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THE MOMENT OF MOMENTUM AND THE PROTON RADIUS

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The theory of nuclear gravitation is used to calculate the moment of momentum of the gravitational field of a proton, which is compared to the corresponding moment of momentum of the electromagnetic field. As a result, the proton radius is estimated and a relation for the moment of momentum of the field is established, which coincides in form with the expression of the virial theorem for energy.

A proton, as a quantum object, possesses its inherent magnetic moment, electric charge, spin, mass, and other characteristics, which are measured with a high accuracy in numerous experiments in elementary particle physics. Obviously, many parameters of a proton may be related to one another by some expressions that follow from the physical nature of interactions. Characteristic examples are the proportionality between the magnetic moment, the spin, and the specific charge, observed for the majority of elementary particles, and the proportionality between the spin and the squared mass for particles on Regge trajectories. In quantum chromodynamics, it is supposed that the integrity of a proton is provided by the strong interaction between its three constituent quarks and the field quanta – gluons. With another approach, nuclear gravitation [1] is introduced by analogy with conventional gravitation where the integrity of cosmic objects is due to the balance of the attracting gravitational forces and the repulsing electromagnetic forces of matter particles. In this paper, in terms of gravitational field theory, a condition is placed on the moment of momentum of a proton and its radius is estimated.

The scalar gravitational potential inside a proton for the case of a homogeneous density distribution of the matter is expressed, with proper boundary conditions, as

$$\psi(r)=\frac{2\pi\Gamma\rho\,(r^2-3R^2)}{3} \quad \text{for} \quad \psi(R)=-\frac{\Gamma M}{R}, \quad \psi(\infty)=0\,.$$

Here, Γ is the nuclear gravitation constant; ρ is the density of the proton matter; r is the moving radius, and R and M are the proton mass and radius, respectively.

In the static case, the acceleration of a matter, $\boldsymbol{G}$, under the action of a field is determined in terms of the potential gradient:

$$\boldsymbol{G}=-\nabla\psi=-\frac{\Gamma M(r)}{r^3}\boldsymbol{r}=-\frac{4\pi\Gamma\rho}{3}\boldsymbol{r}\,, \tag{1}$$

where $M(r)$ is the mass of the matter within the radius r.

For a rotating proton, the gravitational field acquires a moment of momentum whose volumetric density, according to [1], is found by the formula

$$\boldsymbol{g}=-\frac{1}{4\pi\Gamma}\boldsymbol{G}\times\boldsymbol{\Omega}\,, \tag{2}$$

where $\boldsymbol{\Omega}$ is the torsion of the gravitational field.

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To estimate the torsion inside the proton, we perform an instantaneous Lorentz transformation of the gravitational field tensor, whose components are the components of the vectors $\boldsymbol{G}/c$ and $\boldsymbol{\Omega}$, from a resting frame of reference S' into a frame of reference S, which moves with velocity V along the x-axis. Since the torsion $\boldsymbol{\Omega}$ in S' is equal to zero, neglecting the Lorentz factor, we find for the frame of reference S

$$G_x = G'_x\,, \quad G_y = \frac{G'_y + V\,\Omega_z}{\sqrt{1-V^2/c^2}} \approx G'_y\,, \quad G_z = \frac{G'_z - V\,\Omega_y}{\sqrt{1-V^2/c^2}} \approx G'_z\,, \tag{3}$$

$$\Omega_x = 0\,, \quad \Omega_y = \frac{\Omega'_y - V\,G'_z/c^2}{\sqrt{1-V^2/c^2}} \approx -\frac{V\,G'_z}{c^2}\,, \quad \Omega_z = \frac{\Omega'_z + V\,G'_y/c^2}{\sqrt{1-V^2/c^2}} \approx \frac{V\,G'_y}{c^2}\,.$$

