The Collapse of the Wave Function

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In the interpretation of Quantum Mechanics, there were four major mistakes done at different levels:

(1) A misinterpretation of Bell's theorem in which the original intent did not include non-locality, but as a test to see whether or not a particle has a certain property that can be measured.
(2) A misinterpretation of the disagreement between Einstein and Bohr. Einstein's objection to the collapse of the wave function implied a spooky action at a distance, while Bohr insisted on the instantaneous collapse of the wave function which he mistook to be a real wave.
(3) A misinterpretation that the wave function represents a real wave when in actuality it represents the possible states of a quantum system before a measurement.
(4) When Bell's theorem was violated by a quantum system, those violations were misinterpreted as evidence of an instantaneous collapse of the wave function and non-locality.

We will argue: there is no collapse of the wave function. Bell's theorem is not about non-locality. There is no spooky action at a distance. And Quantum Mechanics is about measuring quantities at the microscopic scales and in doing so, these quantities are altered. So what we get is partial knowledge. But in spite of that obstacle, we still get a theory of reality with considerable success.

1. The Heisenberg Uncertainty Principle (HUP) Revisited

A re-interpretation of the HUP is in order.

Here's a thought experiment. Suppose you were God and you could grab an electron and deposit at a certain position. As God, you've just violated the HUP – but that's okay, God can do that. We could depict this as in Fig. 1.

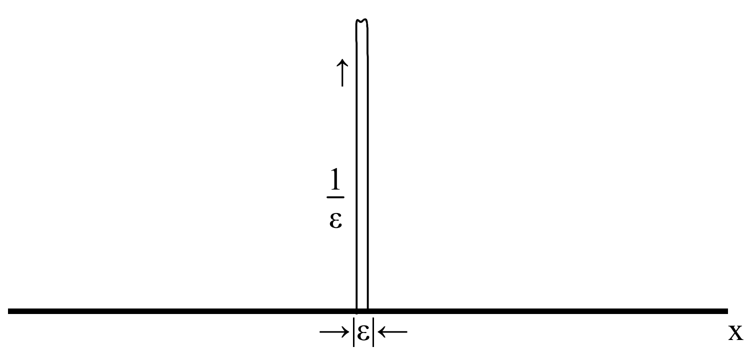


Fig. 1

But as soon as you release the electron, it would look like Fig. 2.

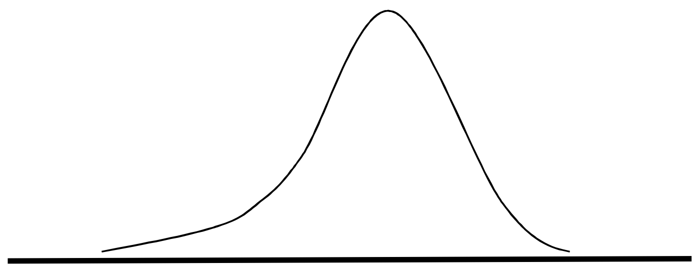


Fig 2.

After a time T, the position of the electron has spread out. The question is: what does that tell us? It looks like the particle is doing some kind of motion, some jiggling. It means that for microscopic particles, they are never at rest. In classical physics, you can have objects at rest. The walls in your room are at rest with you. But in QM, no object is at rest. And that's a fundamental difference with classical physics.

What else does the HUP tells us?

How do we measure the velocity of a car? I see the car because an enormous number of photons are hitting the car in all directions, and some of them reach my eyes. I can then note where it is at a given time, call that x_1, t_1. At a later times, I observe the car at x_2, t_2. I can get a whole set of these points, plot it, get the velocity, determine if it is in uniform motion or accelerate or decelerating, etc.

So it goes for an electron, to find out anything about it, the idea is to shoot a whole bunch of photons. We get lucky as one of those photons hits the electron, and with luck for a second time, it bounces in the right way to reach our eyes. But the photon is telling us, "Sir, that electron is right there," call that position X, even though X is really a smeared area as our electron was jiggling around when it was hit, "but guess what Sir, I've also thrown it off its position, and I haven't a clue in what direction it's going." This is the second thing the HUP is saying: if the position is known with zero uncertainty, then its momentum is unknown. And likewise, if the momentum would be known with absolute certainty, then its position would also be unknown. This is characterized as,

$$\Delta\sigma_x \, \Delta\sigma_p \geq \hbar/2 \qquad (1)$$

Where σ is the standard deviation. Note that in the case of the car (a classical system), we need not to worry that the photons will disturb the trajectory of the car. We will explore more of this meaning later.

