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Some Studies in Neutrosophic Graphs

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Abstract. The main purpose of this paper is to discuss the notion of neutrosophic graphs, weak isomorphisms, co-weak isomorphisms and isomorphisms between two neutrosophic graphs. It is proved that the isomorphism between the two neutrosophic graphs is an equivalence relation. Some other properties of morphisms are also discussed in this paper.

Keywords: Neutrosophic graphs, Weak isomorphisms, Co-weak isomorphisms, Equivalence relation and Isomorphisms.

1 Introduction

Graph theory has its origins in a 1736 paper by the celebrated mathematician Leonhard Euler (10), known as the father of graph theory, when he settled a famous unsolved problem known as Ko¨ nigsburg Bridge problem. Graph theory is considered as a part of combinatorial mathematics. The theory has greatly contributed to our understanding of communication theory, programming, civil engineering, switching circuits, architecture, operational research, economics linguistic, psychology and anthropology. A graph is also used to create a relationship between a given set of objects. Each object can be represented by a vertex and the relationship between them can be represented by an edge.

In 1965, L.A. Zadeh (22) published the first paper on his new theory of fuzzy sets and systems. A fuzzy set is an extension of classical set theory. His work proved to be a mathematical tool for explaining the concept of uncertainty in real life problems. A fuzzy set is characterized by a membership function which assigns to each object a grade of membership ranging between zero and one. Azriel Rosenfeld (18) introduced the of notion of fuzzy graphs in 1975 and proposed another definitions including paths, cycles. connectedness etc. Mordeson and Peng (15) studied operations on fuzzy graphs. Many researchers contributed a lot and gave some more generalized forms of fuzzy graphs which have been studied in (6) and (8). These contributions show a new dimension of graph theory.

F. Smarandache (20) introduced the notion of neutrosophic set which is useful for dealing real life problems having imprecise, indeterminacy and inconsistant data. The theory is generalization of classical sets and fuzzy sets and is applied in decision making problems, control theory, medicines, topology and in many more real life problems. The notion of neutrosphic soft graph is introduced by N. Shah and A. Hussain (19). In the present paper neutrosophic graphs, their types, different operations like union intersection complement are definend. Furthermore different morphisms such as weak isomorphisms, co-weak isomorphism and isomorphisms are defined. Some theorems on morphisms are also proven here. This paper has been arranged as the following; In section 2, some basic concepts about graphs and neutrosophic sets are presented which will be employed in later sections. In section 3, concept of neutrosophic graphs is given and some of their fundamental properties have been studied. In section 4, concept of strong neutrosophic graphs and its complement is studied. Section 5 is devoted for the study of morphisms of neutrosophic graphs. Conclusions are also given at the end of Section 5.

2 PRELIMINARIES

In this section, some definitions about graphs and neutrosophic sets are given. These will be helpful in later sections.

2.1 Definition (21) A graph G^* consists of set of finite objects $V=\{v_1,v_2,v_3....,v_n\}$ called vertices (also called points or nodes) and other set $E=\{e_1,e_2,e_3....,e_n\}$ whose element are called edges (also called lines or arcs).

Usually a graph is denoted as $G^*=(V,E)$. Let G^* be a graph and e $=\{u,v\}$ be an edge of G^* . Since $\{u,v\}$ is 2-element set, we may write $\{u,v\}$ instead of $\{v,u\}$. It is often more convenient to represent this edge by uv or vu.

2.2 Definition (21) An edge of a graph that joins a vertex to itself is called loop.

2.3 Definition (21) In a multigraph no loops are allowed but more than one edge can join two vertices, these edges are called multiple edges or parallel edges and a graph is called multigraph.

2.4 Definition (21) A Graph which has neither loops nor multiple edges is called a simple graph.

2.5 Definition (21) A sub graph H^* of G^* is a graph having all of its vertices and edges in G^*.

2.6 Definition (21) Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two graphs. A function $f : V_1 \to V_2$ is called Isomorphism if

i) f is one to one and onto.

ii) for all $a, b \in V_1, \{a, b\} \in E_1$ if and only if $\{f(a), f(b)\} \in E_2$ when such a function exists, G_1^* and G_2^* are called isomorphic graphs and is written as $G_1^* \cong G_2^*$.

