The current reversal phenomenon of brownian particles in a two-dimensional potential with Lévy noise

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Abstract

Effects of Lévy noise on self-propelled particles in a two-dimensional potential is investigated. The current reversal phenomenon appear in the system. V(x direction average velocity) changes from negative to positive with increasing asymmetry parameter β, and changes from positive to negative with increasing self-propelled velocity v_0. The x direction average velocity V has a maximum with increasing modulation constant λ.

Keywords: Current Reversal, Average Velocity, Self-propelled Particles

1. Introduction

The transport of particles or solitons under zero-average forces (i.e., ratchet transport) has been extensively investigated in the last two decades[1, 2, 3, 4]. Most of the previous studies have mainly dealt with the motion of passive particle. Brownian motion with Gaussian white noise has been regarded as a commonly used model to describe the particles transport in nonlinear systems. Recently, motion of active matter has been studied intensively in theories, simulations and experiments. Active matter has emerged as a paradigm to describe a broad wealth of nonequilibrium collective dynamical behavior, including droplets, bacteria, and microtubule networks driven by molecular motors. Many of studies of active matter focus on one of the simplest and most popular models of self-propelled particles. The transport of active Brownian particles explains the origin of the self-propelled motion, and the self-propelled particles motion is one of the obvious signatures of many living systems. The self-propelled particles moving in potential could exhibit peculiar behavior[5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15].

There are numerous realizations of self-propelled particles in nature ranging from bacteria and spermatozoa to artificial colloidal micro-swimmers. Bialké *et al.* have investigated a two-dimensional suspension of self-propelled colloidal particles crystallizes at sufficiently high densities using Brownian dynamics computer simulations, and found the freezing density is both significantly shifted and depends on the structural or dynamical criterion employed[16]. Pototsky *et al.* found the motion of self-propelled particles can be rectified by asymmetric or ratchet like periodic patterns in space. A nonzero average drift can be induced in a periodic potential with symmetric barriers when the self-propulsion velocity is also symmetric and periodically modulated but phase-shifted against the potential[17]. Ai *et al.* have investigated the rectification and diffusion of self-propelled particles in a two-dimensional corrugated channel with white noise, and found the self-propelled velocity can strongly increase the effective diffusion, and the large rotational diffusion rate can strongly suppress the effective diffusion[18]. All these studies devoted to the self-propelled particles were treating the input noise processes as solely Gaussian and so, had finite variance. We known that Brownian motion is at a disadvantage to depict instantaneous disturbance changes due to its continuity. Lévy noise, which frequently appears in areas of finance, statistical mechanics, and signal processing[19, 22, 23, 24], is more suit able for modeling diversified system noise because it can be decomposed into a continuous part and a jump part by Lévy -Itô decomposition. As a result, Lévy noise extends Gaussian noise to many types of impulsive jump-noise processes found in real and model neurons as well as in models of finance and other random phenomena.

In the present Letter, we investigate the effects of Lévy noise on the self-propelled particles in a two-dimensional potential.

2. Basic model and methods

The dynamic of Brownian particles is governed by the following dimensionless equations[13, 14].

$$\frac{dx}{dt} = v_0 \cos\theta + F_x + \xi(t), \tag{1}$$

$$\frac{dy}{dt} = v_0 \sin\theta + F_y + \xi(t), \tag{2}$$

$$\frac{d\theta}{dt} = \xi_\theta(t), \tag{3}$$

v_0 is the self-propelled velocity. θ is the self-propelled angle. $F_x = -\frac{\partial U(x,y)}{\partial x}$ and $F_y = -\frac{\partial U(x,y)}{\partial y}$.

$$U(x,y) = -\sin(2\pi x) + \frac{1}{2}C_0[1 - \lambda\sin(2\pi x + \phi)]y^2 \tag{4}$$

is the potential, which periodic in x direction and parabolic in y direction. C_0 is the intensity of the y direction potential. ϕ is the phase shift between the x direction potential and the modulation function. λ is the modulation constant with $0 < \lambda < 1$. $\xi_\theta(t)$ is the Gaussian white noise and satisfies the following relations

$$\langle \xi_\theta(t) \rangle = 0, \tag{5}$$

$$\langle \xi_\theta(t)\xi_\theta(t') \rangle = \delta(t - t'). \tag{6}$$

$\xi(t)$ is the Lévy noise and obeys Lévy distribution $L_{\alpha,\beta}(\zeta;\gamma,\mu)$, and the characteristic function is[19]:

$$\Phi(k) = \int_{-\infty}^{+\infty} d\zeta exp(ik\zeta)L_{\alpha,\beta}(\zeta;\gamma,\mu). \tag{7}$$

