Dense Communications using the Engineer's Chaos

Sai Venkatesh Balasubramanian

Sree Sai VidhyaMandhir, Mallasandra, Bengaluru-560109, Karnataka, India.
saivenkateshbalasubramanian@gmail.com

Abstract

With the central focus on a broadened perspective of chaos, this article introduces the concept of the engineer's chaos, characterized by determinism, aperiodicity and sensitive dependence on initial conditions, and exhibiting a marked deviation from conventional chaotic systems by possessing a signal based control parameter. It is seen that multiplying two sinusoidal signals, the simplest possible nonlinear operation generates the engineer's chaos with the frequency ratio of the two signals acting as the control parameter. The nature of the chaos is observed using iterative map and bifurcation plot. Following this, an application of the engineer's chaos to dense communications is demonstrated using a proof-of-concept, where the two chaos generating signals are amplitude modulated before multiplying, and transmitted over a channel represented as a Additive White Gaussian Noise of 18dB. The ability to receive and demodulate the signals with reasonable accuracy highlights the capabilities of the engineer's chaos. The demonstration proves how the engineer's chaos, the result of a broadened perspective on chaos yields triple advantages of minimalist and simple design, increased information carrying capacity, and security inherent to the sensitivity of chaos.

1. Introduction

Sometime during the past century, development in computing power enabled one to simulate various natural and man-made systems and visualize the complex patterns of evolution arising therein [1]. This lead to a surge in interest in the field of nonlinear dynamics, and in particular, chaos theory [2]. Chaotic behaviors of systems are characterized typically by determinism, aperiodicity and extremely sensitive dependence on initial conditions [2].

In due course, understanding of chaotic systems improved, and scientists began to observe that chaos occurs much more frequently in nature, and is much more fundamental than previously thought [2,3]. Within the scope of engineering, chaos theory finds application in, among other areas, system stability, emergence theory and secure communications [4].

However, on observing various research publications in the field, one understands that the while there have been remarkable strides in the mathematics of nonlinear dynamics, the definitions and perceptions of chaos has not broadened enough to keep up with its growing prominence and fundamentalism. Rather than restricting oneself to differential equations and system based control parameters, it is absolutely essential at this juncture to broaden one's definition of chaos, and include for example, signal based control parameters in the generation of chaos.

In this line of thought, the present article focuses on a novel concept termed the 'Engineer's Chaos', with particular focus on the area of communication systems. In sharp contrast to conventional chaotic systems, the engineer's chaos is obtained by operations between signals, represented as sinusoids, using the simplest nonlinear element possible: a multiplier. The presence of chaos and its nature are studied using the bifurcation plot. The emphasis here is to understand that the generated signal possesses all the properties of a chaotic signal, though the generation process is highly unconventional and far more simplistic than state-of-the-art nonlinear circuits.

To explore the usefulness of the engineer's chaos, the present article demonstrates a proof-of-concept communication system using amplitude modulation. In particular, the two carrier signals generating the engineer's chaos are modulated with message signals before being fed to the multiplier. This generates a chaotic signal, as seen from the phase portrait, and since the signal is able to hold two message signals, it is termed the "Dense Chaotic Signal". Channel is simulated using Gaussian noise, and the receiver consists of a local oscillator-multiplier, after which band pass filters are used to demodulate and extract the messages.

From this demonstration, it is seen that the complex nature of the engineer's chaos, as seen by its richness in phase portrait, actually offers a higher than normal capacity to carry information, and this is reflected by the ability to modulate two message signals rather than one. This corresponds to a case of "dense communication", a term borrowed from the 'superdense coding' of quantum computing [5].

The principles and demonstrations outlined in this article can easily be extended to incorporate various forms of analog and digital modulation, encoding and multiplexing systems, and in each case, the increase in system capacity, as well as security inherent to chaos, can be observed. The emphasis of using the engineer's chaos in such applications is the sheer minimality and simplicity of design resulting in the above mentioned advantages, and all this is obtained merely because of a broadened perspective on chaos.