2.7 Definition (21) The union of two simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is the simple graph with the vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2$. The union of G_1^* and G_2^* is denoted by $G^* = G_1^* \cup G_2^* = (V_1 \cup V_2, E_1 \cup E_2) = (V, E)$.

2.8 Definition (21) The join of two simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is the simple graph with the vertex set $V = V_1 \cup V_2$ and edge set $E = E_1 \cup E_2 \cup E'$, where E' is the set of all edges joining the nodes of V_1 and V_2 and assume that $V_1 \cap V_2 \neq \theta$. The join of G_1^* and G_2^* is denoted by $G^* = G_1^* + G_2^* = (V_1 \cup V_2, E_1 \cup E_2 \cup E')$.

2.9 Definition (21) The intersection of two simple graphs $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ is the simple graph with the vertex set $V = V_1 \cap V_2$ and edge set $E = E_1 \cap E_2$. The intersection of G_1^* and G_2^* is denoted by $G^* = G_1^* \cap G_2^* = (V_1 \cap V_2, E_1 \cap E_2) = (V, E)$.

2.10 Definition (20) A neutrosophic set A on the universe of discourse X is defined as $A = \{< x, T_A(x), I_A(x), F_A(x) >, x \in X\}$, where $T, I, F : X \to]^-0, 1^+[$ and $^-0 \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non standard subsets of $]^-0, 1^+[$. But in real life application in scientific and engineering problems it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]^-0, 1^+[$.

3 NEUTROSOPHIC GRAPHS

3.1 Definition Let $G^* = (V, E)$ be a simple graph and $E \subseteq V \times V$. Let $T_f, I_f, F_f : V \to [0, 1]$ denote the truth-membership, indeterminacy- membership and falsity-membership of an element $x \in V$ and $T_g, I_g, F_g : E \to [0, 1]$ denote the truth-membership, indeterminacy-membership and falsity- membership of an element$(x, y) \in E$. By a neutrosophic graphs, we mean a 3-tuple $G = (G^*, f, g)$ such that

$$T_g(x, y) \leq \min\{T_f(x), T_f(y)\}$$
$$I_g(x, y) \leq \min\{I_f(x), I_f(y)\}$$
$$F_g(x, y) \geq \max\{F_f(x), F_f(y)\}$$

For all $x, y \in V$.

3.2 Example Let $G^* = (V, E)$ be a simple graph with $V = \{x_1, x_2, x_3\}$ and $E = \{(x_1, x_2)\ (x_2,, x_3)\ , (x_1, x_3)\}$. A neutrosophic graph G is given in table 1 below and $T(x_i, x_j) = 0$, $I(x_i, x_j) = 0$ and $F(x_i, x_j) = 1$ for all $(x_i, x_j) \in E \setminus \{(x_1, x_2), (x_2, x_3), (x_1, x_3)\}$.

Table1

f	x_1	x_2	x_3
T_f	0.3	0.1	0.1
I_f	0.4	0.3	0.3
F_f	0.5	0.4	0.6

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
T_g	0.1	0.1	0.1
I_g	0.1	0.2	0.2
F_g	0.9	0.8	0.7

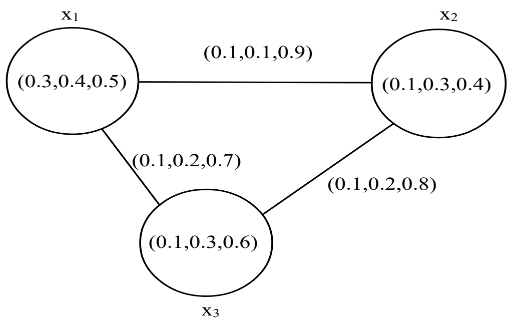

Figure 1

3.3 Definition A neutrosophic graph $G = (G^*, f_1, g_1)$ is called a neutrosophic subgraph of $G = (G^*, f, g)$ if

(i) $T_{f^1}(x) \leq T_f(x), I_{f^1}(x) \leq I_f(x), F_{f^1}(x) \geq F_f(x),$

(ii) $T_{g^1}(x, y) \leq T_g(x, y), I_{g^1}(x, y) \leq I_g(x, y), F_{g^1}(x, y) \geq F_g(x, y).$

for all $x, y \in V$.

