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FUNDAMENTAL PROPERTIES OF MULTIPLICATIVE COUPLED FIBONACCI SEQUENCES OF FOURTH ORDER UNDER TWO SPECIFIC SCHEMES

A. D. Godase*

Department of Mathematics, V. P. College, Vaijapur, Aurangabad – 423701, Maharashtra, India

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ABSTRACT

Coupled Fibonacci sequences involve two sequences of integers in which the elements of one sequence are part of the generalization of the other and vice versa. K. T. Atanassov was first introduced coupled Fibonacci sequences of second order in additive form. In this paper, I present some properties of multiplicative coupled Fibonacci sequences of fourth order under two specific schemes.

Mathematics Subject Classification: *11B39, 11B37.*

Keywords: *Fibonacci sequence, multiplicative Fibonacci sequence.*

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1. INTRODUCTION

In the recent years much work has been done in this field but its multiplicative form is less known. The coupled Fibonacci sequence was first introduced by K. T. Atanassov [4] and also discussed many curious properties and new direction of generalization of Fibonacci sequence in [2], [3] and [5]. He defined and studied about four different ways to generate coupled sequences and called them coupled Fibonacci sequences (or 2-F sequences). The multiplicative Fibonacci Sequences studied by P. Glaister [6] and generalized by P. Hope [7]. K. T. Atanassov [4] notifies four different schemes in multiplicative form for coupled Fibonacci sequences.

Let $\{X_i\}_{i=1}^{i=\infty}$ & $\{Y_i\}_{i=1}^{i=\infty}$ be two infinite sequences and four arbitrary real numbers a, b, c and d be given. The four different multiplicative schemes for 2- Fibonacci sequences are as follows:

First scheme
$$X_{n+2} = X_{n+1}X_n, \quad n \ge 0 \qquad (1.1)$$
$$Y_{n+2} = Y_{n+1}Y_n, \quad n \ge 0$$

Second scheme
$$X_{n+2} = Y_{n+1}X_n, \quad n \ge 0 \qquad (1.2)$$
$$Y_{n+2} = X_{n+1}Y_n, \quad n \ge 0$$

Third scheme
$$X_{n+2} = X_{n+1}Y_n, \quad n \ge 0 \qquad (1.3)$$
$$Y_{n+2} = Y_{n+1}X_n, \quad n \ge 0$$

Fourth scheme
$$X_{n+2} = Y_{n+1}Y_n, \quad n \ge 0 \qquad (1.4)$$
$$Y_{n+2} = X_{n+1}X_n, \quad n \ge 0$$

In this paper, I present some results on multiplicative coupled Fibonacci sequences of fourth order under two specific schemes.

Corresponding author: *A. D. Godase**
Department of Mathematics, V. P. College, Vaijapur, Aurangabad – 423701, Maharashtra, India

2. MULTIPLICATIVE COUPLED FIBONACCI SEQUENCES OF FOURTH ORDER

Let $\{X_i\}_{i=1}^{i=\infty}$ & $\{Y_i\}_{i=1}^{i=\infty}$ be two infinite sequences and eight arbitrary real numbers a, b, c, d, e, f, g, h be given. Multiplicative coupled Fibonacci sequences of fourth order are generated by the following 16 different ways:

The different schemes are as follows:

$$T_1 : \begin{cases} X_{n+4} = X_{n+3}X_{n+2}X_{n+1}X_n \\ Y_{n+4} = Y_{n+3}Y_{n+2}Y_{n+1}Y_n \end{cases},\ T_2 : \begin{cases} X_{n+4} = X_{n+3}X_{n+2}X_{n+1}Y_n \\ Y_{n+4} = Y_{n+3}Y_{n+2}Y_{n+1}X_n \end{cases},\ T_3 : \begin{cases} X_{n+4} = X_{n+3}X_{n+2}Y_{n+1}X_n \\ Y_{n+4} = Y_{n+3}Y_{n+2}X_{n+1}Y_n \end{cases},\ T_4 : \begin{cases} X_{n+4} = X_{n+3}X_{n+2}Y_{n+1}Y_n \\ Y_{n+4} = Y_{n+3}Y_{n+2}X_{n+1}X_n \end{cases},$$