2. The Engineer's Chaos

The very backbone of the present article is a broadened perspective of chaos theory. In accordance with this, a chaotic system is defined to be one that exhibits the following three properties [2]:

a. Determinism
b. Aperiodicity
c. Sensitive Dependence on Initial Conditions

Given a chaotic signal, the determinism can be asserted merely be checking for mathematical consistency as well as ability to express the signal by equations. Aperiodicity can be assessed by examining the time series waveform and looking for absence of periodic repetitions of patterns [2]. Alternatively, one can plot the phase portrait, a graph of a signal vis-à-vis its time derivative, and the presence of a rich pattern covering a certain area, formed by signal trajectories forming different loops each cycle, a phenomenon known as ergodicity. The sensitive dependence can be visually examined by plotting the bifurcation plot, a graph showing the output variations on changing a certain 'control parameter' [2]. Dense and 'grassy' areas indicate values of control parameters that give rise to large variations in output, and hence, chaos.

The most effective way to generate chaos is through a nonlinear system. Keeping in mind the desired objective of simplicity and minimalist design as well as the concept of introducing signal based control parameters, we now explore the simplest nonlinear function possible for two signals: multiplication. Essentially, our chaos generator is a simple multiplier, multiplying two input signals sample by sample. The input signals are chosen as sinusoids, and our control parameter is the ratio of frequencies of the two inputs.

Mathematically, representing the inputs by A and B, and output as C, one can write $C = A \times B$. Substituting the sinusoid expressions and amplitudes,

$$C(t) = A_0 \sin(ft) \times B_0 \sin(rft)$$

Here, 'r' is the ratio of frequencies between the two signals, and hence is the control parameter. The nonlinearity of the multiplication operation is seen by the fact that the signal C(t) contains two frequency components 'rf+f' and 'rf-f', which are new frequency terms different from the inputs 'f' and 'rf'.

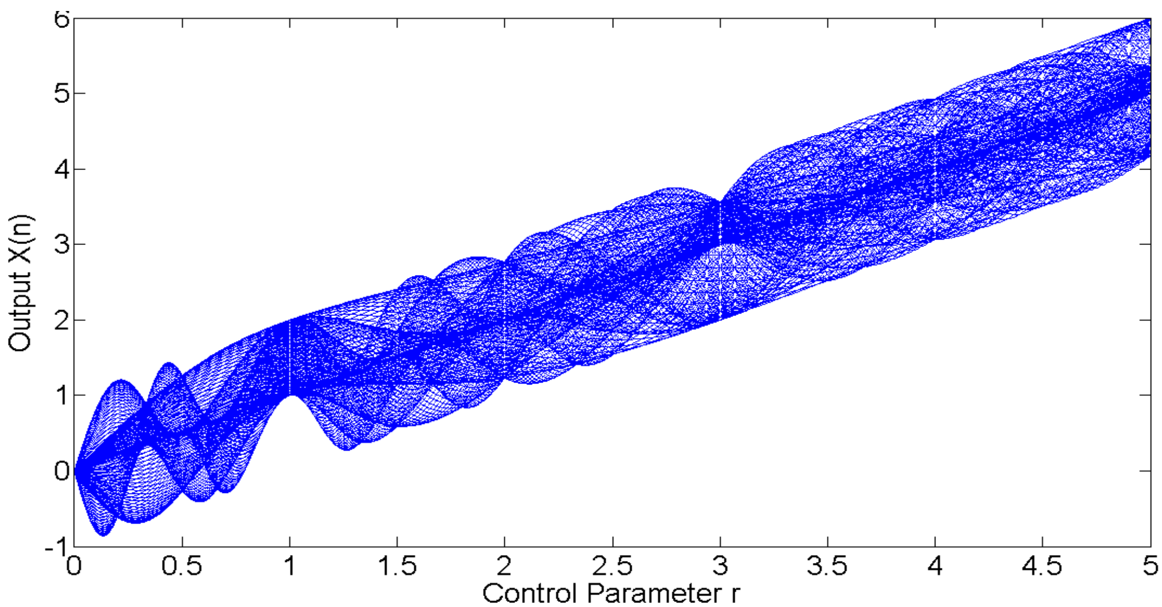
To examine the bifurcation plot, one first needs to express this simple system as an iterative map, which is an equation depicting how each sample of the output depends on its previous output sample, as well as on the control parameter r. To achieve this, we differentiate C(t) to give the time derivative C'(t):