In virtue of the relativity of a motion in the frame of reference S, the proton also moves with the velocity V, but in the reverse direction. If the transformation is performed from the frame of reference S' into S at each point inside the proton, the linear velocity V can be expressed in terms of angular rotational velocity w and spherical coordinates r, θ, and φ as $V = w\,r\sin\theta$, and (1) and (3) can be rewritten:

$$G_x = -\frac{4\pi\Gamma\rho\,r\sin\theta\,\cos\varphi}{3}\,, \quad G_y = -\frac{4\pi\Gamma\rho\,r\sin\theta\,\sin\varphi}{3}\,,$$

$$G_z = -\frac{4\pi\Gamma\rho\,r\cos\theta}{3}\,, \quad \Omega_x = \frac{4\pi\Gamma\rho\,\,w\,r^2\sin\theta\,\cos\theta\cos\varphi}{3c^2}\,, \tag{4}$$

$$\Omega_y = \frac{4\pi\Gamma\rho\,\,w\,r^2\sin\theta\,\cos\theta\sin\varphi}{3c^2}\,, \quad \Omega_z = -\frac{4\pi\Gamma\rho\,\,w\,r^2\sin^2\theta}{3c^2}\,.$$

For not very great velocities, we may neglect the additions to the field components (4) that appear due to the fact that the transformations should be performed, in fact, in frames of reference rotating along the z-axis rather than in locally inertial frames of reference. According to (4), if the proton rotates counterclockwise, the internal torsion Ω_z is directed everywhere opposite to the z-axis, while the projections of the torsion $\boldsymbol{\Omega}$ on the plane z = const are directed away from the z-axis. With expression (2) we find the components of the momentum density vector of the gravitational field inside the proton:

$$g_x = -\frac{1}{4\pi\;\Gamma}(G_y\Omega_z - G_z\Omega_y) = -\frac{4\pi\;\Gamma w\;\;\rho^2 r^3\sin\theta\;\sin\varphi}{9c^2}\,,$$

$$g_y = -\frac{1}{4\pi\;\Gamma}(G_z\Omega_x - G_x\Omega_z) = \frac{4\pi\;\Gamma w\;\;\rho^2 r^3\sin\theta\;\cos\varphi}{9c^2}\,,$$

$$g_z = -\frac{1}{4\pi\;\Gamma}(G_x\Omega_y - G_y\Omega_x) = 0\,.$$

The vector $\boldsymbol{g}$ points in the same direction as the linear velocity of rotation of unit volumes of the proton matter. To calculate the moment of momentum of the gravitational field inside the proton, it is necessary to multiply the modulus of the vector $\boldsymbol{g}$ by the distance from the z-axis, that is, by $r\sin\theta$, and then integrate the result over the proton volume:

$$L = \frac{4\pi\Gamma w\rho^2}{9c^2}\int_0^R r^6 dr \int_0^\pi \sin^3\theta\, d\theta \int_0^{2\pi} d\varphi = \frac{32\pi^2\Gamma w\rho^2 R^7}{189c^2}.$$

In view of the expressions for the proton mass, $M = \frac{4\pi}{3}\rho R^3$, and spin, $I = 0{,}4 M wR^2$, for the case of a uniform distribution of the matter, the quantity L can be written

$$L = \frac{5\Gamma M I}{21 R c^2}, \qquad (5)$$

the vector ***L*** being directed along the spin ***I***.

We assume that there is only one type of degenerate objects in every gravitational field, which have the maximum matter density and, accordingly, the highest gravitational and electromagnetic fields. For nuclear gravitation and conventional gravitation, objects of this type are, respectively, nucleons and neutron stars. Then it should be expected that in (5) the moment of momentum L of the gravitational field is equal to the proton spin I. Actually, if it were the case that $L > I$, then the gravitational field would start the rotation of the proton, thereby increasing its spin. Similarly, the electromagnetic pressure on the matter is directed along the momentum density vector of the electromagnetic field and is proportional to the field energy absorbed by the matter. By reducing the quantities L and I in (5), we can estimate the proton radius:

$$R = \frac{5\Gamma M}{21 c^2} = 6.7\cdot 10^{-16}\ \text{m}. \qquad (6)$$

The nuclear gravitation constant Γ in (6) is found from the condition that in a hydrogen atom, within the Bohr radius R_B, the gravitational force is equal to the electrostatic one:

$$\frac{\Gamma M M_e}{R_\text{B}^2} = \frac{e^2}{4\pi\varepsilon_0 R_\text{B}^2}, \quad \Gamma = \frac{e^2}{4\pi\varepsilon_0 M M_e} = 1.514\cdot 10^{29}\ \text{m}^3\cdot\text{kg}^{-1}\cdot\text{s}^{-2}, \qquad (7)$$

where e is the elementary electric charge; ε_0 is the dielectric constant, and M and M_e are the proton and the electron mass, respectively.