A neutrosophic subgraph of example 3.2 is given in table 2 below and $T_g(x_i, x_j) = 0$,$I_g(x_i, x_j)$ =0 and $F_g(x_i, x_j)$ =1 for all $(x_i, x_j) \in E \setminus \{(x_1, x_2),(x_2, x_3),(x_1, x_3)\}$.

Table 2

f^1	x_1	x_2	x_3
T_{f^1}	0.1	0.1	0.1
I_{f^1}	0.1	0.2	0.2
F_{f^1}	0.5	0.4	0.6

g^1	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
T_{g^1}	0.1	0.1	0.1
I_{g^1}	0.1	0.1	0.1
F_{g^1}	0.9	0.8	0.7

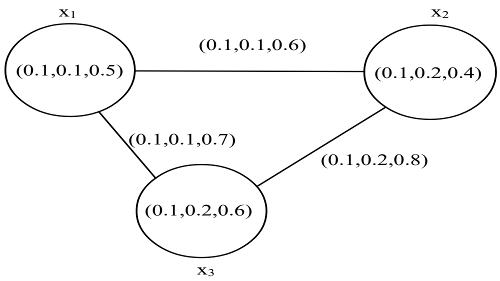

Figure 2

3.4 Definition A neutrosophic graph $G = (G^*, f^1, g^1)$ is said to be spanning neutrosophic subgraph of $G = (G^*, f, g)$ if $T_f(x) = T_f^1(x),\ I_f(x) = I_f^1(x),\ F_f(x) = F_f^1(x)$ for all $x \in V$.

3.5 Definition Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two simple graphs. The union of two neutrosophic graphs $G_1 = (G_1^*, f^1, g^1)$ and $G_2 = (G_2^*, f^2, g^2)$ is denoted by $G = (G^*, f, g)$, *where* $G^* = G_1^* \cup G_2^*$, $f = f^1 \cup f^2$ $g = g^1 \cup g^2$ where the truth-membership, indeterminacy-membership and falsity- membership of union are as follows

$$T_f(x) = \begin{cases} T_f^1(x) & \text{if } x \in V_1 - V_2 \\ T_f^2(x) & \text{if } x \in V_2 - V_1 \\ \max\{T_f^1(x), T_f^2(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases}$$

$$I_f(x) = \begin{cases} I_f^1(x) & \text{if } x \in V_1 - V_2 \\ I_f^2(x) & \text{if } x \in V_2 - V_1 \\ \max\{I_f^1(x), I_f^2(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases}$$

$$F(x) = \begin{cases} F_f^1(x) & \text{if } x \in V_1 - V_2 \\ F_f^2(x) & \text{if } x \in V_2 - V_1 \\ \min\{F_f^1(x), F_f^2(x)\} & \text{if } x \in V_1 \cap V_2 \end{cases}$$

Also

$$T_g(x,y) = \begin{cases} T_{g^1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ T_{g^2}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \max\{T_{g^1}(x,y), T_{g^2}(x,y)\} & \text{if } (x,y) \in E_1 \cap E_2 \end{cases}$$

$$I_g(x,y) = \begin{cases} I_{g^1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ I_{g^2}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \max\{I_{g^1}(x,y), I_{g^2}(x,y)\} & \text{if } (x,y) \in E_1 \cap E_2 \end{cases}$$

$$F_g(x,y) = \begin{cases} F_{g^1}(x,y) & \text{if } (x,y) \in E_1 - E_2 \\ F_{g^2}(x,y) & \text{if } (x,y) \in E_2 - E_1 \\ \min\{F_{g^1}(x,y), F_{g^2}(x,y)\} & \text{if } (x,y) \in E_1 \cap E_2 \end{cases}$$

3.6 Example Let $G_1^* = (V_1, E_1)$ be a simple graph with $V_1 = \{x_1, x_3, x_4\}$ & $E_1 = \{(x_1, x_4), (x_3,, x_4), (x_1, x_3)\}$. A neutrosophic graph G_1 is given in table 3 below and $T_g(x_i, x_j) = 0$, $I_g(x_i, x_j) = 0$ and $F_g(x_i, x_j) = 1$ for all $(x_i, x_j) \in E_1 \setminus \{(x_1, x_4), (x_3, x_4), (x_1, x_3)\}$.

