International J.Math. Combin. Vol.2(2016), 1-15

Smarandache Curves and Applications According to Type-2 Bishop Frame in Euclidean 3-Space

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Abstract: In this paper, we investigate Smarandache curves according to type-2 Bishop frame in Euclidean 3- space and we give some differential geometric properties of Smarandache curves. Also, some characterizations of Smarandache breadth curves in Euclidean 3-space are presented. Besides, we illustrate examples of our results.

Key Words: Smarandache curves, Bishop frame, curves of constant breadth.

AMS(2010): 53A05, 53B25, 53B30.

§1. Introduction

A regular curve in Euclidean 3-space, whose position vector is composed by Frenet frame vectors on another regular curve, is called a Smarandache curve. M. Turgut and S. Yılmaz have defined a special case of such curves and call it Smarandache TB_2 curves in the space E_1^4 [10]. Moreover, special Smarandache curves have been investigated by some differential geometric [6]. A.T.Ali has introduced some special Smarandache curves in the Euclidean space [2]. Special Smarandache curves according to Sabban frame have been studied by [5]. Besides, It has been determined some special Smarandache curves E_1^3 by [12]. Curves of constant breadth were introduced by L.Euler [3].

We investigate position vector of curves and some characterizations case of constant breadth according to type-2 Bishop frame in E^3.

§2. Preliminaries

The Euclidean 3-space E^3 proved with the standard flat metric given by

$$<,>= dx_1^2 + dx_2^2 + dx_3^2$$

[1]Received November 26, 2015, Accepted May 6, 2016.

where (x_1, x_2, x_3) is rectangular coordinate system of E^3. Recall that, the norm of an arbitrary vector $a \in E^3$ given by $\|a\| = \sqrt{<a, a>}$. φ is called a unit speed curve if velocity vector υ of φ satisfied $\|\upsilon\| = 1$

The Bishop frame or parallel transport frame is alternative approach to defining a moving frame that is well defined even when the curve has vanishing second derivative. One can express parallel transport of orthonormal frame along a curve simply by parallel transporting each component of the frame [8]. The type-2 Bishop frame is expressed as

$$\begin{bmatrix} \xi_1' \\ \xi_2' \\ B' \end{bmatrix} = \begin{bmatrix} 0 & 0 & -\varepsilon_1 \\ 0 & 0 & -\varepsilon_2 \\ \varepsilon_1 & \varepsilon_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ B \end{bmatrix} \tag{2.1}$$

In order to investigate type-2 Bishop frame relation with Serret-Frenet frame, first we

$$B' = -\tau N = \varepsilon_1 \xi_1 + \varepsilon_2 \xi_2 \tag{2.2}$$

Taking the norm of both sides, we have

$$\kappa(s) = \frac{d\theta(s)}{ds}, \qquad \tau(s) = \sqrt{\varepsilon_1^2 + \varepsilon_2^2} \tag{2.3}$$

Moreover, we may express

$$\varepsilon_1(s) = -\tau \cos\theta(s), \qquad \varepsilon_2(s) = -\tau \sin\theta(s) \tag{2.4}$$

By this way, we conclude $\theta(s) = Arc\tan\dfrac{\varepsilon_2}{\varepsilon_1}$. The frame $\{\xi_1, \xi_2, B\}$ is properly oriented, and τ and $\theta(s) = \int\limits_0^s \kappa(s)ds$ are polar coordinates for the curve $\alpha(s)$.

We write the tangent vector according to frame $\{\xi_1, \xi_2, B\}$ as

$$T = \sin\theta(s)\xi_1 - \cos\theta(s)\xi_2$$

and differentiate with respect to s

$$\begin{aligned} T' = \kappa N = \quad & \theta'(s)(\cos\theta(s)\xi_1 + \sin\theta(s)\xi_2) \\ & + \sin\theta(s)\xi_1' - \cos\theta(s)\xi_2' \end{aligned} \tag{2.5}$$

Substituting $\xi_1' = -\varepsilon_1 B$ and $\xi_2' = -\varepsilon_2 B$ in equation (2.5) we have

$$\kappa N = \theta'(s)(\cos\theta(s)\xi_1 + \sin\theta(s)\xi_2)$$

In the above equation let us take $\theta'(s) = \kappa(s)$. So we immediately arrive at

$$N = \cos\theta(s)\xi_1 + \sin\theta(s)\xi_2$$

Considering the obtained equations, the relation matrix between Serret-Frenet and the type-2 Bishop frame can be expressed

$$\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} \sin\theta(s) & -\cos\theta(s) & 0 \\ \cos\theta(s) & \sin\theta(s) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ B \end{bmatrix} \tag{2.6}$$

§3. Smarandache Curves According to Type-2 Bishop Frame in $\mathbf{E}^3$

Let $\alpha = \alpha(s)$ be a unit speed regular curve in E^3 and denote by $\{\xi_1^\alpha, \xi_2^\alpha, B^\alpha\}$ the moving Bishop frame along the curve α. The following Bishop formulae is given by

$$\dot{\xi_1^\alpha} = -\varepsilon_1^\alpha B^\alpha, \quad \dot{\xi_2^\alpha} = -\varepsilon_2^\alpha B^\alpha, \quad \dot{B^\alpha} = \varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha$$

3.1 $\xi_1\xi_2$-Smarandache Curves

Definition 3.1 *Let $\alpha = \alpha(s)$ be a unit speed regular curve in E^3 and $\{\xi_1^\alpha, \xi_2^\alpha, B^\alpha\}$ be its moving Bishop frame. $\xi_1\xi_2$-Smarandache curves can be defined by*

$$\beta(s^*) = \frac{1}{\sqrt{2}}(\xi_1^\alpha + \xi_2^\alpha) \tag{3.1}$$

Now, we can investigate Bishop invariants of $\xi_1\xi_2$-Smarandache curves according to $\alpha = \alpha(s)$. Differentiating (3.1.1) with respect to s, we get

$$\begin{aligned} \dot{\beta} &= \frac{d\beta}{ds^*} \cdot \frac{ds^*}{ds} = \frac{-1}{\sqrt{2}}(\varepsilon_1^\alpha + \varepsilon_2^\alpha)B^\alpha \\ T_\beta \cdot \frac{ds^*}{ds} &= \frac{-1}{\sqrt{2}}(\varepsilon_1^\alpha + \varepsilon_2^\alpha)B^\alpha \end{aligned} \tag{3.2}$$

where

$$\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(\varepsilon_1^\alpha + \varepsilon_2^\alpha) \tag{3.3}$$

The tangent vector of curve β can be written as follow;

$$T_\beta = -B^\alpha = -(\varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha) \tag{3.4}$$