$$T_5 : \begin{cases} X_{n+4} = X_{n+3}Y_{n+2}X_{n+1}X_n \\ Y_{n+4} = Y_{n+3}X_{n+2}Y_{n+1}Y_n \end{cases},\ ,T_6 : \begin{cases} X_{n+4} = X_{n+3}Y_{n+2}X_{n+1}Y_n \\ Y_{n+4} = Y_{n+3}X_{n+2}Y_{n+1}X_n \end{cases},\ T_7 : \begin{cases} X_{n+4} = Y_{n+3}X_{n+2}X_{n+1}X_n \\ Y_{n+4} = X_{n+3}Y_{n+2}Y_{n+1}Y_n \end{cases},\ T_8 : \begin{cases} X_{n+4} = Y_{n+3}X_{n+2}X_{n+1}Y_n \\ Y_{n+4} = X_{n+3}Y_{n+2}Y_{n+1}X_n \end{cases},$$

$$T_9 : \begin{cases} X_{n+4} = X_{n+3}Y_{n+2}Y_{n+1}X_n \\ Y_{n+4} = Y_{n+3}X_{n+2}X_{n+1}Y_n \end{cases},\ T_{10} : \begin{cases} X_{n+4} = X_{n+3}Y_{n+2}Y_{n+1}Y_n \\ Y_{n+4} = Y_{n+3}X_{n+2}X_{n+1}X_n \end{cases},\ T_{11} : \begin{cases} X_{n+4} = Y_{n+3}Y_{n+2}X_{n+1}X_n \\ Y_{n+4} = X_{n+3}X_{n+2}Y_{n+1}Y_n \end{cases},\ T_{12} : \begin{cases} X_{n+4} = Y_{n+3}Y_{n+2}X_{n+1}Y_n \\ Y_{n+4} = X_{n+3}X_{n+2}Y_{n+1}X_n \end{cases},$$

$$T_{13} : \begin{cases} X_{n+4} = Y_{n+3}X_{n+2}Y_{n+1}X_n \\ Y_{n+4} = X_{n+3}Y_{n+2}X_{n+1}Y_n \end{cases},\ T_{14} : \begin{cases} X_{n+4} = Y_{n+3}X_{n+2}Y_{n+1}Y_n \\ Y_{n+4} = X_{n+3}Y_{n+2}X_{n+1}X_n \end{cases},\ T_{15} : \begin{cases} X_{n+4} = Y_{n+3}Y_{n+2}Y_{n+1}X_n \\ Y_{n+4} = X_{n+3}X_{n+2}X_{n+1}Y_n \end{cases},\ T_{16} : \begin{cases} X_{n+4} = Y_{n+3}Y_{n+2}Y_{n+1}Y_n \\ Y_{n+4} = X_{n+3}X_{n+2}X_{n+1}X_n \end{cases}.$$

The few terms of schemes (16) and (6) are tabulated below:

	Scheme (16)		**Scheme (6)**	
n	X_n	Y_n	X_n	Y_n
0	a	b	a	b
1	c	d	c	d
2	e	f	e	f
3	g	h	g	h
4	bdfh	aceg	adeh	bcfg
5	dfhaceg	cegbdfh	$bc^2f^2g^2$	$fhad^2e^2$
6	$ac^2e^2g^2f^2h^2$	$bh^2f^2d^2e^2g^2$	$fe^4h^3a^2d^2$	$f^4g^4b^2c^3$
7	$ac^2b^2d^3f^3g4e^4h^4$	$bh^2d^2c^3e^3f^4h^4g^4$	$b^4c^6f^7g^8$	$a^4d^5e^7h^6f^2$

MAIN RESULTS

Now we present some properties under schemes (16) and (6).

3. Scheme (6):

$$Y_{n+4} = X_n X_{n+2} Y_{n+1} Y_{n+3}, \qquad n \geq 0 \qquad (3.1)$$
$$X_{n+4} = X_{n+1} X_{n+3} Y_n Y_{n+2}, \qquad n \geq 0$$

Theorem 3.1: For every integer $n \geq 0$:

(a) $X_{10n}Y_0 = Y_{10n}X_0$,

(b) $X_{10n+2}Y_2 = Y_{10n+2}X_2$,

(c) $X_{10n+4}Y_4 = Y_{10n+4}X_4$

Proof: We prove the above results by induction hypothesis.

(a) If n = 0 then result is true because $Y_0X_0 = X_0Y_0$.