For comparison with the result (6), we find the proton radius by other methods [1]. A neutron and a proton form together an isotopic dublet and are very similar to one another in properties. The difference between the masses of an electrically neutral neutron and a proton of charge e can be ascribed to the mass-energy of the electric field of the proton:

$$M_n - M = \frac{K e^2}{4\pi\varepsilon_0 R c^2}.$$

Putting $K = 0.6$, as for a uniformly charged ball, and substituting the neutron mass M_n and the velocity of light c, we find the proton radius: $R = 6.68\cdot 10^{-16}$ m. In the explanation of the de Broglie waves accompanying moving particles in terms of the electromagnetic field, a condition placed on the particle sizes has been found. For protons we obtain $R = \frac{h}{2Mc} = 6.6\cdot 10^{-16}$ m, where h is Plank's constant.

Experimentally determined values of the proton radius are rather close to the value given by (6). In this case, as a rule, the mean-square charge radius R_q is determined, which can be greater than R. Thus, in experiments on electron scattering by protons [2] it has been found that $R_q = 7.5\cdot 10^{-16}$ m. According to [3], the cross section for the interaction of nucleons with one another that is established at energies over 10 GeV is 38 mb. In the classical limit, this cross section can be assumed to be close to the geometrical cross section of colliding particles that is, to $2\pi R^2$. Then we have $R < 7.8 \cdot 10^{-16}$ m .

Equality (7) allows us to relate the gravitational and the electromagnetic energy in a proton. For these energies, we can write

$$U = -\frac{K_1 \Gamma M^2}{R}, \quad W = -\frac{K_2 e^2}{4\pi\varepsilon_0 R},$$

where K_1 and K_2 are coefficients depending on the mass and the charge distribution, respectively; for a uniform distribution, we have $K_1 = K_2 = 0.6$. Putting $K_1 \approx K_2$, with the help of (7), we get

$$\frac{U}{W} \approx \frac{M}{M_e} = 1836.15,$$

that is, the gravitational-to-electrostatic energy ratio is approximately equal to the proton-to-electron mass ratio.

Let us now return to relation (5) to support the conclusion that the moment of momentum of the gravitational field inside a proton is equal to its spin. Assume that the proton charge is uniformly distributed over a volume of radius *R* and the magnetic moment P_m is concentrated at the center and directed along the *z*-axis. To calculate the momentum density of the electromagnetic field outside the proton for $r > R$, we use the following conventional expressions:

$$g_{\text{out}} = \varepsilon_0 \boldsymbol{E} \times \boldsymbol{B}, \quad \boldsymbol{E} = \frac{e}{4\pi\varepsilon_0 \ r^3}\boldsymbol{r}, \quad \boldsymbol{B} = \frac{\mu_0}{4\pi}\left(\frac{3(\boldsymbol{P_m}\cdot\boldsymbol{r})\boldsymbol{r}}{r^5} - \frac{\boldsymbol{P_m}}{r^3}\right),$$

where $\boldsymbol{E}$ is the electric field strength; $\boldsymbol{B}$ is the magnetic field induction, and μ_0 is the magnetic constant.

The vector $\boldsymbol{g}_{\text{out}}$ lies everywhere in the planes *z* = const, is normal to the *z*-axis, and rotates counterclockwise. For its modulus, we can write in spherical coordinates:

$$|\boldsymbol{g}_{\text{out}}| = \frac{e\mu_0 P_m \sin\theta}{16\pi^2 r^5}.$$

The moment of momentum of the electromagnetic field outside the proton is determined by the volume integral from $r = R$ to infinity:

$$L_{\text{out}} = \int_V |\boldsymbol{r} \times \boldsymbol{g}_{\text{out}}| \, dV = \frac{e\mu_0 P_m}{6\pi R}.$$

The electric field strength inside a uniformly charged proton, the modulus of the momentum density vector, and the moment of momentum of the field are given by

$$\boldsymbol{E} = \frac{e(r)}{4\pi\varepsilon_0 r^3}\boldsymbol{r}, \quad |\boldsymbol{g}_{\text{in}}| = \frac{e\mu_0 P_m \sin\theta}{16\pi^2 r^2 R^3}, \quad L_{\text{in}} = \frac{e\mu_0 P_m}{12\pi R},$$

where *e*(*r*) is the charge within the radius *r*.