Differentiating (3.4) with respect to s, we obtain

$$\frac{dT_\beta}{ds^*} \cdot \frac{ds^*}{ds} = \varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha \tag{3.5}$$

Substituting (3.3) in (3.5), we get

$$T^{'}_{\beta} = \frac{\sqrt{2}}{\varepsilon_1^\alpha + \varepsilon_2^\alpha}(\varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha)$$

Then, the curvature and principal normal vector field of curve β are respectively,

$$\left\| T^{'}_{\beta} \right\| = \kappa_\beta = \frac{\sqrt{2}}{\varepsilon_1^\alpha + \varepsilon_2^\alpha}\sqrt{(\varepsilon_1^\alpha)^2 + (\varepsilon_2^\alpha)^2}$$

$$N_\beta = \frac{1}{\sqrt{(\varepsilon_1^\alpha)^2 + (\varepsilon_2^\alpha)^2}}(\varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha)$$

On the other hand, we express

$$B_\beta = \frac{1}{\sqrt{(\varepsilon_1^\alpha)^2 + (\varepsilon_2^\alpha)^2}} \det \begin{bmatrix} \xi_1^\alpha & \xi_2^\alpha & B^\alpha \\ 0 & 0 & -1 \\ \varepsilon_1^\alpha & \varepsilon_2^\alpha & 0 \end{bmatrix}.$$

So, the binormal vector of curve β is

$$B_\beta = \frac{1}{\sqrt{(\varepsilon_1^\alpha)^2 + (\varepsilon_2^\alpha)^2}}(\varepsilon_2^\alpha \xi_1^\alpha - \varepsilon_1^\alpha \xi_2^\alpha)$$

We differentiate $(3.2)_1$ with respect to s in order to calculate the torsion of curve β

$$\begin{aligned} \ddot{\beta} = \quad & \tfrac{-1}{\sqrt{2}}\{[(\varepsilon_1^\alpha)^2 + \varepsilon_1^\alpha \varepsilon_2^\alpha]\xi_1^\alpha \\ & + [\varepsilon_1^\alpha \varepsilon_2^\alpha + (\varepsilon_2^\alpha)^2]\xi_2^\alpha + [\dot{\varepsilon}_1^\alpha + \dot{\varepsilon}_2^\alpha]\}B^\alpha] \end{aligned}$$

and similarly

$$\dddot{\beta} = \frac{-1}{\sqrt{2}}(\delta_1 \xi_1^\alpha + \delta_2 \xi_2^\alpha + \delta_3 B^\alpha)$$

where

$$\delta_1 = \quad 3\varepsilon_1^\alpha \dot{\varepsilon}_1^\alpha + \varepsilon_1^\alpha \varepsilon_2^\alpha + 2\varepsilon_1^\alpha \dot{\varepsilon}_2^\alpha - (\varepsilon_1^\alpha)^3 - (\varepsilon_1^\alpha)^2 \varepsilon_2^\alpha$$

$$\delta_2 = \quad 2\dot{\varepsilon}_1^\alpha \varepsilon_2^\alpha + \varepsilon_1^\alpha \dot{\varepsilon}_2^\alpha + 3\varepsilon_2^\alpha \dot{\varepsilon}_2^\alpha - \varepsilon_1^\alpha (\varepsilon_2^\alpha)^2 - (\varepsilon_2^\alpha)^3$$

$$\delta_3 = \quad \ddot{\varepsilon}_1^\alpha + \ddot{\varepsilon}_2^\alpha$$

The torsion of curve β is

$$\tau_\beta = \quad \tfrac{\varepsilon_1^\alpha + \varepsilon_2^\alpha}{4\sqrt{2}[(\varepsilon_1^\alpha)^2 + (\varepsilon_2^\alpha)^2]}\{[(\varepsilon_1^\alpha + \varepsilon_2^\alpha)(\varepsilon_1^\alpha \varepsilon_2^\alpha + (\varepsilon_2^\alpha)^2]\delta_1 - [(\varepsilon_1^\alpha + \varepsilon_2^\alpha)((\varepsilon_1^\alpha)^2 + \varepsilon_1^\alpha \varepsilon_2^\alpha)]\delta_2\}$$

3.2 $\xi_1 B$-Smarandache Curves

Definition 3.2 *Let $\alpha = \alpha(s)$ be a unit speed regular curve in E^3 and $\{\xi_1^\alpha, \xi_2^\alpha, B^\alpha\}$ be its moving*

Bishop frame. $\xi_1 B$-Smarandache curves can be defined by

$$\beta(s^*) = \frac{1}{\sqrt{2}}(\xi_1^\alpha + B^\alpha) \tag{3.6}$$

Now, we can investigate Bishop invariants of $\xi_1 B$-Smarandache curves according to $\alpha = \alpha(s)$. Differentiating (3.6) with respect to s, we get

$$\begin{aligned} \dot{\beta} &= \frac{d\beta}{ds^*} \cdot \frac{ds^*}{ds} = \frac{-1}{\sqrt{2}}(\varepsilon_1^\alpha B^\alpha + \varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha) \\ T_\beta \cdot \frac{ds^*}{ds} &= \frac{-1}{\sqrt{2}}(-\varepsilon_1^\alpha B^\alpha + \varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha) \end{aligned} \tag{3.7}$$

where

$$\frac{ds^*}{ds} = \sqrt{\frac{2\left(\varepsilon_1^\alpha\right)^2 + \left(\varepsilon_2^\alpha\right)^2}{2}} \tag{3.8}$$

The tangent vector of curve β can be written as follow;

$$T_\beta = \frac{1}{\sqrt{2\left(\varepsilon_1^\alpha\right)^2 + \left(\varepsilon_2^\alpha\right)^2}}(\varepsilon_1^\alpha \xi_1^\alpha + \varepsilon_2^\alpha \xi_2^\alpha - \varepsilon_1^\alpha B^\alpha) \tag{3.9}$$