The third thing that the HUP says is that if you make a measurement, the very act of making the measurement will alter the system. In our case, we had an electron jiggling about the position X, and now, it's jiggling somewhere else.

To resume, for a quantum system:

(1) A particle is never at rest.
(2) There is an uncertainty in measuring the position and momentum at a given time as indicated by equation 1.
(3) A measurement on the quantum system alters the system in some unpredictable way.

The net result is that we get partial knowledge of the quantum system, and we have to make do with that reality.

2. Incompatible Observables – Conjugate Pairs

Consider a number of plane waves moving towards a slit as in Fig. 3.

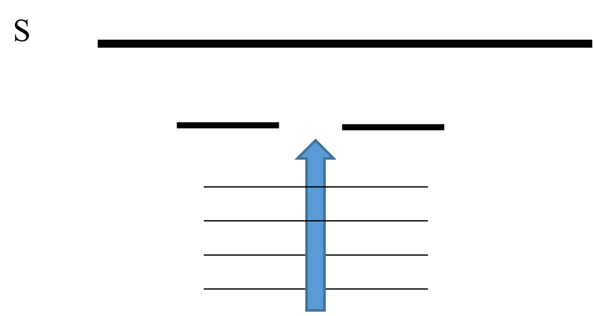


Fig. 3

As they go through the open slit, they start to bend and will hit the screen, leaving on it a series of white and dark fringes Fig. 4.

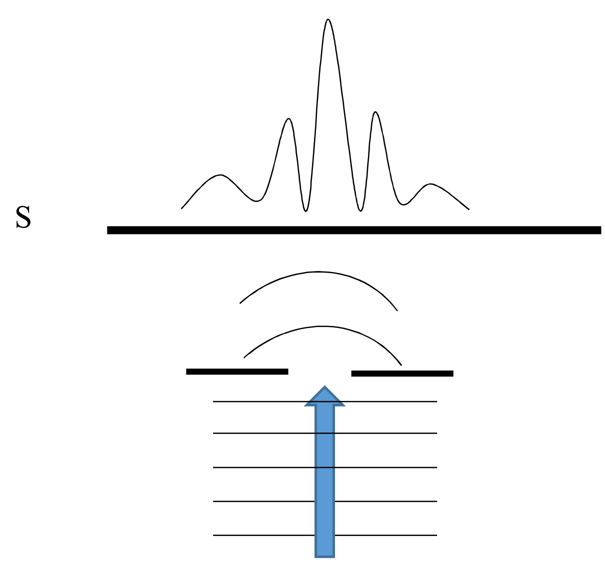


Fig 4.

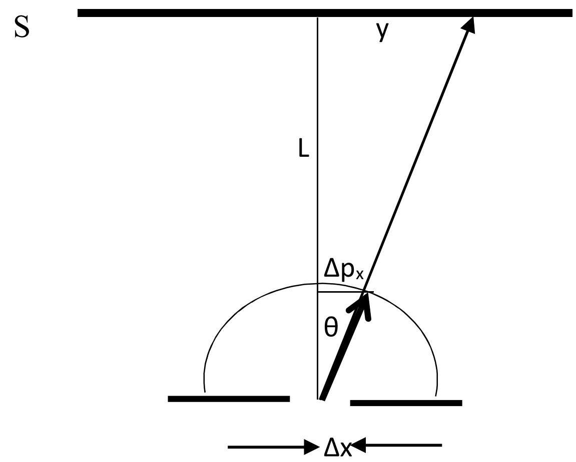


Fig. 5

Consider a point one wavelength away from the slit which will travel in a straight line and hit the screen at point y and note that the wavelength λ is << L, which doesn't show in Fig 5. .

$$\sin\theta \approx \theta = y/L \qquad (2)$$

Also, when that point was entering the slit, it had only momentum along L. But now it's moving at an angle θ and has developed a momentum along the x-direction, Δp_x.

$$\Delta p_x \approx p\,\theta \qquad (3)$$

Also, the point on the wave is one wavelength λ, and Δx/2 away from the line of motion,

$$\lambda \approx (\Delta x/2)\,\theta \qquad (4)$$

Now, historically, de Broglie had proposed that every particle is associated with a wavelength such that,

$$p = h/\lambda \qquad (5)$$

Where h is Planck's constant. Combining 3, 4 and 5, we get

$$\Delta x\,\Delta p_x \approx 2h > h \qquad (6)$$

This is also the HUP, expressed in terms of the uncertainty in the position Δx, and the uncertainty in the momentum Δp. If we want to reduce the uncertainty in the position by passing the waves through a smaller slit, then the bending of the waves will be more pronounced, and so the uncertainty in the momentum will be larger. And vice versa, to decrease the uncertainty in the momentum requires less bending, and to accomplish that the slit must be wider. We say that the position and the momentum are incompatible observables. These are often called conjugate pairs. This is known as Young's experiment.