Therefore, for $\alpha \in (0,1) \cup (1,2]$,

$$\Phi(k) = exp[i\mu k - \gamma^\alpha|k|^\alpha(1 - i\beta sgn(k)tan\frac{\pi\alpha}{2})]. \tag{8}$$

and for $\alpha = 1$

$$\Phi(k) = exp[i\mu k - \gamma|k|(1 + i\beta sgn(k)\frac{2}{\pi}\ln|k|)]. \tag{9}$$

Here $\alpha \in (0,2]$ denotes the stability index that describes an asymptotic power law of the Lévy distribution. When $\alpha \leq 2$, $L_{\alpha,\beta}(\zeta;\gamma,\mu)$ is characterized by a heavy-tail of $|\zeta|^{-(\alpha+1)}$ type with $|\zeta| \gg 1$. The constant β is the asymmetry parameter with $\beta \in [-1,1]$. When β is positive, the distribution is skewed to the right. When it is negative, it is skewed to the left. When $\beta = 0$, the distribution is symmetrical. As α approaches 2, the distribution approaches the symmetrical Gaussian distribution regardless of β. γ is the scale parameter with $\gamma \in (0,\infty)$, $\mu(\mu \in R)$ denotes the location parameter, and $D = \gamma^\alpha$ represents the noise intensity. In this paper we use the Janicki-Weron algorithm to generate the Lévy distribution[19].

As $\alpha \neq 1$, ξ is simulated as

$$\xi = D_{\alpha,\beta,\gamma} B_{\alpha,\beta} [\frac{\cos(M - \alpha(M + C_{\alpha,\beta}))}{W}]^{(1-\alpha)/\alpha} + \mu. \quad (10)$$

As $\alpha = 1$, ξ can be obtained from the formula

$$\xi = \gamma \frac{2}{\pi}[(\frac{\pi}{2} + \beta M)\tan(M) - \beta \ln(\frac{W\cos(M)}{\frac{\pi}{2} + \beta M})] + \frac{2}{\pi}\beta\sigma \ln\sigma + \mu. \quad (11)$$

the constants $B_{\alpha,\beta}$, $C_{\alpha,\beta}$, $D_{\alpha,\beta,\gamma}$ are given by

$$B_{\alpha,\beta} = \frac{\sin(\alpha(M + C_{\alpha,\beta}))}{(\cos(M))^{1/\alpha}}. \quad (12)$$

$$C_{\alpha,\beta} = \frac{\arctan(\beta\tan(\frac{\pi\alpha}{2}))}{\alpha}. \quad (13)$$

$$D_{\alpha,\beta,\gamma} = \gamma[1 + \beta^2\tan^2(\frac{\pi\alpha}{2})]^{1/2\alpha}. \quad (14)$$

M is a random variable uniformly distributed over $(-\frac{\pi}{2}, \frac{\pi}{2})$. W is a random variable exponentially distributed with a unit mean. M and W are statistically independent[19, 20, 21].

The over all long time behavior is a central practical question in the theory of Brownian motors, and the key quantities of particle transport through periodic potential is the particle velocity V. V can be corroborated by Brownian dynamic simulations performed by integration of the Langevin equations using the stochastic Euler algorithm. We only calculate the x direction average velocity as the potential along y-direction is parabolic. The x direction average velocity can be obtained from the following formula:

$$V = \lim_{t\to\infty} \frac{\langle x(t) - x_0 \rangle}{t - t_0} \quad (15)$$

$x(t)$ is the position of particles at time t, and $x(t_0) = x_0$.

3. Results and discussion

In order to analyse the effects of Lévy noise on the system, Eqs.(1), (2) and (3) are integrated using the Euler algorithm with $C_0 = 5.0$, $\phi = \frac{\pi}{2}$ and time step $\Delta t = 5 \times 10^{-3}$. The total integration time was more than 1.0×10^6.

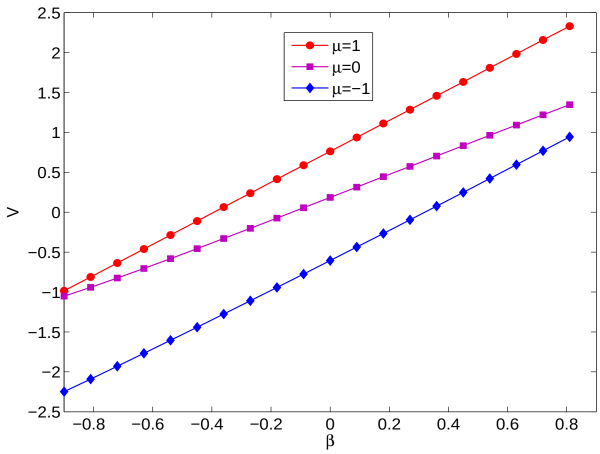

Figure 1: Average velocity V as a function of the asymmetry parameter β with different mean parameter μ. The other parameters are $\alpha = 1.6$, $\lambda = 0.5$, $\gamma = 1.2$ and $v_0 = 2$.