Differentiating (3.9) with respect to s, we obtain

$$\frac{dT_\beta}{ds^*}\frac{ds^*}{ds} = \frac{1}{\left[2\left(\varepsilon_1^\alpha\right)^2 + \left(\varepsilon_2^\alpha\right)^2\right]^{\frac{3}{2}}}(\mu_1 \xi_1^\alpha + \mu_2 \xi_2^\alpha + \mu_3 B^\alpha) \tag{3.10}$$

where

$$\begin{aligned} \mu_1 &= \varepsilon_1^\alpha \varepsilon_2^\alpha \dot{\varepsilon}_2^\alpha + \dot{\varepsilon}_1^\alpha \left(\varepsilon_2^\alpha\right)^2 \\ \mu_2 &= 2\left(\varepsilon_2^\alpha\right)^2 \varepsilon_2^\alpha - 2\varepsilon_1^\alpha \dot{\varepsilon}_1^\alpha \varepsilon_2^\alpha + 2\left(\varepsilon_1^\alpha\right)^2 \varepsilon_2^\alpha - 2\left(\varepsilon_1^\alpha\right)^3 \varepsilon_2^\alpha - \varepsilon_1^\alpha \left(\varepsilon_2^\alpha\right)^3 \\ \mu_3 &= \varepsilon_1^\alpha \varepsilon_2^\alpha \dot{\varepsilon}_2^\alpha - 2\left(\varepsilon_1^\alpha\right)^4 + \left(\varepsilon_1^\alpha\right)^2 \left(\varepsilon_2^\alpha\right)^2 - \dot{\varepsilon}_1^\alpha \left(\varepsilon_2^\alpha\right)^2 \end{aligned}$$

Substituting (3.8) in (3.10), we have

$$T_\beta^{'} = \frac{\sqrt{2}}{\left[2\left(\varepsilon_1^\alpha\right)^2 + \left(\varepsilon_2^\alpha\right)^2\right]^2}(\mu_1 \xi_1^\alpha + \mu_2 \xi_2^\alpha + \mu_3 B^\alpha)$$

Then, the first curvature and principal normal vector field of curve β are respectively

$$\left\|T_\beta^{'}\right\| = \kappa_\beta = \frac{\sqrt{2}}{\left[2\left(\varepsilon_1^\alpha\right)^2 + \left(\varepsilon_2^\alpha\right)^2\right]^2}\sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}$$

$$N_\beta = \frac{1}{\sqrt{\mu_1^2 + \mu_2^2 + \mu_3^2}}(\mu_1 \xi_1^\alpha + \mu_2 \xi_2^\alpha + \mu_3 B^\alpha)$$

On the other hand, we get

$$B_\beta = \frac{1}{\sqrt{\mu_1^2+\mu_2^2+\mu_3^2}\sqrt{2\left(\varepsilon_1^\alpha\right)^2+\left(\varepsilon_2^\alpha\right)^2}}[(\mu_2\varepsilon_1^\alpha+\mu_3\varepsilon_2^\alpha)\xi_1^\alpha$$

$$-\left(\mu_1\xi_1^\alpha+\mu_3\xi_1^\alpha\right)\xi_2^\alpha+\left(\mu_2\varepsilon_1^\alpha\text{-}\mu_1\varepsilon_2^\alpha\right)B^\alpha]$$

We differentiate (3.7) with respect to s in order to calculate the torsion of curve β

$$\ddot{\beta} = \tfrac{-1}{\sqrt{2}}\{[\text{-}2\left(\varepsilon_1^\alpha\right)^2+\dot{\varepsilon}_1^\alpha]\xi_1^\alpha + [-\varepsilon_1^\alpha\varepsilon_2^\alpha+\dot{\varepsilon}_1^\alpha-(\varepsilon_2^\alpha)^2]\xi_2^\alpha-\dot{\varepsilon}_1^\alpha B^\alpha\}$$

and similarly

$$\dddot{\beta} = \frac{-1}{\sqrt{2}}(\Gamma_1\xi_1^\alpha+\Gamma_2\xi_2^\alpha+\Gamma_3 B^\alpha)$$

where

$$\Gamma_1 = \text{-}6\varepsilon_1^\alpha\dot{\varepsilon}_1^\alpha+\ddot{\varepsilon}_1^\alpha+2\left(\varepsilon_1^\alpha\right)^3$$

$$\Gamma_2 = \text{-}2\dot{\varepsilon}_1^\alpha\varepsilon_2^\alpha\text{-}\varepsilon_1^\alpha\dot{\varepsilon}_2^\alpha+\ddot{\varepsilon}_2^\alpha\text{-}2\varepsilon_2^\alpha\varepsilon_2^\alpha+\varepsilon_1^\alpha\left(\varepsilon_2^\alpha\right)^2\text{-}\dot{\varepsilon}_2^\alpha\varepsilon_2^\alpha+\left(\varepsilon_2^\alpha\right)^3$$

$$\Gamma_3 = \text{-}\ddot{\varepsilon}_1^\alpha$$

The torsion of curve β is

$$\tau_\beta = \text{-}\frac{[2\left(\varepsilon_1^\alpha\right)^2+\left(\varepsilon_2^\alpha\right)^2]^4}{4\sqrt{2}(\mu_1^2+\mu_2^2+\mu_3^2)}\{[(\text{-}\varepsilon_1^\alpha\varepsilon_2^\alpha\text{-}\dot{\varepsilon}_2^\alpha+\left(\varepsilon_2^\alpha\right)^2)\Gamma_1$$

$$\text{-}2(\left(\varepsilon_1^\alpha\right)^2-\dot{\varepsilon}_1^\alpha)\Gamma_2+(\text{-}\varepsilon_1^\alpha\varepsilon_2^\alpha\text{-}\dot{\varepsilon}_2^\alpha+\left(\varepsilon_2^\alpha\right)^2)\Gamma_3]\varepsilon_1^\alpha$$

$$\text{-}[(\dot{\varepsilon}_1^\alpha-2\left(\varepsilon_1^\alpha\right)^2)\Gamma_3+\dot{\varepsilon}_1^\alpha\Gamma_1]\varepsilon_2^\alpha\}$$

3.3 $\xi_2 B$-Smarandache Curves

Definition 3.3 *Let $\alpha=\alpha(s)$ be a unit speed regular curve in E^3 and $\{\xi_1^\alpha,\xi_2^\alpha,B^\alpha\}$ be its moving Bishop frame. $\xi_2 B$-Smarandache curves can be defined by*

$$\beta(s^*) = \frac{1}{\sqrt{2}}(\xi_2^\alpha+B^\alpha) \tag{3.11}$$

Now, we can investigate Bishop invariants of $\xi_2 B$-Smarandache curves according to $\alpha=\alpha(s)$. Differentiating (3.11) with respect to s, we get

$$\begin{aligned}\dot{\beta} &= \frac{d\beta}{ds^*}\cdot\frac{ds^*}{ds} = (-\varepsilon_2^\alpha B^\alpha+\varepsilon_1^\alpha\xi_1^\alpha+\varepsilon_2^\alpha\xi_2^\alpha)\\ T_\beta\cdot\frac{ds^*}{ds} &= (\varepsilon_1^\alpha\xi_1^\alpha+\varepsilon_2^\alpha\xi_2^\alpha-\varepsilon_2^\alpha B^\alpha)\end{aligned} \tag{3.12}$$

where

$$\frac{ds^*}{ds}=\sqrt{\frac{\left(\varepsilon_1^\alpha\right)^2+2\left(\varepsilon_2^\alpha\right)^2}{2}} \tag{3.13}$$

