# Candidates for the Slichter Triplet II

Herbert Weidner [a]

**Abstract**: The systematic search for coincident signals with a software-defined receiver in the data sets of fifteen superconducting gravimeters yields six striking signals in the frequency range 35 µHz to 100 µHz that can not be assigned to a known geophysical causes. Some might belong to the long-sought Slichter triplet.

# Introduction

Very strong earthquakes can probably move the inner, solid core, which is surrounded by the liquid outer core. This should respond with a harmonic oscillation around its resting position, leading to measurable fluctuations of gravitation. With the known data of the Earth, the expected period of oscillation is likely to be about five hours. The existence of the ${}_1S_1$ natural mode was postulated 55 years ago by Slichter[1], but could never be confirmed by measurements despite intensive search[2]. The main cause for this failure is that the signal from the Earth's core has very low amplitude and can hardly be detected in the noise of the frequent earthquakes.

In recent decades, sophisticated methods have been developed in the radio technology to detect and demodulate very weak signals in a noisy environment. Without exception, the signal bandwidth must be severely restricted in order to reduce any interference as far as possible. With decreasing signal strength, the demands on the quality of the filters are increasing. In all high quality receivers, the frequency of the signal is reduced by mixing, because at lower frequencies, more precise filters can be constructed. The highest modulation frequency determines the minimum bandwidth. If the movement of the Earth's core is weak attenuated, the amplitude changes very slowly and the typical time constant is expected to be in excess of 1000 hours, leading to a maximum signal bandwidth of 0.5 µHz. A further reduction of bandwidth deletes the high frequency components the modulation, but also improves the signal-to-noise ratio (SNR). In the limit BW → 0, only the existence of a continuous oscillation can be determined. This is enough to prove the existence of the ${}_1S_1$ mode.

If the putative oscillation deep inside the Earth is triggered by changes on the surface, the search should focus on the period after a strong earthquake. The strongest event since the invention of the Superconducting gravimeter was on 2004-12-26.

# The Preparation of the data records

The 2004/2005 CORMIN records of all available SG stations were linked to long data chains. A barometric admittance of 3.8 nm/(s² hPa) decreases the influence of atmosphere pressure variation on the gravity data. Previous studies have shown that this "magic number" minimizes the noise level around 70 µHz. Then, the data reduction is carried out through a series of filters. Their order greatly affects the SNR of the weak signals.

# Programming a superheterodyne receiver

The reproducible search for signals with poor SNR requires narrow-band filters to reduce the noise. They can be realized by electronic components or by software and differ with respect to bandwidth and center frequency. Other features such as group delay and attenuation have no meaning here. Experience has shown that optimum results are achieved without excessive effort by a ratio $10 < \frac{Center\ frequency}{Bandwidth} < 100$ . If the maximum bandwidth is 0.16 µHz, the filter frequency shall be lower than 16 µHz. The known and proven superheterodyne method is the standard method to shift the actual signal frequency to any intermediate frequency (IF) before filtering. In the software-defined receiver described below, the chosen IF of 10 µHz allows the requested bandwidth.

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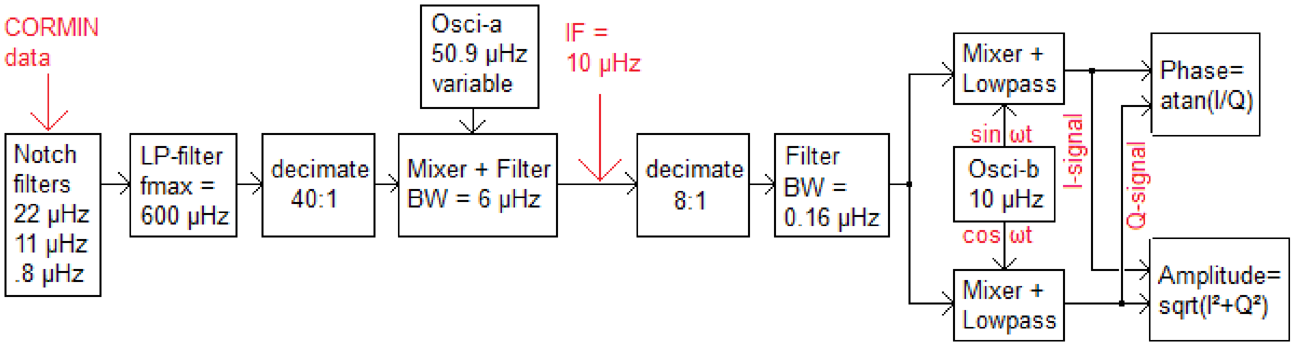
a) 16. August 2016, email: herbertweidner99@gmail.com