Now assume that the result is true for some integer $n \geq 1$

.

i.e. $X_{10n}Y_0 = Y_0X_{10n}$ (3.2)

$$X_{10n+10}Y_0 = X_{10n+9}Y_{10n+8}X_{10n+7}Y_{10n+6}Y_0 \qquad \text{(By scheme 3.1)}$$

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}Y_{10n+5}Y_{10n+8}X_{10n+7}Y_{10n+6}Y_0 \qquad \text{(By scheme 3.1)}$$

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}Y_{10n+4}X_{10n+3}Y_{10n+2}X_{10n+1}Y_{10n+8}X_{10n+7}Y_{10n+6}Y_0 \qquad \text{(By scheme 3.1)}$$

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}Y_{10n+3}X_{10n+2}Y_{10n+1}X_{10n}X_{10n+3}Y_{10n+2}X_{10n+1}Y_{10n+8}X_{10n+7}Y_{10n+6}Y_0$$ (By scheme 3.1)

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}Y_{10n+3}X_{10n+2}Y_{10n+1}X_{10n+3}Y_{10n+2}X_{10n+1}Y_{10n+8}X_{10n+7}Y_{10n+6}(X_{10n}Y_0)$$

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}Y_{10n+3}X_{10n+2}Y_{10n+1}X_{10n+3}Y_{10n+2}X_{10n+1}Y_{10n+8}X_{10n+7}Y_{10n+6}(Y_{10n}X_0)$$ (By equation 3.2)

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}Y_{10n+3}X_{10n+2}Y_{10n+1}(X_{10n+3}Y_{10n+2}X_{10n+1}Y_{10n})Y_{10n+8}X_{10n+7}Y_{10n+6}X_0$$ (By scheme 3.1)

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}Y_{10n+3}X_{10n+2}Y_{10n+1}X_{10n+4}Y_{10n+8}X_{10n+7}Y_{10n+6}X_0$$ (By scheme 3.1)

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}(X_{10n+4}Y_{10n+3}X_{10n+2}Y_{10n+1})Y_{10n+8}X_{10n+7}Y_{10n+6}X_0$$ (By scheme 3.1)

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}X_{10n+5}Y_{10n+8}X_{10n+7}Y_{10n+6}X_0$$ (By scheme 3.1)

$$X_{10n+10}Y_0 = X_{10n+8}Y_{10n+7}X_{10n+6}(Y_{10n+8}X_{10n+7}Y_{10n+6}X_{10n+5})X_0$$ (By scheme 3.1)

$$X_{10n+10}Y_0 = (Y_{10n+9}X_{10n+8}Y_{10n+7}X_{10n+6})X_0$$ (By scheme 3.1)

$$X_{10n+10}Y_0 = Y_{10n+10}X_0$$ (By scheme 3.1)

Hence the result is true for all integers $n \geq 0$.

Similar proofs can be given for remaining parts (b) and (c).

Theorem 3.2: For every integer $n \geq 0$:

a) $Y_{10n+8}Y_{10n+7}Y_{10n+6}Y_{10n+5} = \dfrac{X_{10n+10}}{X_{10n+8}X_{10n+7}X_{10n+6}}$,

b) $X_{10n+8}X_{10n+7}X_{10n+6}X_{10n+5} = \dfrac{Y_{10n+10}}{Y_{10n+8}Y_{10n+7}Y_{10n+6}}$

Theorem 3.3: For every integer $n \geq 4$:

a) $Y_nY_{n+1}Y_{n+2}Y_{n+3} = \dfrac{X_{n+4}X_{n+5}}{X_{n+1}X_{n+2}X_{n+3}X_{n+4}}$,

b) $X_nX_{n+1}X_{n+2}X_{n+3} = \dfrac{Y_{n+4}Y_{n+5}}{Y_{n+1}Y_{n+2}Y_{n+3}Y_{n+4}}$

Theorem 3.4: For every integer $n \geq 4$:

a) $X_nX_{n+1}X^2_{n+2}X_{n+3}X_{n+4} = \dfrac{Y_{n+4}Y_{n+5}Y_{n+6}}{Y_{n+1}Y_{n+2}Y^2_{n+3}Y_{n+4}Y_{n+5}}$,

b) $Y_nY_{n+1}Y^2_{n+2}Y_{n+3}Y_{n+4} = \dfrac{X_{n+4}X_{n+5}X_{n+6}}{X_{n+1}X_{n+2}X^2_{n+3}X_{n+4}X_{n+5}}$