The tangent vector of curve β can be written as follow;

$$T_\beta=\frac{\varepsilon_1^\alpha\xi_1^\alpha+\varepsilon_2^\alpha\xi_2^\alpha-\varepsilon_2^\alpha B^\alpha}{\sqrt{2\left(\varepsilon_1^\alpha\right)^2+\left(\varepsilon_2^\alpha\right)^2}} \tag{3.14}$$

Differentiating (3.14) with respect to s, we obtain

$$\frac{dT_\beta}{ds^*}\frac{ds^*}{ds}=\frac{1}{\left[\left(\varepsilon_1^\alpha\right)^2+2\left(\varepsilon_2^\alpha\right)^2\right]^{\frac{3}{2}}}\left(\eta_1\xi_1^\alpha+\eta_2\xi_2^\alpha+\eta_3B^\alpha\right) \tag{3.15}$$

where

$$\eta_1=\quad 2(\dot{\varepsilon}_1^\alpha\left(\varepsilon_2^\alpha\right)^2-\varepsilon_1^\alpha\varepsilon_2^\alpha)$$

$$\eta_2=\quad \left(\varepsilon_2^\alpha\right)^2\dot{\varepsilon}_2^\alpha+\left(\varepsilon_1^\alpha\right)^2\dot{\varepsilon}_1^\alpha-\varepsilon_1^\alpha\dot{\varepsilon}_1^\alpha\varepsilon_2^\alpha$$

$$\eta_3=\quad \left(\varepsilon_1^\alpha\right)^2\varepsilon_2^\alpha+2\left(\varepsilon_2^\alpha\right)^3-\left(\varepsilon_1^\alpha\right)^4-2\left(\varepsilon_1^\alpha\right)^4-3\left(\varepsilon_1^\alpha\right)^2\left(\varepsilon_2^\alpha\right)^2$$

Substituting (3.13) in (3.15), we have

$$T_\beta^{\prime}=\frac{\sqrt{2}}{\left[2\left(\varepsilon_1^\alpha\right)^2+\left(\varepsilon_2^\alpha\right)^2\right]^2}\left(\eta_1\xi_1^\alpha+\eta_2\xi_2^\alpha+\eta_3B^\alpha\right)$$

Then, the first curvature and principal normal vector field of curve β are respectively

$$\left\|T_\beta^{\prime}\right\|=\kappa_\beta=\frac{\sqrt{2}\sqrt{\eta_1^2+\eta_2^2+\eta_3^2}}{\left[\left(\varepsilon_1^\alpha\right)^2+2\left(\varepsilon_2^\alpha\right)^2\right]^2}$$

$$N_\beta=\frac{1}{\sqrt{\eta_1^2+\eta_2^2+\eta_3^2}}\left(\eta_1\xi_1^\alpha+\eta_2\xi_2^\alpha+\eta_3B^\alpha\right)$$

On the other hand, we express

$$\begin{aligned} B_\beta= \quad & \frac{1}{\sqrt{\eta_1^2+\eta_2^2+\eta_3^2}\sqrt{\left(\varepsilon_1^\alpha\right)^2+2\left(\varepsilon_2^\alpha\right)^2}}[(\eta_2\varepsilon_2^\alpha+\eta_3\varepsilon_2^\alpha)\xi_1^\alpha \\ & -\left(\eta_1\xi_2^\alpha+\eta_3\xi_1^\alpha\right)\xi_2^\alpha+\left(\eta_2\varepsilon_1^\alpha-\eta_1\varepsilon_2^\alpha\right)B^\alpha]\end{aligned}$$

We differentiate $(3.12)_1$ with respect to s in order to calculate the torsion of curve β

$$\begin{aligned}\ddot{\beta}= \quad & \tfrac{1}{\sqrt{2}}\{[\varepsilon_1^\alpha\xi_1^\alpha+\dot{\varepsilon}_1^\alpha-\left(\varepsilon_1^\alpha\right)^2]\xi_1^\alpha \\ & +[\dot{\varepsilon}_2^\alpha-2\left(\varepsilon_2^\alpha\right)^2]\xi_2^\alpha-\dot{\varepsilon}_2^\alpha B^\alpha\}\end{aligned}$$

and similarly

$$\dddot{\beta} = \frac{1}{\sqrt{2}}(\eta_1\xi_1^\alpha + \eta_2\xi_2^\alpha + \eta_3 B^\alpha)$$

where

$$\begin{aligned}
\eta_1 &= -\dot{\varepsilon}_1^\alpha\varepsilon_2^\alpha - 5\dot{\varepsilon}_1^\alpha\varepsilon_1^\alpha + \ddot{\varepsilon}_1^\alpha + (\varepsilon_1^\alpha)^2\varepsilon_2^\alpha + (\varepsilon_1^\alpha)^3 \\
\eta_2 &= -4\dot{\varepsilon}_2^\alpha\varepsilon_2^\alpha + \ddot{\varepsilon}_2^\alpha + 2\varepsilon_2^\alpha \\
\eta_3 &= -\ddot{\varepsilon}_2^\alpha
\end{aligned}$$

The torsion of curve β is

$$\begin{aligned}
\tau_\beta = & -\frac{[(\varepsilon_1^\alpha)^2 + 2(\varepsilon_2^\alpha)^2]^4}{4\sqrt{2}(\eta_1^2+\eta_2^2+\eta_3^2)}\{[\dot{\varepsilon}_2^\alpha\eta_2 + (\dot{\varepsilon}_2^\alpha - 2(\varepsilon_2^\alpha)^2)\eta_3]\varepsilon_1^\alpha \\
& +[2(\varepsilon_2^\alpha)^2\eta_1 + (\varepsilon_1^\alpha\varepsilon_2^\alpha - \dot{\varepsilon}_1^\alpha + (\varepsilon_1^\alpha)^2)\eta_2 \\
& +(-\varepsilon_1^\alpha\varepsilon_2^\alpha + \dot{\varepsilon}_1^\alpha)\eta_3]\varepsilon_2^\alpha\}
\end{aligned}$$

3.4 $\xi_1\xi_2B$-Smarandache Curves

Definition 3.4 *Let $\alpha = \alpha(s)$ be a unit speed regular curve in E^3 and $\{\xi_1^\alpha, \xi_2^\alpha, B^\alpha\}$ be its moving Bishop frame. $\xi_1^\alpha\xi_2B$-Smarandache curves can be defined by*

$$\beta(s^*) = \frac{1}{\sqrt{3}}(\xi_1^\alpha + \xi_2^\alpha + B^\alpha) \tag{3.16}$$