Table 3

f^1	x_1	x_3	x_4
T_{f^1}	0.1	0.2	0.2
I_{f^1}	0.2	0.4	0.5
F_{f^1}	0.3	0.5	0.7

g^1	(x_1, x_4)	(x_3, x_4)	(x_1, x_3)
T_{g^1}	0.1	0.1	0.1
I_{g^1}	0.2	0.3	0.2
F_{g^1}	0.7	0.8	0.5

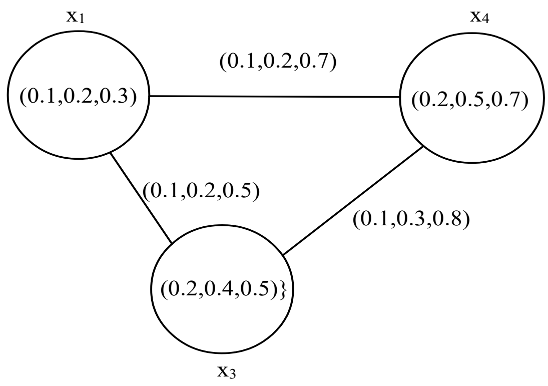

Figure 3

A neutrosophic graph $G_2 = (G_2^*, f^2, g^2)$ is given in table 4 below where $G_2^* = (V_2, E_2)$, $V_2 = \{x_2, x_3, x_4, x_5\}$ and $E_2 = \{(x_2, x_3)\ ,\ (x_3,, x_4)\ ,\ (x_4, x_5)\ ,\ (x_2, x_5)\}$ and $T_g(x_i, x_j) = 0$, $I_g\ (x_i, x_j) = 0$ and $F_g\ (x_i, x_j) = 1$ for all $(x_i, x_j) \in E_2 \setminus \{(x_2, x_3)\ ,(x_2,x_4),(x_3,, x_4)\ ,\ (x_4, x_5)\ ,\ (x_2, x_5)\}$.

Table 4

f^2	x_2	x_3	x_4	x_5
T_{f^2}	0.1	0.2	0.4	0.2
I_{f^2}	0.2	0.3	0.6	0.1
F_{f^2}	0.4	0.4	0.7	0.6

g^2	(x_2,x_3)	(x_3,x_4)	(x_2,x_5)	(x_4,x_5)	(x_2,x_4)
T_{g^2}	0.1	0.2	0.1	0.2	0.1
I_{g^2}	0.2	0.3	0.1	0.1	0.2
F_{g^2}	0.8	0.9	0.9	0.8	0.7

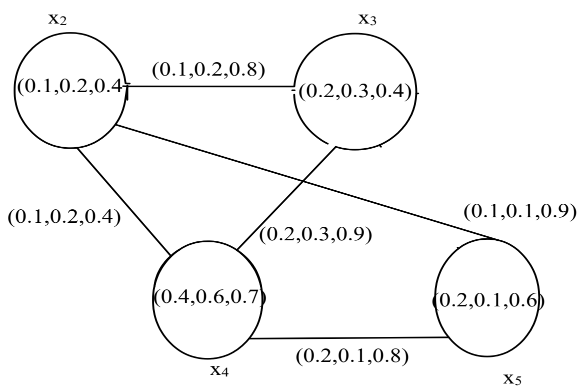

Figure 4

The union $G = (G^*, f, g)$ is given in table 5 below and $T_g\,(x_i, x_j) = 0$, $I_g\,(x_i, x_j) = 0$ and $F_g\,(x_i, x_j) = 1$ for all $(x_i, x_j) \in V \times V \setminus\ \{(x_1, x_4), (x_3, x_4), (x_1, x_3),$ $,(x_2,x_4\ ),(x_2, x_3), (x_4, x_5), (x_2, x_5)\}$.