$$C'(t) = rfsin(ft)\cos(rft) + fcos(ft)\sin(rft)$$

In terms of discrete samples, it is noted that C'(t) can be written as the difference between the next sample and current sample, that is C'(t) = C(i+1) - C(i). Using this relation and rewriting the above equation, we have

$$C(i+1) = C(i) + rfsin(fi)\cos(rfi) + fcos(fi)\sin(rfi)$$

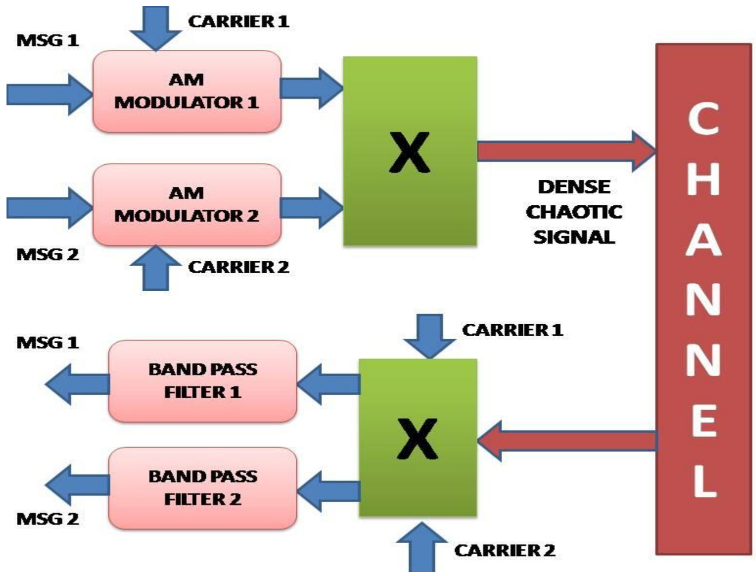
Now, we have the iterative map, using which the bifurcation plot can be plotted as r vs. C(i+1).



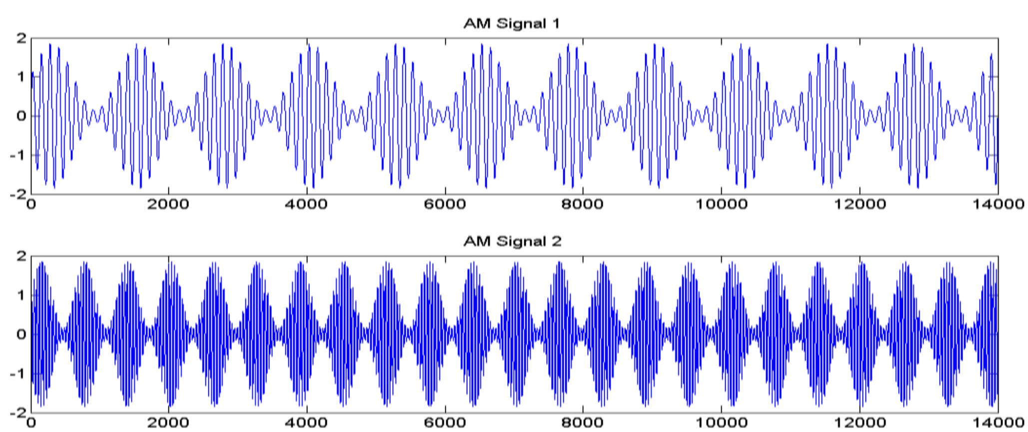
From the plot, apart from the various sparse and dense regions, one can notice a very interesting pattern: exactly in integer values of r, such as 1,2,3… the dense regions vanish and one sees a break in the pattern, with only a few points in the output for these values of r. These are the non-chaotic regimes of our system. For almost all other values of r, one finds patterns of various densities, corresponding to various levels of chaoticity. Noting this pattern, we set the value of r to 'pi' (3.14…), an irrational constant, since this value is seen to generate chaos.