3. Quantum States

A lot of confusion in Quantum Mechanics is the result from not being able to differentiate between the real world and the Hilbert Space. Vectors in real space – like velocities, accelerations, forces, etc. – are objects one can actually measure in the real world. On the other hand, quantum states are represented by vectors (more precisely by rays) in a Hilbert space, but these are NOT subjects of measurement. What we measure for a quantum system are probabilities, and those vectors in that Hilbert space are useful mathematical tools to calculate those probabilities.

Suppose we have a beam of electrons flowing from right to left:

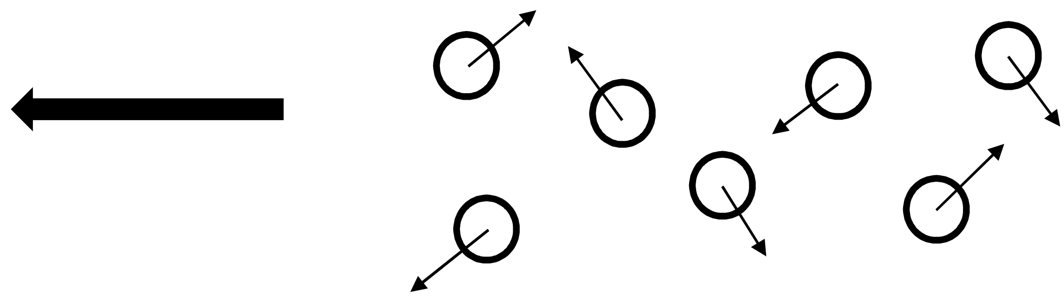


Fig. 6

Notice this is a thought experiment as we really don't know in what direction the spin of each individual electron points. We can safely say that these directions are at random. Now physicists are interested in measuring these spins. So what is needed is some kind of apparatus, and the good news is that there exists one – a magnetic field. Trouble is that these electrons, with their spin, are tiny magnets, and we know that magnets placed in a magnetic field will align (or anti-align) with the magnetic field. Suppose a magnetic field is placed along a certain direction, say the z-axis. Now let's look at one electron as it approaches the magnetic field.

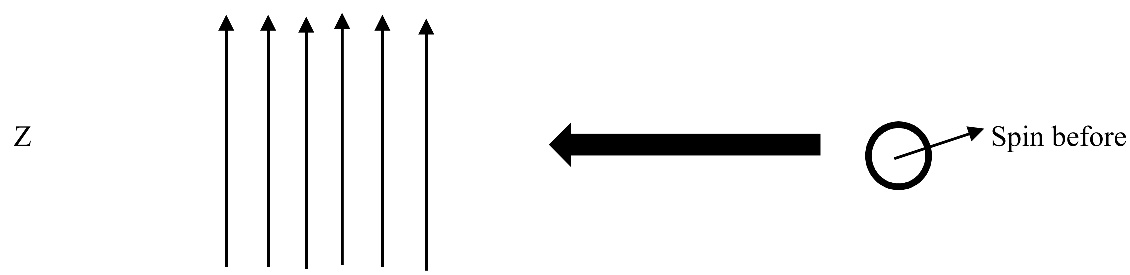


Fig. 7

When that electron penetrates the magnetic field, it will align its spin such that its z-component will yield the value of $+\hbar/2$ along the z-axis, a spin up, which can be represented as in Fig 8:

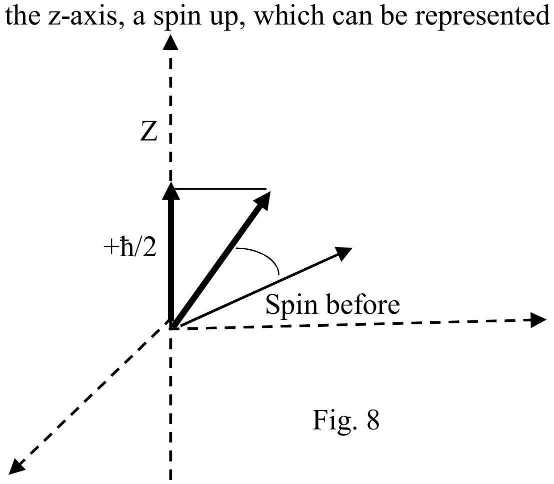


Fig. 8

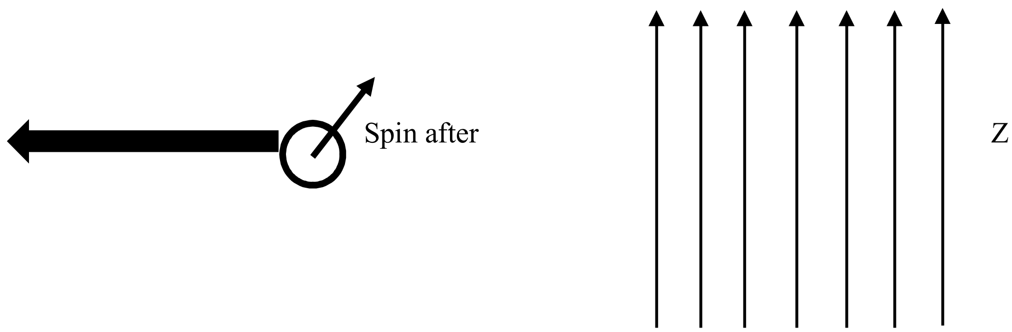
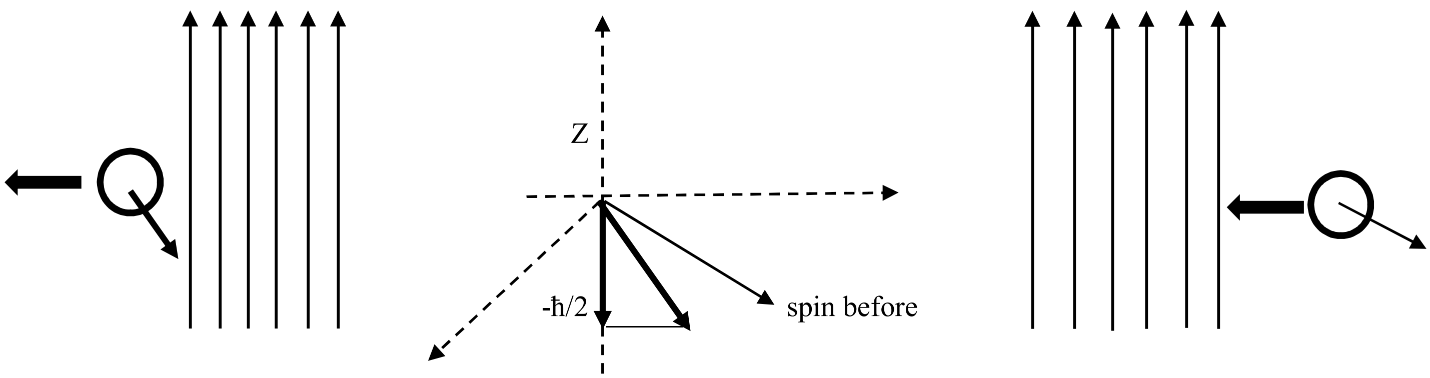


Fig. 9

Note that after passing the magnetic field, the electron's total spin has been altered. Here's another electron about to penetrate the magnetic field (fig. 10a):



(a) (b) (c)

Fig. 10

This time it will anti-align with the magnetic field, with a spin value of $-\hbar/2$, a spin down, (Fig. 10b). We see that again, the total spin has undergone a change in orientation after passing through the magnetic field (10c).

On the whole, 50% of the electrons will align with the magnetic field (spin $=+\hbar/2$, or up), and 50% will anti-align (spin = $-\hbar/2$, or down).

Comments

(1) Before the measurement, the spin of an electron can be in any direction. Passing the electron through the magnetic field forces the electron to change its spin orientation such that it either aligns or anti-aligns with its z-component to be $\pm\,\hbar/2$. This is what distinguishes quantum physics from classical physics: the act of measuring a quantity will disturb the system.

(2) The other components of the spin are indeterminate: if I were to pass these electrons into another magnetic field, say aligned with the x-axis, again it will be found that 50% of the electrons will align

with the magnetic field (spin = $+\hbar/2$), and 50% will anti-align (spin = $-\hbar/2$), this time along the x-axis. On the other hand the spin along the z-axis is no longer known for these particles.