Table 5

f	x_1	x_2	x_3	x_4	x_5
T_f	0.1	0.1	0.2	0.4	0.2
I_f	0.2	0.2	0.4	0.6	0.1
F_f	0.3	0.4	0.4	0.7	0.6

g	(x_1,x_4)	(x_3,x_4)	(x_1,x_3)	(x_2,x_3)	(x_4,x_5)	(x_2,x_4)	(x_2,x_5)
T_g	0.1	0.2	0.1	0.1	0.2	0.1	0.1
I_g	0.2	0.3	0.2	0.2	0.1	0.2	0.1
F_g	0.7	0.8	0.5	0.8	0.8	0.7	0.9

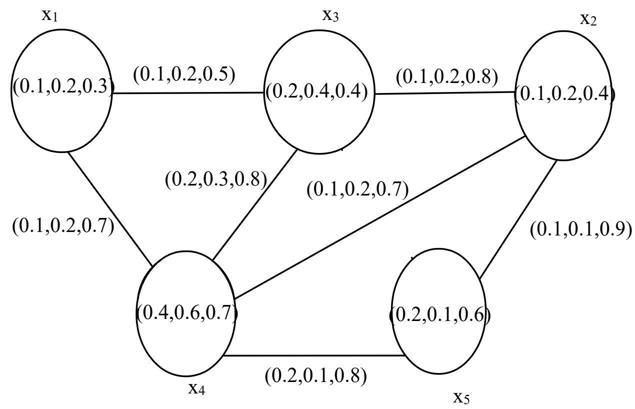

Figure 5

3.7 Proposition The union $G=(G^{*},f,g)$ of two neutrosophic graphs $G_1=(G_1^{*},f^1,g^1)$ and $G_2=(G_2^{*},f^2,g^2)$ is a neutrosophic graph.

Proof

Case i) If $(x,y)\in E_1-E_2$ then

$$T_g(x,y)=T_{g^1}(x,y)\le \min\{T_{f^1}(x),T_{f^2}(y)\}=\min\{T_f(x),T_f(y)\}$$

so $T_g(x,y)\le \min\{T_f(x),T_f(y)\}$

Also $I_g(x,y)=I_{g^1}(x,y)\le \min\{I_{f^1}(x),I_{f^2}(y)\}=\min\{I_f(x),I_f(y)\}$

so $I_g(x,y)\le \min\{I_f(x),I_f(y)\}$

Now $F_g(x,y)=F_{g^1}(x,y)\ge \max\{F_{f^1}(x),F_{f^2}(y)\}=\max\{F_f(x),F_f(y)\}$

Similarly *If* $(x,y)\in E_2-E_1$ *then we can show the same as done above.*

Case ii) *If* $(x,y)\in E_1\cap E_2$, *then*

$$T_g(x,y)=\max\{T_{g^1}(x,y),T_{g^2}(x,y)\}$$
$$\le \max\{\min\{T_{f^1}(x),T_{f^1}(y)\},\min\{T_{f^2}(x),T_{f^2}(y)\}$$
$$\le \min\{\max\{T_{f^1}(x),T_{f^2}(x)\},\max\{T_{f^1}(y),T_{f^2}(y))=\min\{T_f(x),T_f(y)\}$$

Also $I_g(x,y)=\max\{I_{g^1}(x,y),I_{g^2}(x,y)\}$
$$\le \max\{\min\{I_{f^1}(x),I_{f^1}(y)\},\min\{I_{f^2}(x),I_{f^2}(y)\}$$
$$\le \min\{\max\{I_{f^1}(x),I_{f^2}(x)\},\max\{I_{f^1}(y),I_{f^2}(y))=\min\{I_f(x),I_f(y)\}$$

Now $F_g(x,y)=\min\{F_{g^1}(x,y),F_{g^2}(x,y)\}$
$$\ge \min\{\max\{F_{f^1}(x),F_{f^1}(y)\},\max\{F_{f^2}(x),F_{f^2}(y)\}$$
$$\ge \max\{\min\{F_{f^1}(x),F_{f^2}(x)\},\min\{F_{f^1}(y),F_{f^2}(y))$$
$$=\max\{F_f(x),F_f(y)\}$$

Hence the union $G=G_1\cup G_2$ is a neutrosophic graph.