3. Dense Communications

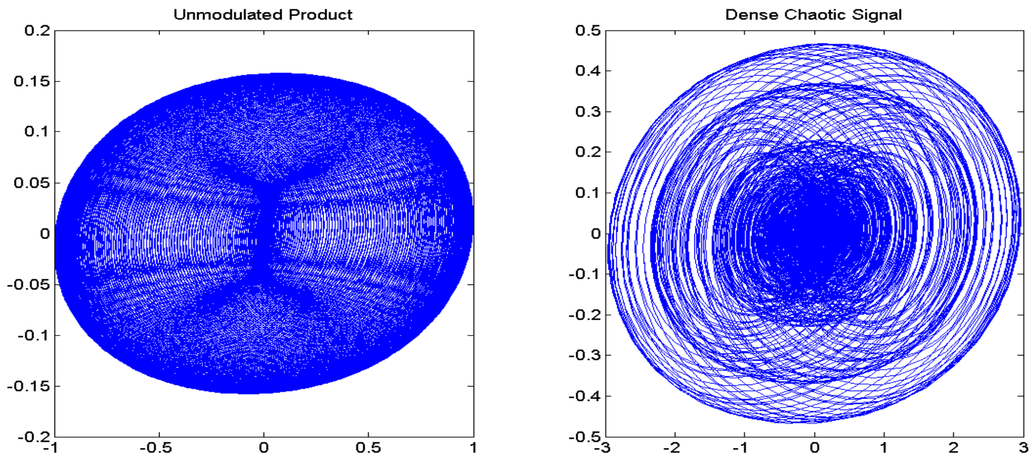
The next step is to demonstrate the usefulness of the engineer's chaos. This is done by proposing a 'dense' communication system. In essence, it is noted that two signals of different frequencies 'f' and 'rf' are used to generate chaos, and hence these can be used as carriers to carry two different message signals. Basic Amplitude Modulation (AM) is used, and the block diagram is shown below.



The dense communication is achieved by modulating the sinusoids of 'f' and 'rf' before multiplying them to generate chaos. These AM modulated signals are shown below:

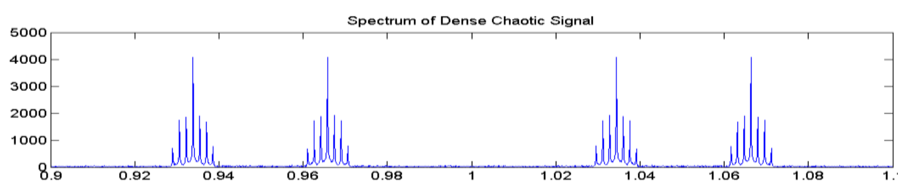


The generated signal is thus called the ‘Dense Chaotic Signal’. The chaotic nature of this signal is observed by plotting its phase portrait, and this is compared with the phase portrait of the engineer’s chaos generated by simply multiplying the carriers without modulating them.