(3) One way to mathematically represent this quantum system (read, the wave function) is this:

$$|\psi\rangle = (1/2)^{½} (|\uparrow\rangle - |\downarrow\rangle) \quad (7)$$

Now as it was already mentioned, this is called a superposition of two quantum states, the up and down states. Note that if we want to calculate the probability that the electron has a spin up, we take the product of the vector $|\uparrow\rangle$ with the wave function $|\psi\rangle$, and square that.

$$P = \| \langle\uparrow|\psi\rangle \|^2 \quad (8)$$

$$= 1/2\, [\langle\uparrow| (|\uparrow\rangle - |\downarrow\rangle)]^2$$

$$= 1/2\, [\langle\uparrow|\uparrow\rangle - \langle\uparrow|\downarrow\rangle]^2$$

Using the orthogonality condition, which is a fundamental property of a Hilbert space,

$$\langle\uparrow|\uparrow\rangle = 1 \; and \; \langle\uparrow|\downarrow\rangle = 0$$

We get,

$$P = 1/2, or\ 50\%, \quad (9)$$

Which is what is observed in the lab.

(4) Now here comes the real crunch. Writing $|\psi\rangle = (1/2)^{½} (|\uparrow\rangle - |\downarrow\rangle)$ is called a superposition but it's not meant to mean that the electron "lives" simultaneously in two states and can't make up its "mind" in which one it wants to live. Those states do not represent ordinary vectors of real objects - like velocities, acceleration, forces, was mentioned above. If it were the case, then since these two vectors are equal in magnitude and opposite in direction I would be able to claim,$|\uparrow\rangle = (-1)|\downarrow\rangle$. And the orthogonality condition would no longer hold, and P would not equal to 50% - actually it would turn out to be 100%!!! What needs to be reminded is that the two vectors, $|\uparrow\rangle$ and $|\downarrow\rangle$ represent possible states before the measurement takes place. And the beauty of it all is that they form a complete set of orthogonal unit vectors, in an abstract space called the Hilbert space, which provides a powerful method of calculating probabilities.

A word on semantics: note that I used the word "apparatus" when that word description is NOT needed. For instance at the LHC, one thinks of two beam interacting (colliding), and not as one beam interacting with an apparatus - the second beam. Similarly, the beam of electrons described above are interacting with a magnetic field (the "apparatus"). Hence, the whole concept of "wave function collapse" is totally unnecessary. The so-called measurement between a microscopic system and a macroscopic system is illusive as it never happens, it is always a microscopic system interacting with another microscopic system. And the wave function cannot collapse as it is not a function of a real wave. QM can do very well without this extra baggage.

4. A Second Look at the Two-Slit Experiment

Two states will evolve, and we can write this process as,

$$|A\rangle \rightarrow |A'\rangle, \qquad |B\rangle \rightarrow |B'\rangle$$

By the principle of linear superposition, these two states will also evolve as,

$$|A> + |B> \rightarrow |A'> + |B'>$$

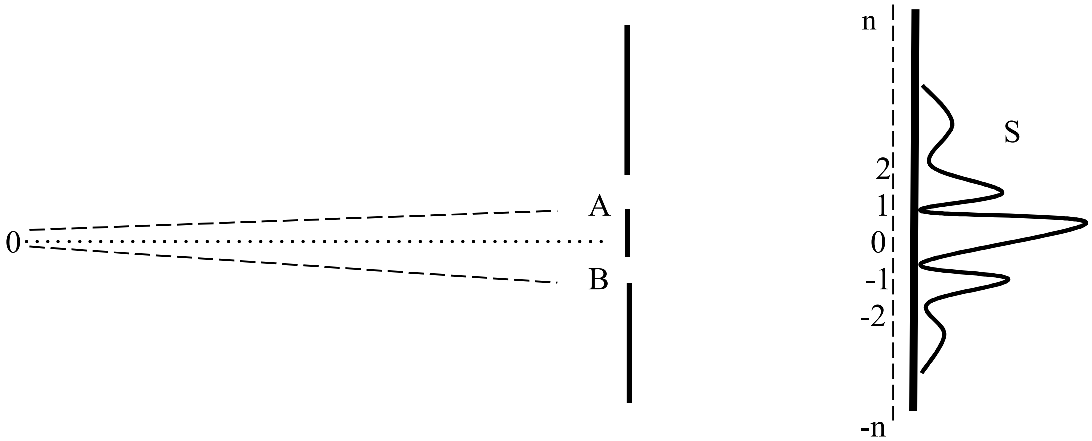


Fig. 11

We label the position from which the electrons pass through as such: where they leave is 0; the slits are labelled A and B; and the screen, S (see fig. 11). The states representing the electron passing in one slit are $|A>$ and going through the second slit, $|B>$. When the electrons leave position 0, from the symmetry of the setup, we can say that they arrive at position A and B with equal probability. So we write,