3.8 Definition The intersection of two neutrosophic graphs $G_1=(G_1^{*},f^1,g^1)$ and $G_2=(G_2^{*},f^2,g^2)$ is denoted by $G=(G^{*},f,g)$, *where* $G=G_1^{*}\cap G_2^{*}$, $f=f_1^1\cap f_1^1$, $g=g^1\cap g^2$, $V=V_1\cap V_2$ and the truth-membership, indeterminacy-membership and falsity- membership of intersection are as follows

$T_f(x)=\min\{T_{f^1}(x),T_{f^2}(x)\}$, $I_f(x)=\min\{I_{f^1}(x),I_{f^2}(x)\}$,

$F_f(x)=\max\{F_{f^1}(x),F_{f^2}(x)\}$

$T_g(x,y)=\min\{T_{g^1}(x,y),T_{g^2}(x,y)\}$, $I_g(x,y)=\min\{I_{g^1}(x,y),I_{g^2}(x,y)\}$

$F_g(x,y)=\max\{F_{g^1}(x,y),F_{g^2}(x,y)\}$

for all $x,y\in V$.

3.9 Example Let $G_1^{*}=(V_1,E_1)$ be a simple graph with $V_1=\{x_1,x_2,x_5\}$ & $E_1=\{(x_1,x_5),(x_1,x_2),(x_2,x_5)\}$.

A neutrosophic graph $G_1=(G_1^{*},f^1,g^1)$ is given in table 6 below and $T_g(x_i,x_j)=0$, $I_g(x_i,x_j)=0$ and $F_g(x_i,x_j)=1$ for all $(x_i,x_j)\in E_1\setminus\{(x_1,x_5),(x_1,x_2),(x_2,x_5)\}$.

Table 6

f^1	x_1	x_2	x_5
T_{f^1}	0.2	0.4	0.3
I_{f^1}	0.3	0.6	0.4
F_{f^1}	0.7	0.7	0.6

g^1	(x_1,x_2)	(x_2,x_5)	(x_1,x_5)
T_{g^1}	0.2	0.3	0.2
I_{g^1}	0.3	0.4	0.3
F_{g^1}	0.7	0.8	0.7

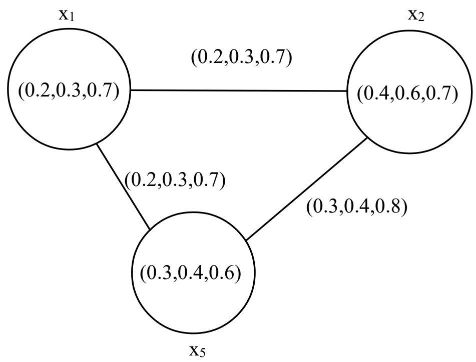

Figure 6

Let $G_2^{*}=(V_2,E_2)$ be a simple graph with $V_2=\{x_2,x_3,x_5\}$ and $E_2=\{(x_2,x_3),(x_3,x_5)(x_2,x_5)\}$.

A neutrosophic graph $G_2=(G_2^{*},f^2,g^2)$ is given in table 7 below and $T_g(x_i,x_j)=0$, $I_g(x_i,x_j)=0$ and $F_g(x_i,x_j)=1$ for all $(x_i,x_j)\in E_2\setminus\{(x_2,x_3),(x_3,x_5)(x_2,x_5)\}$.

Table 7

f^2	x_2	x_3	x_5
T_{f^2}	0.3	0.2	0.4
I_{f^2}	0.5	0.4	0.5
F_{f^2}	0.6	0.6	0.9

g^2	(x_2, x_3)	(x_3, x_5)	(x_2, x_5)
T_{g^2}	0.1	0.2	0.2
I_{g^2}	0.3	0.4	0.4
F_{g^2}	0.7	0.9	0.9

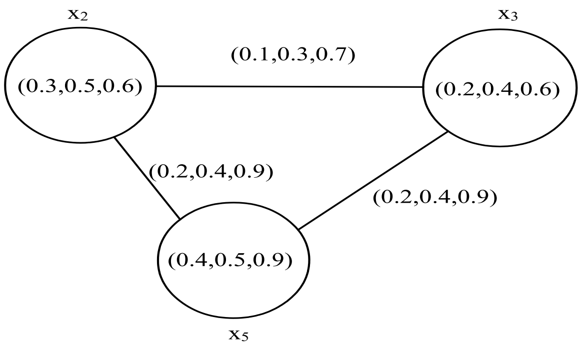

Figure 7

Let $V = V_1 \cap V_2 = \{x_2, x_5\}$, $E = E_1 \cap E_2 = \{(x_2, x_5)\}$. The intersection of the above two graphs G_1 and G_2 is given in the table 8 below and figure 8.