The figure shows the block diagram of the receiver for 50.9 μHz. If another frequency is to be received, it is sufficient to adapt the frequency of the oscillator-a.



First, a triple narrow-band notch filter reduces the amplitude of the strongest spectral lines (calculated with quadruple precision) and a windowed-sinc filter (WSF) limits the bandwidth of the recorded data to the range 33 μHz – 600 μHz. After increasing the sampling time to 10 minutes, a second WSF narrows the bandwidth to the range 35 μHz – 160 μHz, creating the intermediate records of the SG stations H1, H2, M1, M2, MB, MC, ST, W1, W2, CB, KA, MA, S1, S2, TC. Each of these fifteen files contains the gravity data of a twelve month long period after the 2004-12-26 earthquake. These are the basic data for all further investigations.

Previous measurements[3] have shown that it is sufficient to limit the search to the first 60 days after the earthquake. Thereafter, most possible oscillations have become so weak that the further search can be terminated. Therefore, the file length of each recording is limited to 2000 samples and the sampling period is extended to 40 minutes. This 55 days period following the earthquake is systematically examined for hidden signals.

In order to to avoid problems with the image frequency, a frequency-transforming band filter reduces the signal frequency instead of a conventional mixer. This serves two purposes: It filters a 6 μHz wide frequency band around 50.9 μHz and reduces the average frequency to 10 μHz. The subsequent extension of the sampling period to 320 minutes simplifies the construction of the final WSF with bandwidth 0.16 μHz or less.

The demodulator is a combination of two direct-conversion receivers, because both amplitude and phase can be calculated from the I-Q signals. Similar to a frequency-locked loop, a control circuit varies the frequency of oscillator-a to ensure that the phase at the output of the demodulator is time-independent. Then, oscillator-a (near 50.9 μHz) has exactly the same frequency as a signal that is hidden in the noisy CORMIN recordings, but there may be a constant phase difference.

## How to detect signals in the noise?

Experience has shown that it makes little sense to inspect the data of a few or even only a single SG station for extremely weak signals of unknown frequency. Between 35 μHz and 100 μHz, the spectrum of every station shows several hundred conspicuous spots and each of them could be part of the Slichter triplet. Every individual observation can arise at random and is of limited value. Only the agreement in multiple independent data sets can convince.

As it means endless work to detect coincidences between different stations manually, the search for clues was automated. First, it is necessary to specify, which signals should be accepted and which should be rejected. A harmonic oscillation has exactly three testable criteria: amplitude, frequency and phase. In a noisy environment, it takes some attempts to define at least one clear criterion.

The assumption that the ${}_1S_1$ resonance frequency is invariable, may lead to the unrealistic expectation $\Delta f = 0$, because the inevitable noise will always simulate minor frequency changes. Pairwise comparison of the results from different stations can only succeed if small frequency deviations are accepted. A broader bandwidth than 0.2 μHz decreases the SNR so much that the desired spectral line can not be clearly identified in all records. A too narrow filter bandwidth increases the rise time to such an extent that the detection of short signals gets problematic. After several attempts, a bandwidth of 0.16 µHz was selected – small enough to exclude coincidences, large enough to at least discover some matches.