$$|0> \rightarrow |A> + |B>$$

When an electron has arrived at A or B, what happens after that when they hit the screen? Experiments show that they can land on any of the points on the screen, so we write,

$$|A> \rightarrow \sum_n \Psi_n |n>$$

$$|B> \rightarrow \sum_n \Phi_n |n>$$

Where $|n>$ forms a complete set of orthonormal vectors. The whole process can be described as,

$$|0> \rightarrow |A> + |B> \equiv \chi_n$$

The probability that an electron will arrive at the mth point on the screen is (equ. 8),

$$P_m = \| < m | \chi_n > \|^2$$

$$= < \chi_n \,|\, m > < m \,|\, \chi_n >$$

$$= \sum_{nn'} (\Psi_{n'}^* + \Phi_{n'}^*) < n' \,|\, m > < m \,|\, n > (\Psi_n + \Phi_n)$$

Using the orthogonality condition,

$$< n' \,|\, m > = \delta_{n'm} \;;\; < m \,|\, n > = \delta_{nm}$$

$$P_m = (\Psi_m^* + \Phi_m^*)(\Psi_m + \Phi_m)$$

$$= \Psi_m^* \Psi_m + \Phi_m^* \Phi_m + \Psi_m^* \Phi_m + \Phi_m^* \Psi_m \quad (11)$$

The first term $\Psi_m^* \Psi_m$ represents the probability if only the first slit was open. Similarly, the second term $\Phi_m^* \Phi_m$ represents the probability if only the second slit was open. Classically, we should get the sum of these two terms if both slits were open. But we do not observe that. The interesting aspect of this result from quantum physics is that we get two extra terms, $\Psi_m^* \Phi_m$ and$\Phi_m^* \Psi_m$, that correctly explains the interference pattern of the double-slit experiment. Another major difference between classical physics and quantum physics is that in the first, probabilities are added, while in the second, the amplitudes are added first and then we square the amplitudes to get the probabilities.

5. The Act of Measuring

Suppose we want to know through which slit the electron has passed. This can be done by inserting a detector at position A. Furthermore, we prepare the electron at position 0 with a down spin. When it passes through A, its spin is flipped to an up spin, and when it passes through position B, nothing happens to the electron and it remains with a spin down (see fig. 12).

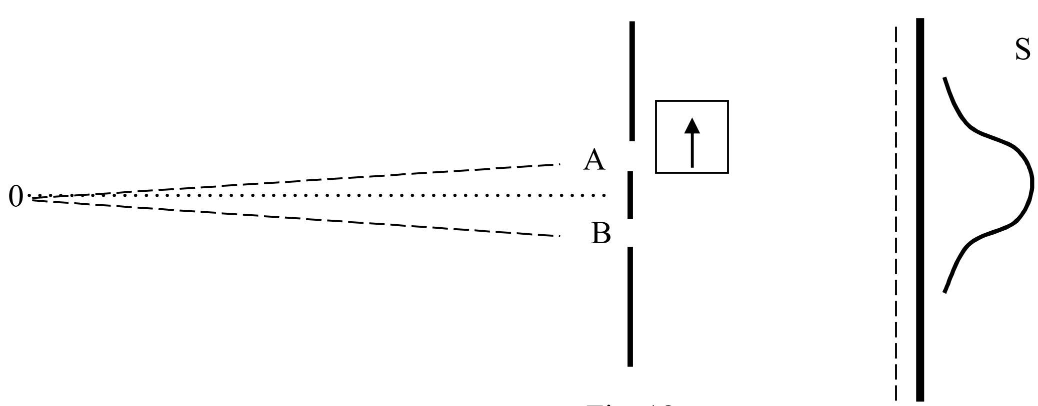


Fig. 12

We need two labels for the states: one for position, and the other for the spin. We describe the process as,

$$|\,0,d\ > \rightarrow\ \ |\,A,u\ >\ +\ |\,B,d\ >\ \rightarrow\ \sum_n \Psi_n\,|\,n,u\ >\ +\sum_n \Phi_n\,|\,n,d\ >$$

Due to the presence of the detector, the electrons are entangled through their spins: one is up, the other is down. Entanglement means that if we know a certain property of one particle, we also know that property of a second particle. In this case, we know that if the spin at A is up after passing through the detector, we also know that if it passed at B, it is spin down. Again to calculate the probability of finding the electron at the m^{th} position, we square the amplitudes, or multiply the amplitude with their complex conjugate (equ. 8).