Theorem 3.5: For every integer $n \geq 0$:

a) $X_{5n+4} = \dfrac{Y_{5n}Y_{5n+5}}{Y_{5n+4}}$,

b) $Y_{5n+4} = \dfrac{X_{5n}X_{5n+5}}{X_{5n+4}}$

Theorem 3.6: For every integer $n \geq 4$:

a) $X_n X_{n+1} X_{n+2}^2 X_{n+3}^2 X_{n+4} X_{n+5} = \dfrac{Y_{n+4} Y_{n+5} Y_{n+6} Y_{n+7}}{Y_{n+1} Y_{n+2} Y_{n+3}^2 Y_{n+4}^2 Y_{n+5} Y_{n+6}}$,

b) $Y_n Y_{n+1} Y_{n+2}^2 Y_{n+3}^2 Y_{n+4} Y_{n+5} = \dfrac{X_{n+4} X_{n+5} X_{n+6} X_{n+7}}{X_{n+1} X_{n+2} X_{n+3}^2 X_{n+4}^2 X_{n+5} X_{n+6}}$

Theorem 3.7: For every integer $n \geq 4$:

a) $Y_n Y_{n+1} Y_{n+2}^2 Y_{n+3}^2 Y_{n+4}^2 Y_{n+5} Y_{n+6} = \dfrac{X_{n+4} X_{n+5} X_{n+6} X_{n+7} X_{n+8}}{X_{n+1} X_{n+2} X_{n+3}^2 X_{n+4}^2 X_{n+5}^2 X_{n+6} X_{n+7}}$,

b) $X_n X_{n+1} X_{n+2}^2 X_{n+3}^2 X_{n+4}^2 X_{n+5} X_{n+6} = \dfrac{Y_{n+4} Y_{n+5} Y_{n+6} Y_{n+7} Y_{n+8}}{Y_{n+1} Y_{n+2} Y_{n+3}^2 Y_{n+4}^2 Y_{n+5}^2 Y_{n+6} Y_{n+7}}$

Theorem 3.8: For every integer m, n ≥ 4:

a) $\dfrac{\prod_{i=4}^{m} X_{n+i}}{(\prod_{i=3}^{m-3} X_{n+i}^2)(\prod_{i=2}^{m-4} Y_{n+i}^2)} = (Y_n Y_{n+1} Y_{m+n-3} Y_{m+n-2})(X_{n+1} X_{n+2} X_{m+n-2} X_{m+n-1})$,

b) $\dfrac{\prod_{i=4}^{m} Y_{n+i}}{(\prod_{i=3}^{m-3} Y_{n+i}^2)(\prod_{i=2}^{m-4} X_{n+i}^2)} = (X_n X_{n+1} X_{m+n-3} X_{m+n-2})(Y_{n+1} Y_{n+2} Y_{m+n-2} Y_{m+n-1})$

Theorem 3.9:

a) $X_4 + X_6 + X_8 + X_{10} + X_{12} + X_{14} + ---- = (Y_0 X_1 + Y_4 X_5) Y_2 X_3 + (Y_4 X_5 + Y_8 X_9) Y_6 X_7 + ----$

b) $X_5 + X_7 + X_9 + X_{11} + X_{13} + X_{15} + ---- = (Y_1 X_2 + Y_5 X_6) Y_3 X_4 + (Y_5 X_6 + Y_9 X_{10}) Y_7 X_8 + ---$

Theorem 3.10: For every integer $n \geq 0$:

a) $\dfrac{X_{n+4}}{X_{n+6}} = \dfrac{X_{n+1} Y_n}{X_{n+4} Y_{n+5}}$,

b) $\dfrac{Y_{n+4}}{Y_{n+6}} = \dfrac{Y_{n+1} X_n}{Y_{n+4} X_{n+5}}$

Theorem 3.11: For every integer $n \geq 0$:

$$(X_{n+4} Y_{n+4})^2 = \frac{(X_n Y_n)(X_{n+1} Y_{n+1})(X_{n+6} Y_{n+6})}{(X_{n+5} Y_{n+5})}$$