The demand for constant amplitude is meaningless for several reasons: Firstly, the sidebands are removed to reduce the noise. Without sidebands, the amplitude is always fixed. Secondly, a narrow filter causes a slow increase in amplitude (transient response), which is determined by the selected bandwidth. Finally, the attenuation factor of the ${}_1S_1$ resonance is unknown.

Under no circumstances, the verification of phase may be omitted, that is the central point of the proof. Some (unfortunately few) stations measure the gravity with two superconducting spheres in one instrument. Both detectors should register in-phase signals from the core. Real signals can not cause large phase deviations or even phase opposition. Below, this subject is treated in more detail. By comparing the signals of neighboring stations, small deviations must be tolerated. Large deviations of the phases or even anti-phase are a reason for rejection. Phase relations can not be checked with the standard Fourier analysis.

# Dragnet for conspicuous frequencies

A program was written that identifies all frequencies in the range 35 µHz to 100 µHz, which are subject to the above criteria. In principle, this is a software-defined receiver with extremely low bandwidth, analyzing gradually all frequencies in this range. The data of the fifteen superconducting gravimeters are sequentially connected to the input of the receiver. The search frequency is incremented in tiny steps of 0.03 µHz and each record is filtered with the bandwidth 0.2 µHz. The analysis of a total of 65 million samples lasts about three hours.

For each of the 2000 data points in every record, the instantaneous frequency is calculated. If the resulting time course meets the following requirements, the search frequency is classified as significant.

- Within the bandwidth of 0.2 µHz, there must be no systematic frequency drift.
- The average frequency must be independent of the center frequency and bandwidth of the filter.
- The average frequency must be the nearly the same at 2/3 of all investigated stations.
- As the nine European stations are in close proximity, even 80% must match. If a SG instrument contains two superconducting spheres, it is assumed that the results are independent of each other.

Finally, six distinctive frequencies remained who met all the requirements. They cannot be assigned to the tides[4] and deserve further investigation:

| Frequency µHz | 39.414 | 50.895 | 53.134 | 63.237 | 85.505 | 91.589 |
|---|---|---|---|---|---|---|
| Uncertainty | ± 0.010 | ± 0.026 | ± 0.016 | ± 0.032 | ± 0.024 | ± 0.028 |

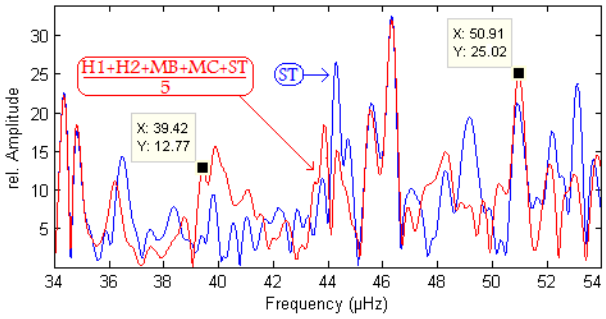
# Detailed analysis near 39.4 µHz

The gravimeters of some stations contain *two* superconducting spheres and in an undisturbed environment, both should measure identical amplitudes *and* phases. With the data these stations, each three calculations have been performed: once for each station separately and then with the vector sum of the data. The comparison confirms that the coincidence of amplitude, peak time and phase is almost always perfect – confirming that a real signal is measured and no noise.