$$P_m = \left(\Psi_m^*\ < m,u\,|\ +\ \Phi_m^* < m,d\,|\,\right)\left(\Psi_m\,|\,m,u > \ + \Phi_m|\,m,d >\right)$$

$$= \ \Psi_m^*\Psi_m < \ m,u\,|\,m,u\ > \ \ + \Phi_m^*\Phi m < \ m,d\,|\,m,d\ >$$

$$+\,\Psi_m^*\Phi_m\ < \ m,u\,|\,m,d\ >\ +\ \Phi_m^*\Psi_m\ < \ m,d\,|\,m,u\ >$$

Note that the up and down vector states, because the electrons have opposite spins, are now orthogonal to each other.

$$< \ m,u\,|\,m,d\ > = \ 0\ = < \ m,d\,|\,m,u\ >$$

$$< \ m,u\,|\,m,u\ > \ = \ 1\ = < \ m,d\,|\,m,d\ >$$

Therefore,

$$P_m = \ \Psi_m^*\,\Psi_m\ +\ \Phi_m^*\,\Phi_m \quad (12)$$

This result is completely different from equ. 11. We see now that the very act of detecting the spin of one electron, that is, making some sort of measurement, destroys the interference pattern. Again, this is another markedly difference between a classical system, in which we can always make a measurement without disturbing it, and a quantum system, in which a measurement entails disturbing the system and getting a different result.

6. Bell’s Theorem Revisited

In Bell's theorem[1], we make two assumptions in the proof. These are:

A. Logic is a valid way to reason.

B. Body either ad a property or doesn’t have property A.

This is important to understand. The parameter in question is not necessarily non-locality. It can be anything that a particle possesses and can be measured. Consider the set of all measurements, for which A, B and C are any three measurements, and are independent property. Examples: A is up or down, B is head or tail, C is red or green, etc. Secondly, the theorem is not about hidden parameters but whether it has a property or not. Making it about non-local hidden parameters is to doubly compound the error in misinterpreting Bell's theorem.

Derivation of Bell's inequality

Definition: if an object has property A, we denote that as A+; if not, we denote it by A-

(i) $N\ (A+, B-)\ =\ N\ (A+, B-, C+)\ +\ N\ (A+, B-, C-)$; this is true since an object must have the property C or does not have it.

(ii) So $N\ (A+, B-)\ \geq\ N\ (A+, B-, C-);\ since\ N\ (A+, B-, C+)$ cannot be smaller than zero.

(iii) $(B+, C-)\ =\ N\ (A+, B+, C-)\ +\ N\ (A-, B+, C-)$; similar reasoning to step i, an object must have the property A or does not have it.

(iv) $N\ (B+, C-)\ \geq\ N\ (A+, B+, C-)$; similar reasoning to step ii.

(v) So $(A+, B-)\ +\ N\ (B+, C-)\ \geq\ N\ (A+, B-, C-)\ +\ N\ (A+, B+, C-)$; adding inequalities ii and iv together.

(vi) But the RHS of v gives: $(A+, B-, C-)\ +\ N\ (A+, B+, C-)\ =\ N\ (A+, C-)$; similar reasoning to steps i and iii, an object must have the property B or does not have it.

(vii) Substituting vi into v, we get, $N\ (A+, B-)\ +\ N\ (B+, C-)\ \geq\ N\ (A+, C-)$; which completes the proof.

To reiterate: a body has a proper or it does not have it. For instance, looking at the earth at a distance, one can observe its spin around an axis and measure it. On the other hand, looking at the moon, we observe it has no spin. So either a body has spin (the earth) or it doesn't have spin (the moon). On the other hand, the electron has spin, but only one component can be measured. The other two components remain indeterminate once one component is measured as it was discussed through fig. 6-7-8-9-10. It is in that mind frame that we must interpret Bell's theorem. It goes without saying that a classical system will not violate Bell's theorem, while a quantum system will. In Alain Aspect experiment[2], Bell's inequality theorem was tested by measuring the polarization of photons along different axes. The inequality was violated as the system understudied was a quantum system. To attribute this violation to non-locality is a major blunder. The violation is strictly due to the HUP: along the three axes, the components of the spin are incompatible observables, and Bell's theorem applies only to a classical system. So applying Bell's theorem to a quantum system will inevitably result into a violation.

The EPR Revisited

It's time to go back to the 1927 Solvay Conference when the disagreement between Einstein and Bohr first surfaced and began the debate that has lasted ever since.

The first disagreement centered on the notion of a wave collapse. Einstein was on the right side as he correctly deduced that such collapse would mean the existence of a spooky action at a distance.

The EPR[3] that came subsequently proposed that there were hidden parameters to explain what Einstein thought was the unexplainable. On that, history has shown that Einstein was wrong.