Now, we can investigate Bishop invariants of $\xi_1^\alpha\xi_2B$-Smarandache curves according to $\alpha = \alpha(s)$. Differentiating (3.16) with respect to s, we get

$$\begin{aligned}
\dot{\beta} &= \frac{d\beta}{ds^*}\cdot\frac{ds^*}{ds} = \tfrac{1}{\sqrt{3}}[(\varepsilon_1^\alpha + \varepsilon_2^\alpha)B^\alpha - \varepsilon_1^\alpha\xi_1^\alpha - \varepsilon_2^\alpha\xi_2^\alpha] \\
T_\beta\cdot\frac{ds^*}{ds} &= \tfrac{1}{\sqrt{3}}[(\varepsilon_1^\alpha + \varepsilon_2^\alpha)B^\alpha - \varepsilon_1^\alpha\xi_1^\alpha - \varepsilon_2^\alpha\xi_2^\alpha)]
\end{aligned} \tag{3.17}$$

where

$$\frac{ds^*}{ds} = \sqrt{\frac{2[(\varepsilon_1^\alpha)^2 + \varepsilon_1^\alpha\varepsilon_2^\alpha + (\varepsilon_2^\alpha)^2]}{3}} \tag{3.18}$$

The tangent vector of curve β can be written as follow;

$$T_\beta = \frac{\varepsilon_1^\alpha\xi_1^\alpha + \varepsilon_2^\alpha\xi_2^\alpha - (\varepsilon_1^\alpha + \varepsilon_2^\alpha)B^\alpha}{\sqrt{2[(\varepsilon_1^\alpha)^2 + \varepsilon_1^\alpha\varepsilon_2^\alpha + (\varepsilon_2^\alpha)^2]}} \tag{3.19}$$

Differentiating (3.19) with respect to s, we get

$$\frac{dT_\beta}{ds^*}\frac{ds^*}{ds}=\frac{(\lambda_1\xi_1^\alpha+\lambda_2\xi_2^\alpha+\lambda_3B^\alpha)}{2\sqrt{2}\left[(\varepsilon_1^\alpha)^2+\varepsilon_1^\alpha\varepsilon_2^\alpha+(\varepsilon_2^\alpha)^2\right]^{\frac{3}{2}}} \tag{3.20}$$

where

$$\begin{aligned}
\lambda_1= & [\dot{\varepsilon}_1^\alpha\text{-}2\,(\varepsilon_1^\alpha)^2\text{-}\varepsilon_1^\alpha\varepsilon_2^\alpha]u(s)\text{-}\varepsilon_1^\alpha[2\varepsilon_1^\alpha\dot{\varepsilon}_1^\alpha+\dot{\varepsilon}_1^\alpha\varepsilon_2^\alpha+\varepsilon_1^\alpha\varepsilon_2^\alpha+2\dot{\varepsilon}_2^\alpha\varepsilon_2^\alpha] \\
\lambda_2= & [\dot{\varepsilon}_2^\alpha\text{-}2\,(\varepsilon_2^\alpha)^2\text{-}\varepsilon_1^\alpha\varepsilon_2^\alpha]u(s)\text{-}\varepsilon_2^\alpha[\dot{\varepsilon}_1^\alpha+\varepsilon_1^\alpha\dot{\varepsilon}_2^\alpha+2\varepsilon_2^\alpha\dot{\varepsilon}_2^\alpha] \\
\lambda_3= & [\text{-}\dot{\varepsilon}_1^\alpha\text{-}\dot{\varepsilon}_2^\alpha]u(s)+\varepsilon_1^\alpha[2\varepsilon_1^\alpha\dot{\varepsilon}_1^\alpha+3\dot{\varepsilon}_1^\alpha\varepsilon_2^\alpha+\varepsilon_1^\alpha\dot{\varepsilon}_2^\alpha+2\dot{\varepsilon}_2^\alpha\varepsilon_2^\alpha] \\
& +\dot{\varepsilon}_2^\alpha[\dot{\varepsilon}_1^\alpha\,(\varepsilon_2^\alpha)^2+2\,(\varepsilon_2^\alpha)^2]
\end{aligned}$$

Substituting (3.18) in (3.20), we have

$$T_\beta^{'}=\frac{\sqrt{3}(\lambda_1\xi_1^\alpha+\lambda_2\xi_2^\alpha+\lambda_3B^\alpha)}{4\left[(\varepsilon_1^\alpha)^2+\varepsilon_1^\alpha\varepsilon_2^\alpha+(\varepsilon_2^\alpha)^2\right]^2}$$

Then, the first curvature and principal normal vector field of curve β are respectively

$$\begin{aligned}
&\left\|T_\beta^{'}\right\|=\kappa_\beta=\frac{\sqrt{3}\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}{4\left[(\varepsilon_1^\alpha)^2+\varepsilon_1^\alpha\varepsilon_2^\alpha+(\varepsilon_2^\alpha)^2\right]^2} \\
&N_\beta=\frac{1}{\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}(\lambda_1\xi_1^\alpha+\lambda_2\xi_2^\alpha+\lambda_3B^\alpha)
\end{aligned} \tag{3.21}$$

On the other hand, we express

$$B_\beta=\frac{1}{\sqrt{2[(\varepsilon_1^\alpha)^2+\varepsilon_1^\alpha\varepsilon_2^\alpha+(\varepsilon_2^\alpha)^2]}\cdot\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}\det\begin{bmatrix}\xi_1^\alpha & \xi_2^\alpha & B^\alpha\\ \varepsilon_1^\alpha & \varepsilon_2^\alpha & \text{-}(\varepsilon_1^\alpha+\varepsilon_2^\alpha)\\ \lambda_1 & \lambda_2 & \lambda_3\end{bmatrix}$$

So, the binormal vector field of curve β is

$$\begin{aligned}
B_\beta=&\frac{1}{\sqrt{2[(\varepsilon_1^\alpha)^2+\varepsilon_1^\alpha\varepsilon_2^\alpha+(\varepsilon_2^\alpha)^2]}\cdot\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}\{[(\varepsilon_1^\alpha+\varepsilon_2^\alpha)\lambda_1 \\
&-\varepsilon_2^\alpha\lambda_3]\xi_1^\alpha+[-\varepsilon_1^\alpha\lambda_3\text{-}(\varepsilon_1^\alpha+\varepsilon_2^\alpha)]\xi_2^\alpha+[\varepsilon_1^\alpha\lambda_2\text{-}\varepsilon_2^\alpha\lambda_1]B^\alpha\}
\end{aligned}$$