| SG Station | Frequency (µHz) | Rel. Amplitude | Peak time past the earthquake (Days) | Phase φ (Radians) |
|---|---|---|---|---|
| M1 | 39,4065 | 0,72 | 23,73 | -0,3976 |
| M2 | 39,4612 | 0,62 | 16,62 | -0,1007 |
| (M1+M2)/2 | 39,4245 | 0,65 | 20,79 | -0,3067 |
| H2 | 39,4069 | 1,68 | 28,42 | -0,1450 |
| H1 | 39,4556 | 0,97 | 23,81 | 0,3219 |
| (H1+H2)/2 | 39,4132 | 1,31 | 26,7 | -0,1158 |
| ST | 39,4053 | 0,66 | 16,92 | 0,2957 |
| MB | 39,3642 | 0,82 | 22,75 | 0,1014 |
| (ST+MB)/2 | 39,3822 | 0,73 | 19,58 | 0,2188 |
| MC | 39,3761 | 1,06 | 29,29 | -0,0008 |
| W1 | 39,2401 very noisy | 4,82 | 28,11 | 2,4427 |
| W2 | 39,4543 noisy | 2,4 | 21,03 | -2,9973 |
| Average EU (*without* W1 and W2) | 39,4096 ± 0,0100 | | | |
| CB | 39,3082 | 0,84 | 36,00 | -3,0459 |
| KA | 39,4950 | 1,51 | 28,05 | -1,0946 |
| MA | 39,3905 | 0,34 | 40,95 | -0,7391 |
| TC | 39,4234 | 3,94 | 13,68 | -1,5111 |
| Average non-EU (*without* S1 and S2) | 39,4043 ± 0,0387 | | | |

All amplitudes reach their maximum about 25 days after the earthquake. Strictly speaking, this delay is a systematic measurement error which is caused by the slow transient response of the band-pass filter (BW = 0.16 µHz). As both values are linked by the formula $BW \cdot t_r \approx 0.34$ , the expected rise time is $t_r$ = 24.6 days. This good match between the two values allows two conclusions: The natural resonance near 39.4 µHz was triggered by the 2004 earthquake and the decay time of this oscillation is significantly longer than 25 days (Q>280).

The shown spectrum between 34 µHz and 54 µHz illustrates how necessary the phase information is to detect signals in the noise. The blue curve is the spectrum of the data which were recorded by the station ST. The tiny peak at 39.4 µHz disappears (as with some other spectra) in the noise and is easily overseen. The addition of the spectra of several stations can not change appreciably the SNR,



because the total amplitude can only rise and never fall. It is hard to find a very weak spectral line with this method.

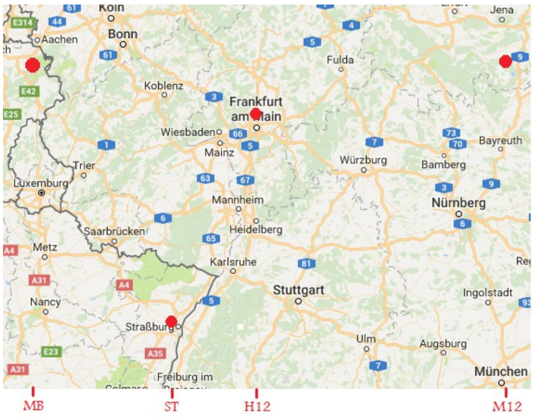
For comparison, the raw data of the stations H1, H2, MB, MC and ST were added and normalized. Then, the spectrum of this sum was calculated, drawn in red color. At several frequencies, the red curve is below the blue one, as expected. The reason is simple: Sometimes, the instantaneous amplitudes of the noise vectors partially compensate each other, sometimes they add up. The strong spectral lines of the tides around 34 µHz and 46 µHz do not depend on the geographical position of the recording station, and the two curves coincide. For all other frequencies, both results differ. A large difference is found near 39.4 µHz, where the mean amplitude of all five stations exceeds the amplitude measured is Strasbourg by a factor of about three. If one were to analyze only the data of ST, this spectral line would likely be overlooked.