Theorem 3.12: For every integer $n \geq 1$:

$$\prod_{i=1}^{n} X_{4i} = \prod_{i=0}^{2n-1} (X_{2i+1} Y_{2i})$$

Theorem 3.13: For every integer $n \geq 4$:

$$\prod_{i=1}^{n} X_i = \frac{X_1 Y_2}{X_4 Y_1} \prod_{i=0}^{n} Y_i^2$$

Theorem 3.14: For every integer $n \geq 0$:

a) $X_n X_{n+5} = Y_n Y_{n+5}$,

b) $X_{n+1} X_{n+6} = Y_{n+1} Y_{n+6}$,

c) $X_{n+2}X_{n+7} = Y_{n+2}Y_{n+7}$,

d) $X_{n+3}X_{n+8} = Y_{n+3}Y_{n+8}$,

e) $X_{n+4}X_{n+9} = Y_{n+4}Y_{n+9}$,

f) $X_{n+5}X_{n+10} = Y_{n+5}Y_{n+10}$

Theorem 3.15: For every integer $n \geq 0$:

a) $X_nX_{n+1}X_{n+5}X_{n+6} = Y_nY_{n+1}Y_{n+5}Y_{n+6}$,

b) $X_{n+1}X_{n+2}X_{n+6}X_{n+7} = Y_{n+1}Y_{n+2}Y_{n+6}Y_{n+7}$,

c) $X_{n+2}X_{n+3}X_{n+7}X_{n+8} = Y_{n+2}Y_{n+3}Y_{n+7}Y_{n+8}$,

d) $X_{n+3}X_{n+4}X_{n+8}X_{n+9} = Y_{n+3}Y_{n+4}Y_{n+8}Y_{n+9}$,

e) $X_{n+4}X_{n+5}X_{n+9}X_{n+10} = Y_{n+4}Y_{n+5}Y_{n+9}Y_{n+10}$

Theorem 3.16: For every integer $n \geq 0$:

a) $X_nX_{n+1}Y_{n+10}Y_{n+11} = Y_nY_{n+1}X_{n+10}X_{n+11}$,

b) $X_{n+1}X_{n+2}Y_{n+11}Y_{n+12} = Y_{n+1}Y_{n+2}X_{n+11}X_{n+12}$,

c) $X_{n+2}X_{n+3}Y_{n+12}Y_{n+13} = Y_{n+2}Y_{n+3}X_{n+12}X_{n+13}$,

d) $X_{n+3}X_{n+4}Y_{n+13}Y_{n+14} = Y_{n+3}Y_{n+4}X_{n+13}X_{n+14}$,

e) $X_{n+4}X_{n+5}Y_{n+14}Y_{n+15} = Y_{n+4}Y_{n+5}X_{n+14}X_{n+15}$

Theorem 3.17: For every integer $n \geq 0$:

$$X_{n+10}X_{n+11}X_{n+15}X_{n+16} = Y_{n+10}Y_{n+11}Y_{n+15}Y_{n+16}$$

Theorem 3.18: For every integer $n \geq 5$:

a) $(\prod_{i=5}^{n} X_i)(\prod_{i=0}^{n-5} X_i) = (\prod_{i=5}^{n} Y_i)(\prod_{i=0}^{n-5} Y_i)$,

b) $\dfrac{(\prod_{i=5}^{n-5} X_i^2)}{(\prod_{i=5}^{n-5} Y_i^2)} = \dfrac{Y_0Y_1Y_2Y_3Y_4Y_{n-4}Y_{n-3}Y_{n-2}Y_{n-1}Y_n}{X_0X_1X_2X_3X_4X_{n-4}X_{n-3}X_{n-2}X_{n-1}X_n}$

Theorem 3.19: For every integer $n \geq 5$:

$X_nY_n = (X_{n-5}Y_{n-5})(X_0Y_0)^{2n_1}(X_1Y_1)^{2n_2}(X_2Y_2)^{2n_3}(X_3Y_3)^{2n_4}$

Where $n_1, n_2, n_3, n_4 \geq 1$ such that $n_1 + n_2 + n_3 + n_4 = 3.2^{n-5}$.

4. Scheme (16):

$$Y_{n+4} = X_nX_{n+1}X_{n+2}X_{n+3}, \qquad n \geq 0 \tag{4.1}$$
$$X_{n+4} = Y_nY_{n+1}Y_{n+2}Y_{n+3}, \qquad n \geq 0$$

Theorem 4.1: For every integer $n \geq 0$:

a) $X_{5n}Y_0 = Y_{5n}X_0$,

b) $Y_{5n+1}X_1 = Y_1X_{5n+1}$,

c) $Y_{5n+2}X_2 = Y_2X_{5n+2}$,

d) $Y_{5n+3}X_3 = Y_3X_{5n+3}$,

e) $Y_{5n+4}X_4 = Y_4X_{5n+4}$,

Proof: We prove the above results by induction hypothesis.
(a) If n = 0 then result is true because $Y_0 X_0 = X_0 Y_0$.