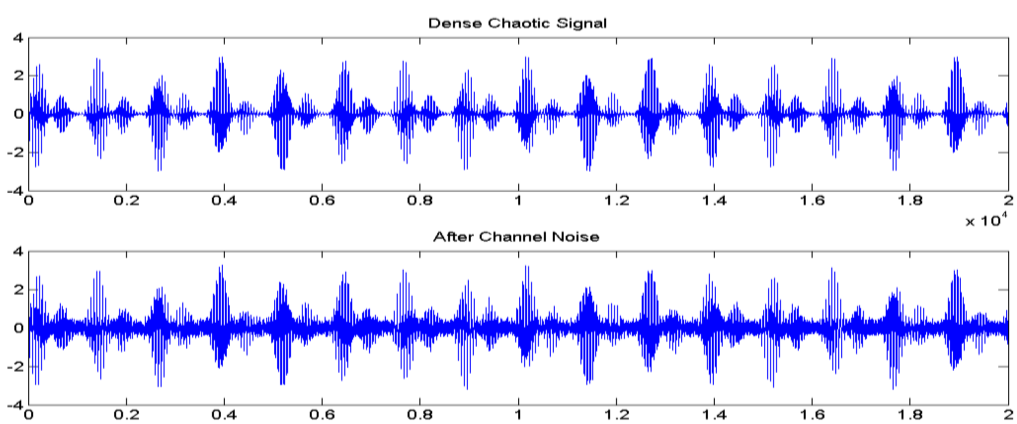


From the phase portraits, one can see the rich patterns in the engineer’s chaos, and thus its capacity to hold information in the phase space. One can also see how modulating the carrier signals before multiplying them changes its nature, to provide the phase portrait of the dense chaotic signal.

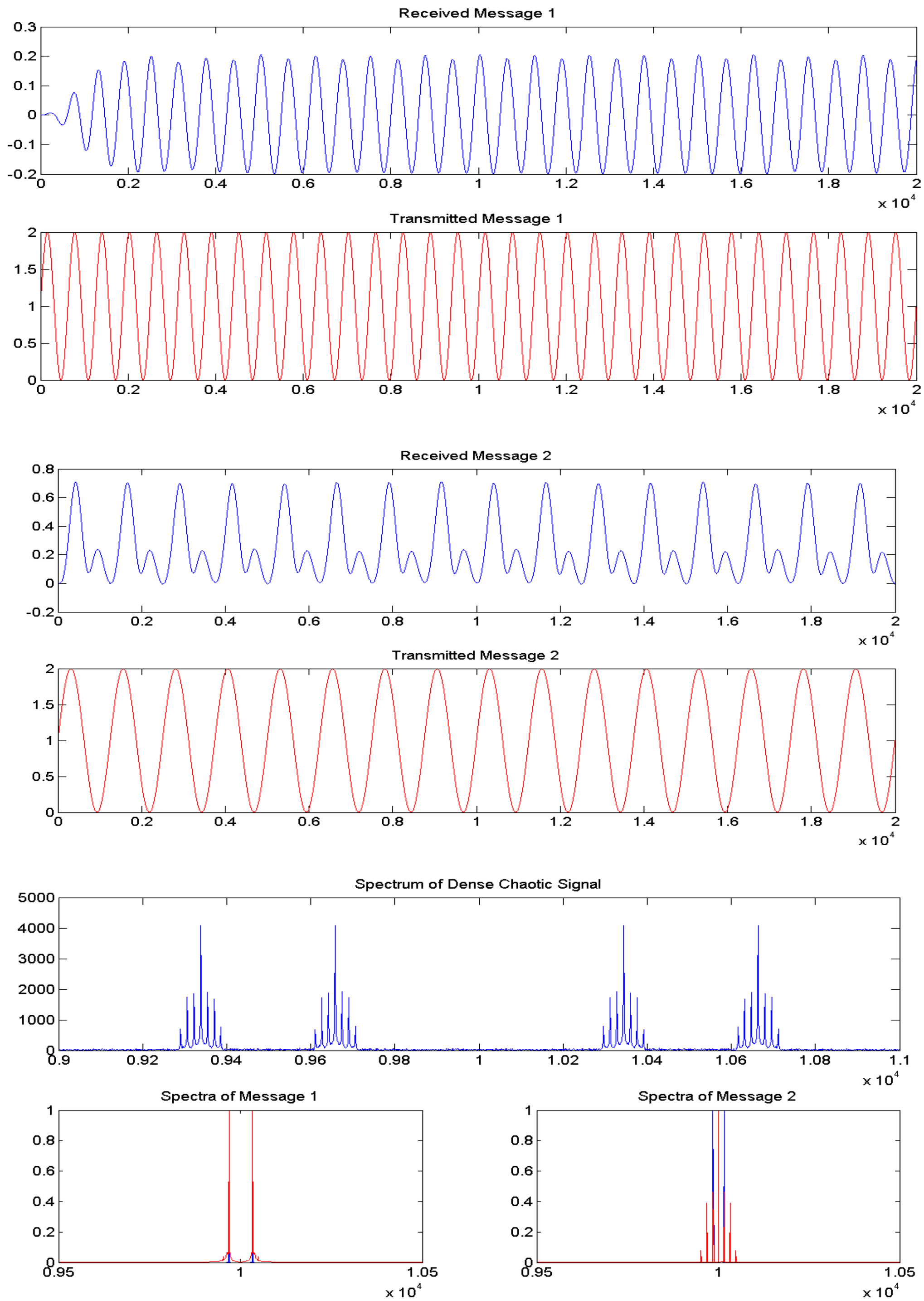
Since Double Sideband AM is used, the composite signal consists of carriers and sidebands of both the AM signals. This spectrum is plotted as follows:



This dense chaotic signal thus forms the composite signal, and is the one transmitted through the channel. The channel is represented as Additive White Gaussian Noise (AWGN) with a nominal Signal-to-Noise Ratio (SNR) of 18dB [6]. The signals before and after adding noise are shown below:



At the receiver end, the dense chaotic signal is fed to a multiplier, which multiplies the incoming signal by the product of the carrier signals generated locally. Following this, band pass filters with suitable specifications are used to extract the two message signals from the composite signal. The generated signals are shown as follows, compared with the transmitted messages both in time waveform and frequency spectrum:



It is seen that one is able to successfully demodulate the message signals with reasonable amount of accuracy, even in spite of the 18dB channel noise.

The above MATLAB simulation demonstrates a proof-of-concept dense communication system using basic AM, facilitated through the minimalist design proposed earlier for the engineer's chaos.

It can also be easily verified that if the carrier frequencies and thus the frequency ratio generated locally at the receiver for demodulation do not exactly match the transmitted carrier frequencies, there will be considerable distortion and error in the demodulated message. This is the security advantage due to a property inherent to the engineer's chaos: sensitive dependence on the control parameter.

4. Conclusion

With the central focus on a broadened perspective of chaos, this article introduces the concept of the engineer's chaos, characterized by determinism, aperiodicity and sensitive dependence on initial conditions, and exhibiting a marked deviation from conventional chaotic systems by possessing a signal based control parameter. It is seen that multiplying two sinusoidal signals, the simplest possible nonlinear operation generates the engineer's chaos with the frequency ratio of the two signals acting as the control parameter. The nature of the chaos is observed using iterative map and bifurcation plot.

Following this, an application of the engineer's chaos to dense communications is demonstrated using a proof-of-concept, where the two chaos generating signals are amplitude modulated before multiplying, and transmitted over a channel represented as a Additive White Gaussian Noise of 18dB. The ability to receive and demodulate the signals with reasonable accuracy highlights the capabilities of the engineer's chaos. The demonstration proves how the engineer's chaos, the result of a broadened perspective on chaos yields triple advantages of minimalist and simple design, increased information carrying capacity, and security inherent to the sensitivity of chaos.

References

[1] Wolfram, Stephen. A new kind of science. Vol. 5. Champaign: Wolfram media, 2002.

[2] Strogatz, Steven H. Nonlinear dynamics and chaos: with applications to physics, biology, chemistry, and engineering. Westview press, 2014.

[3] Devaney, Robert L. A first course in chaotic dynamical systems. Westview Press, 1992.

[4] Bilotta, Eleonora, and Pietro Pantano. A gallery of Chua attractors. Singapore: World Scientific, 2008.

[5] McMahon, David. Quantum computing explained. John Wiley & Sons, 2007.

[6] Gibson, Jerry D., ed. Mobile communications handbook. CRC press, 2012.