We have obtained that the moment of momentum of the electromagnetic field inside a proton is half that outside the proton:

$$\boldsymbol{L}_{\text{in}} = \frac{1}{2}\boldsymbol{L}_{\text{out}} . \tag{8}$$

In addition, the law of conservation of moment of momentum relates the mechanical moment of momentum L_q of the charges moving inside a proton, which create the magnetic field of the proton, and the total moment of momentum of the electromagnetic field, L_f :

$$\boldsymbol{L}_f = \boldsymbol{L}_{\text{in}} + \boldsymbol{L}_{\text{out}} , \quad \boldsymbol{L}_q + \boldsymbol{L}_f = 0 . \tag{9}$$

If we assume that the magnetic moment of a proton is concentrated at its center and is directed along the *z*-axis, from (8) and (9) it follows that $\boldsymbol{L}_{\text{q}}$ and $\boldsymbol{P}_{\text{m}}$ are oppositely directed and the magnetic field of the proton is such as if it had been formed due to the motion of negative charges clockwise about the *z*-axis. In this case, the following equality should be fulfilled: $\boldsymbol{L}_q = -\frac{3}{2}\boldsymbol{L}_{\text{out}}$.

In another, opposite, case, the magnetic moment is not localized at the center of a proton, but is uniformly distributed over its volume. Among the well-known objects, the closest analog of a proton is a neutron star whose magnetic field, matter density, and degree of degeneracy are not much less than those of a proton. In a neutron star, the magnetic field should be frozen in the matter, being supported by the ordered state of the magnetic moments of the neutrons whose magnetic moment and the spin are counterdirected. Let us imagine that the magnetic moment of a proton, which we earlier considered to be located at the center, now occupies the whole of the volume so that the amplitude of the magnetic field both inside and outside the proton could be considered invariable. Then the magnetic field pattern outside the proton will be the same; however, the magnetic field inside the proton will change its sign, and, instead of (8) and (9), we shall get

$$\boldsymbol{L}_{\text{in}} = -\frac{1}{2}\boldsymbol{L}_{\text{out}} , \quad \boldsymbol{L}_f = \boldsymbol{L}_{\text{in}} + \boldsymbol{L}_{\text{out}} = \frac{1}{2} \boldsymbol{L}_{\text{out}} = -\boldsymbol{L}_{\text{in}} , \quad \boldsymbol{L}_q = \boldsymbol{L}_{\text{in}} = -\frac{1}{2}\boldsymbol{L}_{\text{out}} . \tag{10}$$

Irrespective of the character of the magnetic moment distribution over the volume of a proton, its total magnetic moment appears to be opposite in direction to the mechanical moment of momentum of the particles creating the magnetic moment. This is also valid for neutron stars, so that there is one more indication of similarity.

If we compare in pairs the electrical and gravitational quantities, $\boldsymbol{L}_{\text{q}}$ and $\boldsymbol{I}$ and $\boldsymbol{L}_{\text{in}}$ and $\boldsymbol{L}$, then from (10) just follows the equality of the moment of momentum of the rotating masses of a proton or its spin to the moment of momentum of the gravitational field inside the proton: $\boldsymbol{I} = \boldsymbol{L}$, and this has been used in (5) to estimate the proton radius.

In accordance with the foregoing, for the case where the magnetic field or mass sources are distributed uniformly, the internal moment of momentum of the electromagnetic or, respectively, gravitational field is equal, accurate to the sign, to half the moment of momentum of the field outside the object. Relation (10) remarkably has something in common with the well-known virial theorem according to which the work of irrelevant forces on the creation of an object is executed so that one half the expended energy goes into the kinetic energy of the object particles, while the other goes into the energy of the field and generally is carried away by radiation.

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