## Detailed analysis near 50.9 µHz

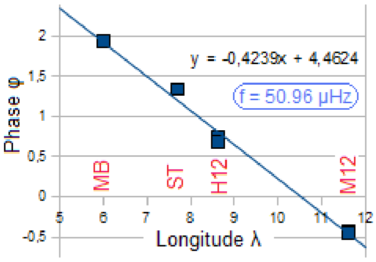
The range around 50.9 µHz was examined with the above-described superheterodyne receiver. Thanks to the very low bandwidth BW = 0.10 µHz, the sought spectral line could be easily identified even in poor SNR. The results are summarized in the following table.

| SG Station | Frequency (µHz) | Rel. Amplitude | Peak time past the earthquake (Days) | Phase φ (Radians) |
|---|---|---|---|---|
| M1 | 50,8800 | 1,11 | 21,15 | -0,4252 |
| M2 | 50,8864 | 1,27 | 25,07 | -0,4456 |
| (M1+M2)/2 | 50,8831 | 1,18 | 23,21 | -0,4214 |
| H2 | 50,9644 | 2,02 | 27,33 | 0,6797 |
| H1 | 50,9702 | 1,54 | 25,11 | 0,7472 |
| (H1+H2)/2 | 50,9671 | 1,77 | 26,34 | 0,7126 |
| ST | 50,9067 | 0,81 | 26,22 | 1,3388 |
| MB | 51,0082 | 1,44 | 26,02 | 1,9287 |
| MC | 50,9932 | 1,38 | 37,22 | 2,4086 |
| W1 | 51,0288 | 2,23 | 24,63 | 1,6960 |
| W2 | 51,0161 | 1,47 | 23,32 | 2,4639 |
| (W1+W2)/2 | 51,0246 | 1,65 | 23,98 | 2,0129 |
| Average EU | 50,9551 ± 0,0165 |  | 24,73 ± 0,49 |  |
| CB | 50,9849 | 0,6328 | 41,10 | -0,8997 |
| KA | 51,0100 | 2,2396 | 86,48 very noisy | -2,3207 |
| MA | 50,8593 | 1,5415 | 63,54 | 1,0906 |
| S1 | 51,0023 | 2,6415 | 42,07 | -3,0120 |
| S2 | 51,0267 | 1,6796 | 20,84 | -2,3365 |
| (S1+S2)/2 | 51,01 | 2,0745 | 33,21 | -2,7374 |
| TC | 50,9586 | 2,5993 | 32,04 | 1,1512 |
| Average non-EU | 50,9788 ± 0,0216 |  |  |  |

The table provides another potentially very interesting feature of this oscillation. Each signal has at least three characteristic average values: frequency, phase and amplitude. In communication technology, at least one of these values is regularly changed by modulation, but in geophysics, unmodulated signals can be also very interesting.



The measurement of 51 μHz spectral line with the phase-sensitive demodulator of the receiver delivers a curious result. Five of the six SG spheres in Central Europe (see the map) are almost exactly on the same latitude, only Strasbourg is significantly further south.



The measured phases (see the turquoise colored fields in the table above) are almost exactly proportional to the longitude λ (and independent of the latitude) and seem to be not random. The mathematical relationship is $\varphi=4.4624-0.4239\,\lambda$ .

Differentiating this equation with respect to time, one obtains

$$\frac{d\varphi}{dt}=-0.4239\frac{d\lambda}{dt}=-0.4239\frac{360^0}{24\,hours}=\frac{-6.4^0}{1\,hour}\quad.$$