We differentiate (3.20) with respect to s in order to calculate the torsion of curve β

$$\begin{aligned}
\ddot{\beta}=&\text{-}\tfrac{1}{\sqrt{3}}\{[2\,(\varepsilon_1^\alpha)^2+\varepsilon_1^\alpha\xi_1^\alpha\text{-}\dot{\varepsilon}_1^\alpha]\xi_1^\alpha \\
&+[2\,(\varepsilon_2^\alpha)^2+\varepsilon_1^\alpha\varepsilon_2^\alpha\text{-}\dot{\varepsilon}_2^\alpha]\xi_2^\alpha+[\dot{\varepsilon}_1^\alpha+\dot{\varepsilon}_2^\alpha]B^\alpha\}
\end{aligned}$$

and similarly

$$\dddot{\beta} = -\frac{1}{\sqrt{3}}(\sigma_1\xi_1^\alpha + \sigma_2\xi_2^\alpha + \sigma_3 B^\alpha)$$

where

$$\begin{aligned}
\eta_1 = &\quad 4\dot{\varepsilon}_1^\alpha\varepsilon_1^\alpha + 3\dot{\varepsilon}_1^\alpha\varepsilon_2^\alpha - \ddot{\varepsilon}_1^\alpha - 2\left(\varepsilon_1^\alpha\right)^3 - \left(\varepsilon_1^\alpha\right)^2\varepsilon_2^\alpha \\
\eta_2 = &\quad 5\dot{\varepsilon}_2^\alpha\varepsilon_2^\alpha + \dot{\varepsilon}_1^\alpha\varepsilon_2^\alpha + \varepsilon_1^\alpha\varepsilon_2^\alpha - \ddot{\varepsilon}_2^\alpha - 2\left(\varepsilon_2^\alpha\right)^3 - \varepsilon_1^\alpha\left(\varepsilon_2^\alpha\right)^2 \\
\eta_3 = &\quad \ddot{\varepsilon}_2^\alpha + \ddot{\varepsilon}_2^\alpha
\end{aligned}$$

The torsion of curve β is

$$\begin{aligned}
\tau_\beta = &\quad -\frac{16[(\varepsilon_1^\alpha)^2 + \varepsilon_1^\alpha\varepsilon_2^\alpha + (\varepsilon_2^\alpha)^2]^2}{9\sqrt{3}\sqrt{\lambda_1^2+\lambda_2^2+\lambda_3^2}}\{[(2\left(\varepsilon_2^\alpha\right)^2 + \varepsilon_1^\alpha\varepsilon_2^\alpha - \dot{\varepsilon}_2^\alpha)\sigma_1 + (-\dot{\varepsilon}_2^\alpha - 2\left(\varepsilon_1^\alpha\right)^2 - \varepsilon_1^\alpha\varepsilon_2^\alpha)\sigma_2 \\
&+ (2\left(\varepsilon_2^\alpha\right)^2 + \varepsilon_1^\alpha\varepsilon_2^\alpha - \dot{\varepsilon}_2^\alpha)\sigma_3]\varepsilon_1^\alpha + [-\dot{\varepsilon}_1^\alpha - 2\dot{\varepsilon}_2^\alpha + 2\left(\varepsilon_2^\alpha\right)^2 + \varepsilon_1^\alpha\varepsilon_2^\alpha)\sigma_1 \\
&+ (-2\left(\varepsilon_1^\alpha\right)^2 - \varepsilon_1^\alpha\varepsilon_2^\alpha + \dot{\varepsilon}_1^\alpha)\sigma_2 + (2\left(\varepsilon_1^\alpha\right)^2 + \varepsilon_1^\alpha\varepsilon_2^\alpha - \dot{\varepsilon}_1^\alpha)\sigma_3]\varepsilon_2^\alpha\}.
\end{aligned}$$

§4. Smarandache Breadth Curves According to Type-2 Bishop Frame in $\mathbf{E}^3$

A regular curve with more than 2 breadths in Euclidean 3-space is called Smarandache breadth curve.

Let $\alpha = \alpha(s)$ be a Smarandache breadth curve. Moreover, let us suppose $\alpha = \alpha(s)$ simple closed space-like curve in the space E^3. These curves will be denoted by (C). The normal plane at every point P on the curve meets the curve at a single point Q other than P.

We call the point Q the opposite point P. We consider a curve in the class Γ as in having parallel tangents ξ_1 and ξ_1^* opposite directions at opposite points α and $\alpha^*$of the curves.

A simple closed curve having parallel tangents in opposite directions at opposite points can be represented with respect to type-2 Bishop frame by the equation

$$\alpha^*(s) \quad = \quad \alpha(s) + \lambda\xi_1 + \varphi\xi_2 + \eta B \tag{4.1}$$

where $\lambda(s), \varphi(s)$ and $\eta(s)$ are arbitrary functions also α and α^* are opposite points.

Differentiating both sides of (4.1) and considering type-2 Bishop equations, we have

$$\begin{aligned}
\frac{d\alpha^*}{ds} = \xi_1^*\frac{ds^*}{ds} = &\quad (\frac{d\lambda}{ds} + \eta\varepsilon_1 + 1)\xi_1 + (\frac{d\varphi}{ds} + \eta\varepsilon_2)\xi_2 \\
&+ (-\lambda\varepsilon_1 - \varphi\varepsilon_2 + \frac{d\eta}{ds})B
\end{aligned} \tag{4.2}$$

Since $\xi_1^* = -\xi_1$ rewriting (4.2) we have

$$\begin{aligned} \frac{d\lambda}{ds} &= -\eta\varepsilon_1 - 1 - \frac{ds^*}{ds} \\ \frac{d\varphi}{ds} &= -\varphi\varepsilon_2 \\ \frac{d\eta}{ds} &= \lambda\varepsilon_1 + \varphi\varepsilon_2 \end{aligned} \tag{4.3}$$

