, or to another one.

Our model also satisfies the brain free-energy constraints required by the most influential current brain theories (Sengupta 2013a; Friston). Indeed, spontaneous activity displays a lower energetic level than the evoked one. As stated above, we know that each spike has a certain consumption of ATP (Attwell) and that each spike is formed by an oscillation, equipped with both an amplitude and a frequency. For the Ohm's law, the energy consumption due to the AMPLITUDE of the oscillation is negligible, compared with the energy consumption due to its FREQUENCY. It means that the evoked brain activity, equipped with a mean spike frequency higher than the spontaneous activity, spends more energy. It is in touch with our results, because the increase of spike frequency (axis  $x$ ) leads to a "known" perception. When, for example, an external stimulus enters the visual field, high frequency spikes are elicited and transmitted through the nerves and the free-energy of the energy landscape increases, compared with its basal level. The message (that we may compare with a particle trajectory) is conveyed towards a functional funnel (Watanabe). When the particle trajectory falls into the (fractal) funnel, the perception is neat and the stimulus is clearly recognized. The trajectory goes towards lower energetic levels, until, at the bottom of the funnel, the free energy is the lowest and the perception is the "surest" possible. There are two alternative ways to elicit a brain response: the first are the spontaneous, high frequency neuronal avalanches (Beggs), which might lead to the imagination of an object (without its presence in the environment). The second alternative way is a different spontaneous, not evoked activity, which could be the physiological counterpart of the default mode network with its psychological correlates (such as mind wandering or self-generated thoughts) (Andrews-Hanna). This low-energy activity is characterized by low-frequency oscillations equipped with power laws, infinite correlation length and (probably) falling into strange attractors and phase transitions. During a sensation, a funnel-like strange attractor occurs, thus decreasing the peak of energy towards a lower basin. When the sensation disappears, the non-dissipative torus is restored.

Of course, ours is just an in-progress, provisional scheme. As suggested by Sprott himself (personal communication), we might also not to stop at three dimensions plus time, and we might take into account noninteger dimensions too. For example, two interacting particles can be described as the motion of a single particle in a 6-dimensional space, where three of the dimensions characterize the particle position, and the other three characterize the components of velocity in each of the three spatial directions. A dynamical phase space's landscape (such as the brain) that consists of  $N$  interacting microareas could be described by a single point moving in a  $2N$ -dimensional phase space, although it will typically be the case that the dynamics is confined to a lower-dimensional subset of that very high-dimension space. It must also be kept into account that the torus, being structurally unstable in dynamical systems with dimensions

greater than three, tends to evolve into a strange attractor. It means that, in long timescales, the trajectory inexorably falls into a funnel-like attractor equipped with the lowest possible energetic level, as proposed by protein-folding models based on the principle of minimum frustration (Sutto).

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