Now assume that the result is true for some integer $n \ge 1$.

i.e. $X_{5n} Y_0 = Y_{5n} X_0$ (4.2)

$X_{5n+5} Y_0 = Y_{5n+4} Y_{5n+3} Y_{5n+2} Y_{5n+1} Y_0$ (By scheme4.1)

$X_{5n+5} Y_0 = X_{5n+3} X_{5n+2} X_{5n+1} X_{5n} Y_{5n+3} Y_{5n+2} Y_{5n+1} Y_0$ (By scheme4.1)

$X_{5n+5} Y_0 = X_{5n+3} X_{5n+2} X_{5n+1} Y_{5n+3} Y_{5n+2} Y_{5n+1} (X_{5n} Y_0)$

$X_{5n+5} Y_0 = X_{5n+3} X_{5n+2} X_{5n+1} Y_{5n+3} Y_{5n+2} Y_{5n+1} (Y_{5n} X_0)$ (By equation 4.2)

$X_{5n+5} Y_0 = X_{5n+3} X_{5n+2} X_{5n+1} (Y_{5n+3} Y_{5n+2} Y_{5n+1} Y_{5n}) X_0$

$X_{5n+5} Y_0 = X_{5n+3} X_{5n+2} X_{5n+1} X_{5n+4} X_0$ (By scheme4.1)

$X_{10n+10} Y_0 = Y_{10n+10} X_0$ (By scheme4.1)

Hence the result is true for all integers $n \ge 0$.

Similar proofs can be given for remaining parts (b), (c), (d) and (e).

Theorem 4.2: For every integer $n \ge 1$:

a) $X_{5n+4} X_0 X_1 X_2 X_3 X_4 = Y_{5n+4} Y_0 Y_1 Y_2 Y_3 Y_4$

b) $X_{5n+4} X_0 X_1 X_2 = Y_{5n+4} Y_0 Y_1 Y_2$,

c) $X_{5n} X_0 X_1 X_2 X_3 X_4 = Y_{5n} Y_0 Y_1 Y_2 Y_3 Y_4$,

d) $X_{5n+1} X_0 X_1 X_2 X_3 X_4 = Y_{5n+1} Y_0 Y_1 Y_2 Y_3 Y_4$,

e) $X_{5n+2} X_0 X_1 X_2 X_3 X_4 = Y_{5n+2} Y_0 Y_1 Y_2 Y_3 Y_4$,

f) $X_{5n+3} X_0 X_1 X_2 X_3 X_4 = Y_{5n+3} Y_0 Y_1 Y_2 Y_3 Y_4$

g) $X_{5n+4} X_1^2 X_3^2 = Y_{5n+4} Y_1^2 Y_3^2$

Theorem 4.3: For every integer $n \ge 0$:

a) $X_{n+5} X_{n+6} = Y_{n+1} Y_{n+2}^2 Y_{n+3}^2 Y_{n+4}^2 Y_{n+5}$

b) $Y_{n+5} Y_{n+6} = X_{n+1} X_{n+2}^2 X_{n+3}^2 X_{n+4}^2 X_{n+5}$

Theorem 4.4: For every integer $n \ge 0$:

a) $Y_{n+5} Y_{n+6} Y_{n+7} = X_{n+1} X_{n+2}^2 X_{n+3}^3 X_{n+4}^3 X_{n+5}^2 X_{n+6}$

b) $X_{n+5} X_{n+6} X_{n+7} = Y_{n+1} Y_{n+2}^2 Y_{n+3}^3 Y_{n+4}^3 Y_{n+5}^2 Y_{n+6}$

Theorem 4.5: For every integer $n \ge 0$:

a) $X_{n+5} X_{n+6} X_{n+7} X_{n+8} = Y_{n+1} Y_{n+2}^2 Y_{n+3}^3 Y_{n+4}^4 Y_{n+5}^3 Y_{n+6}^2 Y_{n+7}$

b) $Y_{n+5} Y_{n+6} Y_{n+7} Y_{n+8} = X_{n+1} X_{n+2}^2 X_{n+3}^3 X_{n+4}^4 X_{n+5}^3 X_{n+6}^2 X_{n+7}$