The stochastic averages reported above were obtained as ensemble averages over 2×10^4 trajectories with random initial conditions.

Fig.1 shows the dependence of average velocity V on the asymmetry parameter β with different mean parameter μ. We find V changes from negative value to positive value with increasing asymmetry parameter β. We know, $V < 0$, means the particles moving in the anti x direction, $V > 0$, means the particles moving in the x direction, so the system appear current reversal phenomenon as increasing β. In view of of the property of Lévy noise, we find the slope of $V - \beta$ with $\mu = 1$ is equal to the slope with $\mu = -1$, and larger than the slope with $\mu = 0$.

Fig.2 shows the average velocity V as a function of mean parameter μ with different asymmetry parameter β. Overall, V increases with increasing mean parameter μ, but the $V - \mu$ curve is not a smooth curve. There exist break points as $|\mu| = 4$ and $|\mu| = 3$.

The average velocity V as a function of stability index α with different modulation constant λ is reported in Fig.3. As shown, we find $\alpha = 1$ is a break point for the $V - \alpha$ curve, and V with $\alpha = 1$ is much larger than V with $\alpha \neq 1$. So Lévy noise at the break point is more likely to induce particles transport than the noise at other point.

Fig.4 shows the average velocity V as a function of the modulation constant λ with different α. We find there exist a maximum with increasing λ, so the average velocity takes its maximal value as the modulation constant

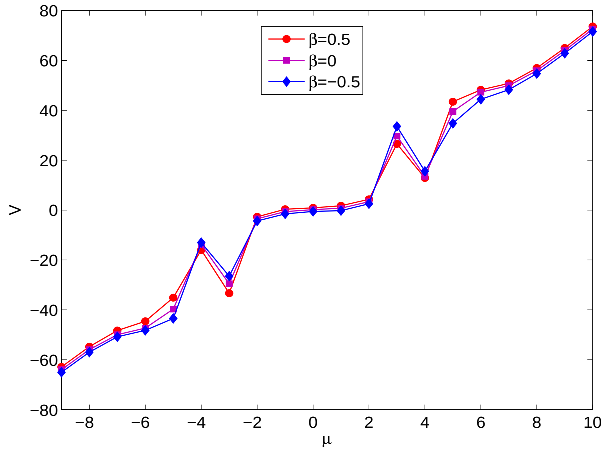

Figure 2: Average velocity V as a function of mean parameter μ with different asymmetry parameter β. The other parameters are $\alpha = 1.6$, $\lambda = 0.5$, $\gamma = 1.2$ and $v_0 = 2$.

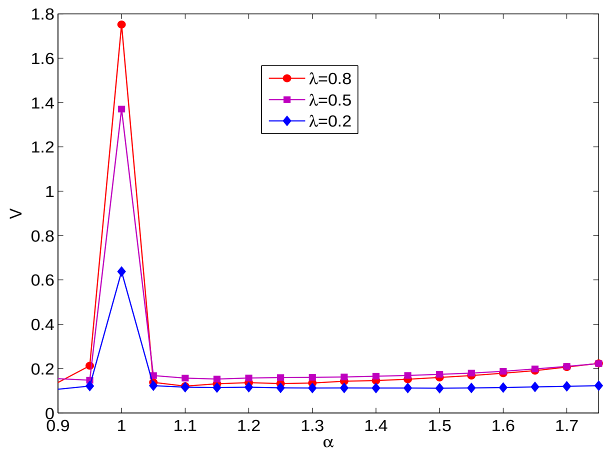

Figure 3: Average velocity V as a function of stability index α with different modulation constant λ. The other parameters are $\beta = 0$, $\mu = 0$, $\gamma = 1.2$ and $v_0 = 2$.

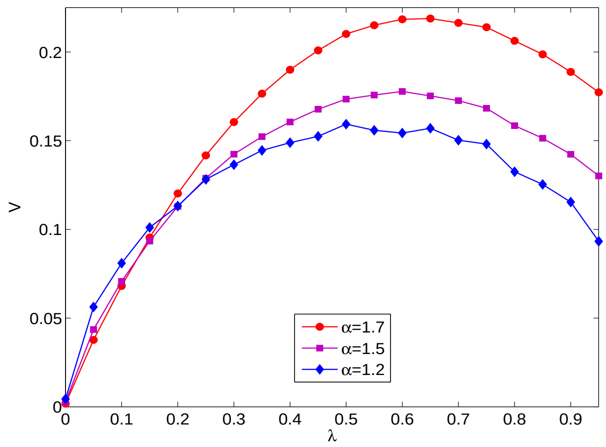

Figure 4: Average velocity V as a function of the modulation constant λ with different α. The other parameters are $\beta = 0$, $\mu = 0$, $\gamma = 1.2$ and $v_0 = 2$.