If we call θ as the angle between the tangent of the curve (C) at point $\alpha(s)$ with a given direction and consider $\dfrac{d\theta}{ds} = \kappa$, we have (4.3) as follow:

$$\begin{aligned} \frac{d\lambda}{d\theta} &= -\eta\frac{\varepsilon_1}{\kappa} - f(\theta) \\ \frac{d\varphi}{d\theta} &= -\varphi\frac{\varepsilon_2}{\kappa} \\ \frac{d\eta}{d\theta} &= \lambda\frac{\varepsilon_1}{\kappa} + \varphi\frac{\varepsilon_2}{\kappa} \end{aligned} \tag{4.4}$$

where $f(\theta) = \delta + \delta^*$, $\delta = \dfrac{1}{\kappa}$, $\delta^* = \dfrac{1}{\kappa^*}$ denote the radius of curvature at α and α^* respectively. And using system (4.4), we have the following differential equation with respect to λ as

$$\begin{aligned} &\frac{d^3\lambda}{d\theta^3} - [\frac{\kappa}{\varepsilon_1}\frac{d}{d\theta}(\frac{\varepsilon_1}{\kappa})]\frac{d^2\lambda}{d\theta^2} + [\frac{\varepsilon_1^2}{\kappa^2} - \frac{\varepsilon_1}{\kappa} - \frac{d}{d\theta}(\frac{\kappa}{\varepsilon_1})\frac{d}{d\theta}(\frac{\varepsilon_1}{\kappa}) \\ &-\frac{\kappa}{\varepsilon_1}\frac{d^2}{d\theta^2}(\frac{\varepsilon_1}{\kappa})]\frac{d\lambda}{d\theta} + [\frac{\varepsilon_1}{\kappa}\frac{d}{d\theta}(\frac{\varepsilon_1}{\kappa}) - \frac{\varepsilon_1^2}{\varepsilon_2\kappa}]\lambda + \\ &+[-\frac{\kappa}{\varepsilon_2} - \frac{\kappa}{\varepsilon_1}\frac{d}{d\theta}(\frac{\varepsilon_1}{\kappa})]\frac{d^2 f}{d\theta^2} - [\frac{\kappa}{\varepsilon_2} + 2\frac{\kappa}{\varepsilon_1}\frac{d}{d\theta}(\frac{\varepsilon_1}{\kappa})]\frac{df}{d\theta} \\ &-[\frac{\varepsilon_2^2}{\varepsilon_1\kappa} + \frac{\varepsilon_1}{\varepsilon_2} + 2\frac{d}{d\theta}(\frac{\kappa}{\varepsilon_1})\frac{d}{d\theta}(\frac{\varepsilon_1}{\kappa}) + \frac{\kappa}{\varepsilon_1}\frac{d^2}{d\theta^2}(\frac{\varepsilon_1}{\kappa})]f(\theta) = 0 \end{aligned} \tag{4.5}$$

Equation (4.5) is characterization for α^*. If the distance between opposite points of (C) and (C^*) is constant, then we can write that

$$\|\alpha^* - \alpha\| = \lambda^2 + \varphi^2 + \eta^2 = l^2 = \text{constant} \tag{4.6}$$

Hence, we write

$$\lambda\frac{d\lambda}{d\theta} + \varphi\frac{d\varphi}{d\theta} + \eta\frac{d\eta}{d\theta} = 0 \tag{4.7}$$

Considering system (4.4) we obtain

$$\lambda \cdot f(\theta) = 0 \tag{4.8}$$

We write $\lambda = 0$ or $f(\theta) = 0$. Thus, we shall study in the following subcases.

Case 1. $\lambda = 0$. Then we obtain

$$\eta = -\int_0^\theta \frac{\kappa}{\varepsilon_1} f(\theta)d\theta, \qquad \varphi = \int_0^\theta (\int_0^\theta \eta \frac{\varepsilon_2}{\kappa} d\theta) \frac{\varepsilon_2}{\kappa} d\theta \tag{4.9}$$

and

$$\frac{d^2 f}{d\theta^2} - \frac{df}{d\theta} - [(\frac{\tau}{\kappa})^2 \frac{\sin^3\theta}{\cos\theta} - \frac{\tau}{\kappa}\cos\theta]f = 0 \tag{4.10}$$

General solution of (4.10) depends on character of $\frac{\tau}{\kappa}$. Due to this, we distinguish following subcases.

Subcase 1.1 $f(\theta) = 0$. then we obtain

$$\begin{aligned} \lambda &= \int_0^\theta \eta \frac{\varepsilon_1}{\kappa} d\theta \\ \varphi &= -\int_0^\theta \eta \frac{\varepsilon_2}{\kappa} d\theta \\ \eta &= \int_0^\theta \lambda \frac{\varepsilon_1}{\kappa} d\theta + \int_0^\theta \varphi \frac{\varepsilon_2}{\kappa} d\theta \end{aligned} \tag{4.11}$$

Case 2. Let us suppose that $\lambda \neq 0$, $\varphi \neq 0$,$\eta \neq 0$ and λ, φ, η constant. Thus the equation (4.4) we obtain $\frac{\varepsilon_1}{\kappa} = 0$ and $\frac{\varepsilon_2}{\kappa} = 0$.

Moreover, the equation (4.5) has the form $\frac{d^3\lambda}{d\theta^3} = 0$ The solution (4.12) is $\lambda = L_1\frac{\theta^2}{2} + L_2\theta + L_3$ where L_1, L_2 and L_3 real numbers. And therefore we write the position vector ant the curvature

$$\alpha^* = \alpha + A_1\xi_1 + A_2\xi_2 + A_3 B$$

where $A_1 = \lambda$, $A_2 = \varphi$ and $A_3 = \eta$ real numbers. And the distance between the opposite points of (C) and (C^*) is

$$\|\alpha^* - \alpha\| = A_1^2 + A_2^2 + A_3^2 = \text{constant}$$

§5. Examples

In this section, we show two examples of Smarandache curves according to Bishop frame in E^3.

Example 5.1 First, let us consider a unit speed curve of E^3 by

$$\begin{aligned} \beta(s) = \; & (\frac{25}{306}\sin(9s) - \frac{9}{850}\sin(25s), \\ & -\frac{25}{306}\cos(9s) + \frac{9}{850}\cos(25s), \frac{15}{136}\sin(8s)) \end{aligned}$$

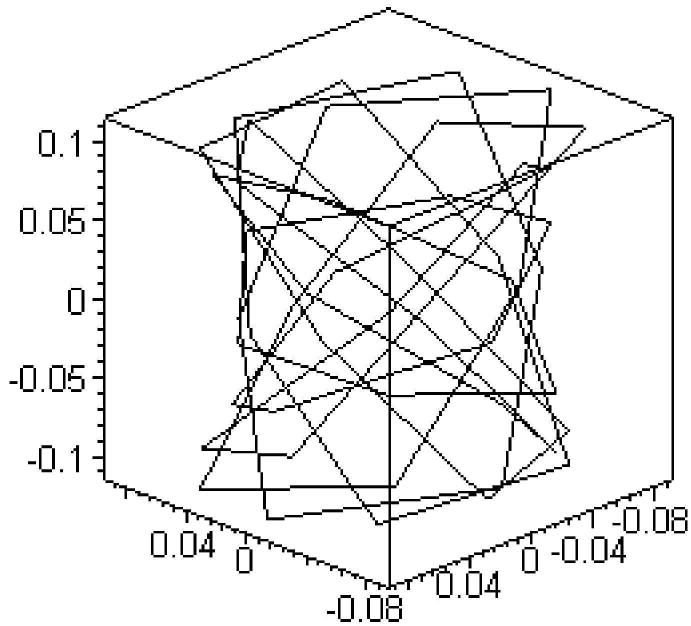

Fig.1 The curve $\beta = \beta(s)$

See the curve $\beta(s)$ in Fig.1. One can calculate its Serret-Frenet apparatus as the following