Theorem 4.6: For every integer m, $n \geq 0$:

a) $X_{n+5}X_{n+6}X_{n+7}X_{n+8}X_{n+9} = Y_{n+1}Y_{n+2}^2Y_{n+3}^3Y_{n+4}^4Y_{n+5}^4Y_{n+6}^3Y_{n+7}^2Y_{n+8}$

b) $Y_{n+5}Y_{n+6}Y_{n+7}Y_{n+8}Y_{n+9} = X_{n+1}X_{n+2}^2X_{n+3}^3X_{n+4}^4X_{n+5}^4X_{n+6}^3X_{n+7}^2X_{n+8}$

Theorem 4.7: For every integer m, $n \geq 0$:

a) $X_{n+5}X_{n+6}X_{n+7}X_{n+8}X_{n+9} ----X_{n+m} = Y_{n+1}Y_{n+2}^2Y_{n+3}^3Y_{n+4}^4Y_{n+5}^4 ---Y_{n+m-6}^4Y_{n+m-5}^4Y_{n+m-4}^4Y_{n+m-3}^3Y_{n+m-2}^2Y_{n+m-1}$

b) $Y_{n+5}Y_{n+6}Y_{n+7}Y_{n+8}Y_{n+9} ----Y_{n+m} = X_{n+1}X_{n+2}^2X_{n+3}^3X_{n+4}^4X_{n+5}^4 ---X_{n+m-6}^4X_{n+m-5}^4X_{n+m-4}^4X_{n+m-3}^3X_{n+m-2}^2X_{n+m-1}$

Theorem 4.8: For every integer m, $n \geq 0$:

$X_{n+1}X_{n+2}^2X_{n+3}^3X_{n+4}^4X_{n+5}^5X_{n+6}^5 --X_{n+m-4}^5X_{n+m-3}^4X_{n+m-2}^3X_{n+m-1}^2X_{n+m} = Y_{n+1}Y_{n+2}^2Y_{n+3}^3Y_{n+4}^4Y_{n+5}^5Y_{n+6}^5 --Y_{n+m-4}^5Y_{n+m-3}^4Y_{n+m-2}^3Y_{n+m-1}^2Y_{n+m}$

Theorem 4.9: For every integer m, $n \geq 0$:

$$\frac{\prod_{i=1}^{m} X_{n+i}^5}{\prod_{i=1}^{m} Y_{n+i}^5} = \frac{X_{n+1}^4X_{n+2}^3X_{n+3}^2X_{n+4}^1X_{n+m-3}^1X_{n+m-2}^2X_{n+m-1}^3X_{n+m}^4}{Y_{n+1}^4Y_{n+2}^3Y_{n+3}^2Y_{n+4}^1Y_{n+m-3}^1Y_{n+m-2}^2Y_{n+m-1}^3Y_{n+m}^4}$$

Theorem 4.10: For every integer $n \geq 1$:

a) $\prod_{i=1}^{n} X_{4i} = \prod_{i=0}^{4n-1} Y_i$

b) $\prod_{i=1}^{n} Y_{4i} = \prod_{i=0}^{4n-1} X_i$

Theorem 4.11: For every integer $n \geq 1$:

a) $\prod_{i=1}^{n} X_{4i+1} = \prod_{i=0}^{4n} Y_i$

b) $\prod_{i=1}^{n} Y_{4i+1} = \prod_{i=0}^{4n} X_i$

Theorem 4.12: For every integer $n \geq 2$:

a) $\prod_{i=2}^{2n-3} X_i^2 = \frac{1}{X_0X_1X_{2n-2}X_{2n-1}}\prod_{i=2}^{n} Y_{2i}$

b) $\prod_{i=2}^{2n-3} Y_i^2 = \frac{1}{Y_0Y_1Y_{2n-2}Y_{2n-1}}\prod_{i=2}^{n} X_{2i}$

Theorem 4.13: For every integer $n \geq 2$:

a) $\prod_{i=3}^{2n-2} X_i^2 = \frac{1}{X_1X_2X_{2n}X_{2n-1}}\prod_{i=2}^{n} Y_{2i+1}$

b) $\prod_{i=3}^{2n-2} Y_i^2 = \frac{1}{Y_1Y_2Y_{2n}Y_{2n-1}}\prod_{i=2}^{n} X_{2i+1}$