Here's your typical argument that has come through the decades since this disagreement started.

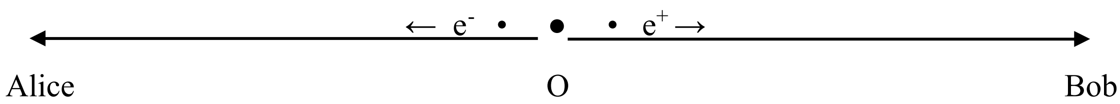


Fig. 13

A particle at O decays and sends two particles: an electron e^- towards Alice, and a positron e^+ towards Bob.

Each particle flies off in opposite direction with opposite momentum (conservation of momentum) and opposite spin (conservation of angular momentum).

Case A

Alice is going to measure the spin of her particles with a magnetic field along the z-axis, likewise for Bob. So both are performing an experiment depicted in Fig 6-7-8-9-10. In each case, consider one particle at a time. As the electron (positron) approaches the magnetic field, the orientation of the spin with the magnetic field is totally unknown to our two observers. Now the particle goes through the magnetic field. There are only two possibilities for each particle,

(1A) Alice measures a spin up and Bob measures a spin down.

(2A) Alice measures a spin down and Bob measures a spin up.

There are NO other alternatives.

There is no mystery here, there is no spooky action at a distance, there is no weirdness, there is no communication traveling faster than light. There are only two possibilities, and this is what will be observed, which is explained entirely by the conservation laws.

On the whole, Alice will measure 50% of all the electrons coming her way with spin up, and 50% with spin down. Bob will measure similar results for his positrons.

Case B

Alice is going to measure the spin of her particles with a magnetic field along the z-axis, but this time, Bob will measure his particles along a different axis, say the x-axis.

The situation doesn't change in regard to the particle approaching the magnetic field: the orientation of the particle's spin is still unknown to both Alice and Bob.

Consider one particle at a time.

(1B) Alice measures the first particle with a spin up along the z-axis.

(2B) Bob measures his first particle with a spin up along the x-axis.

Can Bob conclude that he also knows that his particle has a spin down along the z-axis, since Alice measured her particle with a spin up along that axis?

No, he doesn't know. His experiment is different than Alice's as his particle's orientation was forced along the x-axis, by what amount is unknown. And Alice's particle was forced to align along the z-axis by also an unknown quantity. The only conclusion that Bob can make is what he measured: a spin up along the x-axis. Secondly the components of the spin of his particle along the y and z axis remains unknown to him, just as Alice doesn't know the x and y components of her particle.

As in case A, Alice will measure 50% of all the electrons coming her way with spin up, and 50% with spin down, but keep in mind, she has only measured the spin along the z-axis. She has no knowledge of the other components of the spin of her particles – the x and y components.

Likewise Bob will also measure 50% of all the electrons coming his way with spin up, and 50% with spin down, but keep in mind, he has only measured the spin along the x-axis. He has no knowledge of the other components of the spin of his particles – the y and z components.

Again there is no mystery here, there is no spooky action at a distance, there is no weirdness, there is no communication traveling faster than light.

Conclusion

How can we explain the confusion that has reigned for more than nine decades?

There were mistakes done at different levels:

(5) A misinterpretation of Bell's theorem in which the original intent did not include non-locality, but as a test to see whether or not a particle has a certain property that can be measured.
(6) A misinterpretation of the disagreement between Einstein and Bohr. Einstein's objection to the collapse of the wave function implied a spooky action at a distance, and Bohr should have listened to that.
(7) A misinterpretation that the wave function represents a real wave when in actuality it represents the possible states of a quantum system before a measurement.
(8) When Bell's theorem was violated by a quantum system, those violations were misinterpreted as evidence instantaneous collapse of the wave function and non-locality.

Those who were carrying the torch for Einstein thought that Bell's theorem confirmed non-locality (which Bell's theorem doesn't really say) because that also confirmed that Einstein was right (a wave function collapse implied a spooky action at a distance, but the wave function isn't a real wave to begin with) leading to the idea that an instantaneous collapse (nothing can travel faster than the speed of light) makes the universe weird.

Here's the real deal: there is no instantaneous collapse of the wave function, and there is no spooky action at a distance.

References

[1] J S Bell, *Physics* **1** 195–200 (1964)

[2] A Aspect, P Grangier and G Roger, *Phys. Rev. Lett.* **49 (2)** 91–4 (1982)

[3] A Einstein, B Podolsky and N Rosen, *Physical Review* **47(10)** 777-780 (1935)