λ takes suitable value. We also find the peak moves to right with increasing α.

Fig.5 depicts average velocity V as a function of the self-propelled velocity v_0 with different scale parameter γ. As shown, V decreases monotonically with increasing v_0. We also find V changes from positive value to negative value with increasing v_0. The system appear current reversal phenomenon with increasing self-propelled velocity v_0 under the influence of the Lévy noise,

The average velocity V as a function of the scale parameter γ with different asymmetry parameter β is reported in Fig.6. We find V increases monotonically with increasing γ as β is large($\beta = 0$, $\beta = 0.25$, $\beta = 0.5$ in Fig.6(a)). As β is small($\beta = -0.3$, $\beta = -0.4$, $\beta = -0.5$ in Fig.6(b)), V decreases first and reaches a minimum value and then increases with increasing γ; so for small β, the anti x particles transport velocity takes its maximal value as the scale parameter γ takes suitable value.

4. Conclusions

Lévy noise is more suit able for modeling diversified system noise because it can be decomposed into a continuous part and a jump part by Lévy - Itô decomposition. In this paper, we have investigated the effects of Lévy noise on self-propelled particles in a two-dimensional potential. The potential

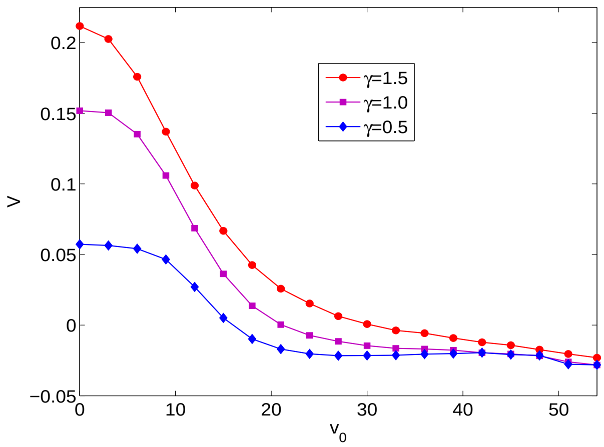

Figure 5: Average velocity V as a function of the self-propelled velocity v_0 with different γ. The other parameters are $\alpha = 1.2$, $\beta = 0$, $\mu = 0$ and $\lambda = 0.5$.

periodic in x direction and parabolic in y direction. We find the current reversal phenomenon appear in the system. The average velocity V changes from negative value to positive value with increasing asymmetry parameter β. V changes from positive to negative with increasing self-propelled velocity v_0. V has a maximum with increasing modulation constant λ. As β is large, V increases with increasing scale parameter γ, but V has a minimum value with increasing γ as β is small.

5. Acknowledgments

Project supported by Natural Science Foundation of Anhui Province(Grant No:1408085QA11, 1508085QF140).

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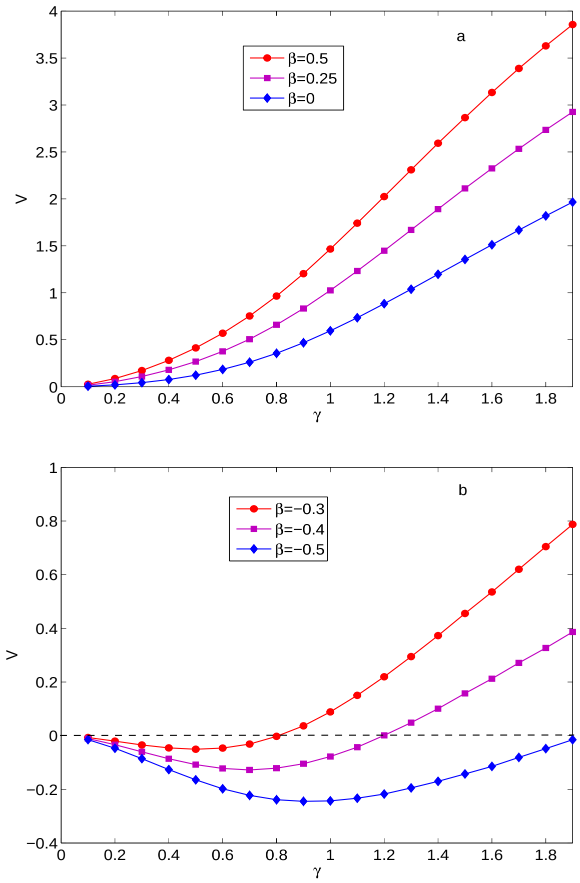

Figure 6: Average velocity V as a function of the scale parameterγ with different β. The other parameters are $\alpha = 1.5$, $\mu = 1$, $\lambda = 0.5$ and $v_0 = 3$.

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