# Detailed analysis near 53.1 μHz

| SG Station | Frequency (μHz) | Rel. Amplitude | Peak time past the earthquake (Days) | Phase φ (Radians) |
|---|---|---|---|---|
| M1 | 53,1432 | 0,64 | 27,72 | 0,3593 |
| M2 | 53,1422 | 0,87 | 27,84 | -0,0187 |
| (M1+M2)/2 | 53,1427 | 0,74 | 27,77 | 0,1433 |
| H2 | 53,0341 | 0,86 | 18,94 | -2,2312 |
| H1 | 53,0830 | 0,48 | 9,7 | -2,0406 |
| (H1+H2)/2 | 52,8949 | 0,74 | 18,56 | 2,9565 |
| ST | 53,1248 | 0,99 | 27,23 | -0,7097 |
| MB | 53,0922 | 0,91 | 30,59 | -1,6750 |
| MC | 53,1858 | 0,21 | 22,64 | 1,0615 |
| W1 | 53,2421 | 2,93 | 20,76 | -0,6492 |
| W2 | 53,1325 | 1,59 | 25,97 | -1,5524 |
| (W1+W2)/2 | 53,21 | 2,11 | 20,58 | -0,9340 |
| Average EU | 53,1186 ± 0,0259 | | | |
| CB | 53,0624 | 1,18 | 24,07 | 0,1358 |
| KA | 53,0906 | 0,68 | 22,54 | -2,8183 |
| MA | 53,2011 | 1,12 | 41,25 | -1,4213 |
| S1 | 53,1497 | 2,41 | 18,91 | 3,1923 |
| S2 | 53,1539 | 2,35 | 22,85 | 3,0645 |
| (S1+S2)/2 | 53,1529 | 2,37 | 20,85 | 3,1388 |

| | | | | |
|---|---|---|---|---|
| TC | 53,0761 | 4,19 | 25,71 | -0,8137 |
| Average non-EU | 53,1267 ± 0,0192 | | | |

## Detailed analysis near 63.2 µHz

| SG Station | Frequency (µHz) | Rel. Amplitude | Peak time past the earthquake (Days) | Phase φ (Radians) |
|---|---|---|---|---|
| M1 | 63,372 | 1,15 | 25,17 | -2,7556 |
| M2 | 63,1717 | 1,16 | 33,56 | 1,4465 |
| (M1+M2)/2 | 63,1845 | 1,02 | 29,59 | 1,3787 |
| H2 | 63,2721 | 1,31 | 27,5 | -1,5731 |
| H1 | 63,2676 | 0,96 | 31,37 | -1,6603 |
| (H1+H2)/2 | 63,2701 | 1,13 | 29,14 | -1,6094 |
| ST | 63,2797 | 0,83 | 29,38 | 2,9798 |
| MB | 63,1156 | 1,18 | 36,93 | -2,1624 |
| MC | 63,1835 | 1,01 | 16,96 | 0,8279 |
| W1 | 63,27 | 2,04 | 17,27 | -2,4563 |
| W2 | 63,18 | 1,27 very noisy | 4,44 | -2,3113 |
| (W1+W2)/2 | 63,27 | 1,59 , noisy | 13,79 | -2,3549 |
| Average EU | 63,2359 ± 0,0202 | | | |
| CB | 63,2175 | 1 | 62,64 | 0,0641 |
| KA | 63,1961 | 1,83 | 42,89 | 0,7302 |
| MA | 63,1663 | 2,39 | 58,56 | -0,7219 |
| S1 | 63,2107 | 1,18 noisy | 15,44 | -1,8573 |
| S2 | 63,1661 | 2,41 noisy | 2,2 | -1,7218 |
| (S1+S2)/2 | 63,1611 | 1,74 , noisy | 2,2 | -1,8452 |
| TC | 63,2089 | 1,52 noisy | 2,2 | 2,1395 |
| Average non-EU | 63,1895 ± 0,0092 | | | |

## Detailed analysis near 85.5 µHz

| SG Station | Frequency (µHz) | Rel. Amplitude | Peak time past the earthquake (Days) | Phase φ (Radians) |
|---|---|---|---|---|
| M1 | 85,6991 | 0,90 | 23,21 | 2,6177 |
| M2 | 85,3784 | 0,56 | 32,51 | -0,0549 |
| (M1+M2)/2 | 85,3683 | 0,53 | 33,26 | -0,0928 |
| H2 | 85,4909 | 1,09 | 20,36 | -1,8420 |
| H1 | 85,4879 | 1,00 | 15,58 | -1,7967 |