Theorem 4.14: For every integer $n \geq 4$:

a) $X_nY_n = Y_{n-4}X_{n+1}$

b) $X_nY_n = X_{n+1}X_{n-3}X_{n-4}X_{n-5}X_{n-6}$

c) $X_nY_n = Y_nY_{n-1}Y_{n-2}Y_{n-3}Y_{n-4}$

Theorem 4.15: For integer $n \geq 5$ assume that,

$$X_n = X_{n-5}(X_0Y_0)^{n_0^1}(X_1Y_1)^{n_1^1}(X_2Y_2)^{n_2^1}(X_3Y_3)^{n_3^1},$$
$$X_{n+1} = X_{n-4}(X_0Y_0)^{n_0^2}(X_1Y_1)^{n_1^2}(X_2Y_2)^{n_2^2}(X_3Y_3)^{n_3^2},$$
$$X_{n+2} = X_{n-3}(X_0Y_0)^{n_0^3}(X_1Y_1)^{n_1^3}(X_2Y_2)^{n_2^3}(X_3Y_3)^{n_3^3},$$
$$X_{n+3} = X_{n-2}(X_0Y_0)^{n_0^4}(X_1Y_1)^{n_1^4}(X_2Y_2)^{n_2^4}(X_3Y_3)^{n_3^4},$$
$$X_{n+4} = X_{n-1}(X_0Y_0)^{n_0^5}(X_1Y_1)^{n_1^5}(X_2Y_2)^{n_2^5}(X_3Y_3)^{n_3^5}$$

where $n_0^1, n_1^1, n_2^1, n_3^1, n_0^2, n_1^2, n_2^2, n_3^2, n_0^3, n_1^3, n_2^3, n_3^3, n_0^4, n_1^4, n_2^4, n_3^4 \in \mathbb{Z}^+$

Then,

$$X_{n+5} = X_n(X_0Y_0)^{2n_0^5-n_0^1}(X_1Y_1)^{2n_1^5-n_1^1}(X_2Y_2)^{2n_2^5-n_2^1}(X_3Y_3)^{2n_3^5-n_3^1}$$
$$X_{n+6} = X_{n+1}(X_0Y_0)^{\{2[2n_0^5-n_0^1]-n_0^2\}}(X_1Y_1)^{\{2[2n_1^5-n_1^1]-n_1^2\}}(X_2Y_2)^{\{2[2n_2^5-n_2^1]-n_2^2\}}(X_3Y_3)^{\{2[2n_3^5-n_3^1]-n_3^2\}}$$
$$X_{n+7} = X_{n+2}(X_0Y_0)^{2\{2[2n_0^5-n_0^1]-n_0^2\}-n_0^3}(X_1Y_1)^{2\{2[2n_1^5-n_1^1]-n_1^2\}-n_1^3}(X_2Y_2)^{2\{2[2n_2^5-n_2^1]-n_2^2\}-n_2^3}(X_3Y_3)^{2\{2[2n_3^5-n_3^1]-n_3^2\}-n_3^3}$$
$$X_{n+8} = X_{n+3}(X_0Y_0)^{2[2\{2[2n_0^5-n_0^1]-n_0^2\}-n_0^3]-n_0^4}(X_1Y_1)^{2[2\{2[2n_1^5-n_1^1]-n_1^2\}-n_1^3]-n_1^4}(X_2Y_2)^{2[2\{2[2n_2^5-n_2^1]-n_2^2\}-n_2^3]-n_2^4}(X_3Y_3)^{2[2\{2[2n_3^5-n_3^1]-n_3^2\}-n_3^3]-n_3^4}$$
$$X_{n+9} = X_{n+4}(X_0Y_0)^{2\{2[2\{2[2n_0^5-n_0^1]-n_0^2\}-n_0^3]-n_0^4\}-n_0^5}(X_1Y_1)^{2\{2[2\{2[2n_1^5-n_1^1]-n_1^2\}-n_1^3]-n_1^4\}-n_1^5}(X_2Y_2)^{2\{2[2\{2[2n_2^5-n_2^1]-n_2^2\}-n_2^3]-n_2^4\}-n_2^5}(X_3Y_3)^{2\{2[2\{2[2n_3^5-n_3^1]-n_3^2\}-n_3^3]-n_3^4\}-n_3^5}$$

CONCLUSION

In this paper I described and extended Multiplicative coupled Fibonacci sequences of fourth order under two specific schemes. Similar results can be developed for other schemes.

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