$$T = (\tfrac{25}{34}\cos 9s + \tfrac{9}{34}\cos 25s, \tfrac{25}{34}\sin 9s - \tfrac{9}{34}\sin 25s, \tfrac{15}{17}\cos 8s)$$

$$N = (\tfrac{15}{34}\csc 8s(\sin 9s - \sin 25s), -\tfrac{15}{34}\csc 8s(\cos 9s - \cos 25s), \tfrac{8}{17})$$

$$B = (\tfrac{1}{34}(25\sin 9s - 9\sin 25s), -\tfrac{1}{34}(25\cos 9s + 9\cos 25s), -\tfrac{15}{17}\sin 8s)$$

$$\kappa = -15\sin 8s \text{ and } \tau = 15\cos 8s$$

In order to compare our main results with Smarandache curves according to Serret-Frenet frame, we first plot classical Smarandache curve of β Fig.1.

Now we focus on the type-2 Bishop trihedral. In order to form the transformation matrix (2.6), let us express

$$\theta(s) = -\int_0^s 15\sin(8s)ds = \frac{15}{8}\cos(8s)$$

Since, we can write the transformation matrix

$$\begin{bmatrix} T \\ N \\ B \end{bmatrix} = \begin{bmatrix} \sin(\frac{15}{8}\cos 8s) & -\cos(\frac{15}{8}\cos 8s) & 0 \\ \cos(\frac{15}{8}\cos 8s) & \sin(\frac{15}{8}\cos 8s) & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \xi_1 \\ \xi_2 \\ B \end{bmatrix}$$

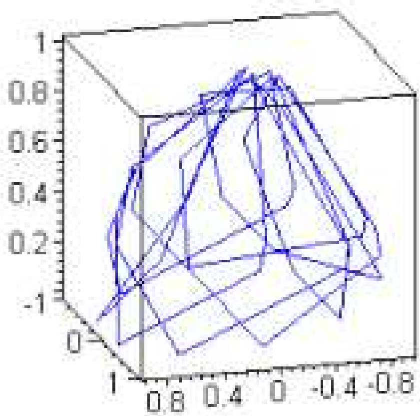

Fig.2 $\xi_1\xi_1$ Smarandache curve

By the method of Cramer, one can obtain type-2 Bishop frame of β as follows

$$\begin{aligned}\xi_1= \quad &(\sin\theta(\tfrac{25}{34}\cos 9s-\tfrac{9}{34}\cos 25s)+\tfrac{15}{34}\cos\theta\csc 8s(\sin 9s-\sin 25s),\\ &\sin\theta(\tfrac{25}{34}\sin 9s-\tfrac{9}{34}\sin 25s)-\tfrac{15}{34}\cos\theta\csc 8s(\cos 9s-\cos 25s),\\ &\tfrac{15}{17}\sin\theta\cos 8s+\tfrac{8}{17}\cos\theta)\end{aligned}$$

$$\begin{aligned}\xi_2= \quad &(-\cos\theta(\tfrac{25}{34}\cos 9s-\tfrac{9}{34}\cos 25s)+\tfrac{15}{34}\sin\theta\csc 8s(\sin 9s-\sin 25s),\\ &-\cos\theta(\tfrac{25}{34}\sin 9s-\tfrac{9}{34}\sin 25s)-\tfrac{15}{34}\sin\theta\csc 8s(\cos 9s-\cos 25s),\\ &-\tfrac{15}{17}\cos\theta\cos 8s+\tfrac{8}{17}\sin\theta)\end{aligned}$$

$$B= \quad (\tfrac{1}{34}(25\sin 9s-9\sin 25s),-\tfrac{1}{34}(25\cos 9s+9\cos 25s),-\tfrac{15}{17}\sin 8s)$$

where $\theta=\frac{15}{8}\cos(8s)$. So, we have Smarandache curves according to type-2 Bishop frame of the unit speed curve $\beta=\alpha(s)$, see Fig.2-4 and Fig.5.

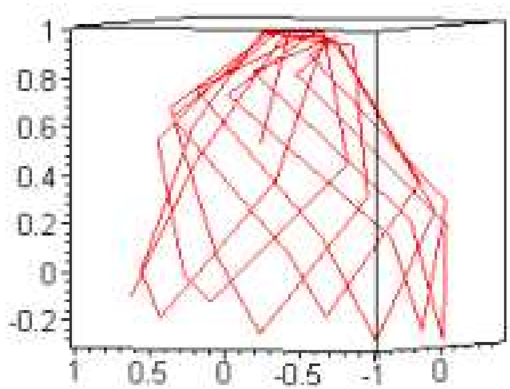

Fig.3 $\xi_1 B$ Smarandache curve

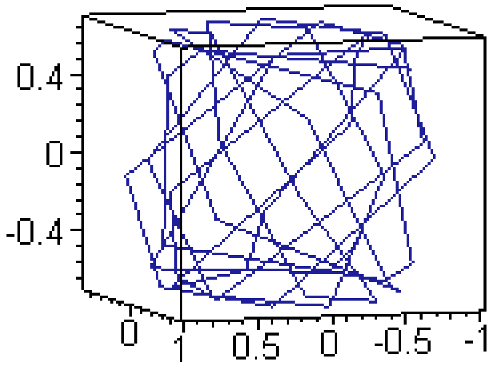

Fig.4 $\xi_2 B$ Smarandache curve

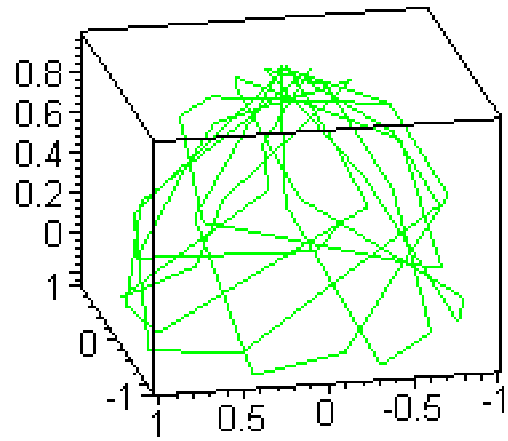

Fig.5 $\xi_1\xi_2 B$ Smarandache curve

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