| | | | | |
|---|---|---|---|---|
| (H1+H2)/2 | 85,4903 | 1,04 | 18,20 | -1,8146 |
| ST | 85,4768 | 0,75 | 26,15 | -2,3561 |
| MB | 85,5320 | 1,52 | 32,23 | -0,7574 |
| MC | 85,5180 | 0,79 | 20,50 | 2,9037 |
| W1 | 85,5050 | 2,30 | 20,00 | 1,3743 |
| W2 | 85,4870 | 1,84 | 19,96 | 0,8312 |
| (W1+W2)/2 | 85,4966 | 2,03 | 20,03 | 1,1282 |
| Average EU | 85,4942 ± 0,0236 | | | |
| CB | 85,5027 | 0,78 | 26,15 | 2,9406 |
| KA | 85,4812 | 0,97 | 27,76 | 0,4991 |
| MA | 85,7576 | 1,18 | 30,46 | -2,2992 |
| S1 | 85,5585 | 1,72 | 16,09 | -0,3408 |
| S2 | 85,5546 | 1,63 | 19,10 | -0,2417 |
| (S1+S2)/2 | 85,5559 | 1,67 | 17,56 | -0,2971 |
| TC | 85,3359 | 2,50 | 19,34 | 2,0405 |
| Average non-EU | 85,5352 ± 0,0474 | | | |

# Detailed analysis near 91.6 µHz

| SG Station | Frequency (µHz) | Rel. Amplitude | Peak time past the earthquake (Days) | Phase φ (Radians) |
|---|---|---|---|---|
| M1 | 91,7398 | 1,44 | 26,55 | 0,8159 |
| M2 | 91,7222 | 1,38 | 25,67 | 0,5544 |
| (M1+M2)/2 | 91,7311 | 1,4 | 26,08 | 0,6458 |
| H2 | 91,4870 | 1,13 | 25,60 | 1,6264 |
| H1 | 91,4980 | 1,14 | 23,27 | 1,5236 |
| (H1+H2)/2 | 91,4905 | 1,13 | 24,17 | 1,5508 |
| ST | 91,6010 | 0,44 | 19,66 | 0,2818 |
| MB | 91,7901 | 0,95 | 32,09 | 0,3464 |
| MC | 91,6808 | 1,33 | 26,51 | -0,5432 |
| W1 | 91,5964 | 1,03 | 18,73 | -2,7847 |
| W2 | 91,5481 | 1,90 | 19,43 | -3,1141 |
| (W1+W2)/2 | 91,5607 | 1,45 | 18,25 | -3,0332 |
| Average EU | 91,6205 ± 0,0312 | | | |
| CB | 91,4985 | 1,38 | 32,08 | -1,1147 |
| KA | 91,4776 | 0,61 | 17,4 | -2,4589 |
| MA | 91,6228 | 1,40 | 26,25 | 0,9424 |
| S1 | 91,7705 | 1,98 | 11,66 | -2,4442 |

| | | | | |
|---|---|---|---|---|
| S2 | 91,7279 | 1,39 | 7,54 | -2,8062 |
| (S1+S2)/2 | 91,7598 | 1,68 | 9,87 | -2,5795 |
| TC | 91,5380 | 2,26 | 31,89 | -1,9368 |
| Average non-EU | 91,6279 ± 0,0476 | | | |

# Summary

With a few exceptions, the globally distributed Gravimeters measure six enigmatic signals with good matching frequencies and amplitudes. As the signals hardly exceed the noise level, three of them could belong to the Slichter triplet. Further studies require sound predictions about the expected phases depending on the geographical location of the station.

# Acknowledgments

Thanks to the operators of the GGP stations for the excellent gravity data. The underlying data of this examination were measured by a net of about twenty SG distributed over all continents, the data are collected in the Global Geodynamic Project[5].

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[4] T. Hartmann, H. Wenzel, The HW95 tidal potential catalogue, 1995
[5] The "Global Geodynamics Project", http://www.eas.slu.edu/GGP/ggphome.html