Table 8

f	x_2	x_5	g	(x_2, x_5)
T_f	0.3	0.3	T_g	0.2
I_f	0.5	0.4	I_g	0.4
F_f	0.7	0.9	F_g	0.9

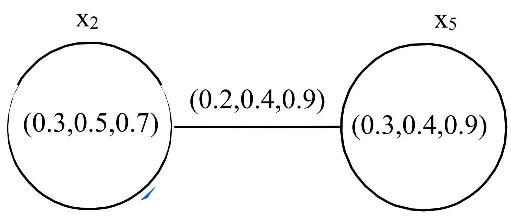

Figure 8

3.10 Proposition The intersection $G = (G^*, f, g)$ of two neutrosophic graphs $G_1 = (G_1^*, f^1, g^1)$ and $G_2 = (G_2^*, f^2, g^2)$ is a neutrosophic graph where $V = V_1 \cap V_2$.

Proof

Let $x, y \in V = V_1 \bigcap V_2$ and $(x, y) \in E = E_1 \bigcap E_2$,

then $T_g(x, y) = \min\{T_{g^1}(x, y), T_{g^2}(x, y)$

$\leq \min\{\min\{T_{f^1}(x), T_{f^1}(y)\}, \min\{T_{f^2}(x), T_{f^2}(y)\}$

$\leq \min\{\min\{T_{f^1}(x), T_{f^2}(x)\}, \min\{T_{f^1}(y), T_{f^2}(y))$

$= \min\{T_f(x), T_f(y)\}$

Also $I_g(x, y) = \min\{I_{g^1}(x, y), I_{g^2}(x, y)\}$

$\leq \min\{\min\{I_{f^1}(x), I_{f^1}(y)\}, \min\{I_{f^2}(x), I_{f^2}(y)\}$

$\leq \min\{\min\{I_{f^1}(x), I_{f^2}(x)\}, \min\{I_{f^1}(y), I_{f^2}(y)) = \min\{I_f(x), I_f(y)\}$

Now $F_g(x, y) = \max\{F_{g^1}(x, y), F_{g^2}(x, y)\}$

$\geq \max\{\max\{F_{f^1}(x), F_{f^1}(y)\}, \max\{F_{f^2}(x), F_{f^2}(y)\}\}$

$\geq \max\{\max\{F_{f^1}(x), F_{f^2}(x)\}, \max\{F_{f^1}(y), F_{f^2}(y)\}\} = \max\{F_f(x), F_f(y)\}$

Hence the intersection $G = G_1 \cap G_2$ is a neutrosophic graph.

4 Strong Neutrosophic Graphs

In this section we will study the notion of strong neutrosophic graphs and complement of such graphs.

4.1 Definition A neutrosophic graph $G = (G^*, f, g)$ is called strong if

$T_g(x, y) = \min\{T_f(x), T_f(y)\}$

$I_g(x, y) = \min\{I_f(x), I_f(y)\}$

$F_g(x, y) = \max\{F_f(x), F_f(y)\}$

for all $(x, y) \in E$.

4.2 Example Let $V = \{x_1, x_2, x_3\}$ and $E=\{(x_1, x_2), (x_2, x_3), (x_2, x_3)\}$. A strong neutrosophic graph $G = (G^*, f, g)$ where $G^* = (V, E)$ is simple graph, is given in table 9 below and $T_g(x_i, x_j) = 0$, $I_g(x_i, x_j) = 0$ and $F_g(x_i, x_j) = 1$ for all $(x_i, x_j) \in V \times V \setminus \{(x_1, x_2), (x_2, x_3), (x_2, x_3)\}$.

Table 9

f	x_1	x_2	x_3
T_f	0.1	0.2	0.3
I_f	0.2	0.3	0.4
F_f	0.4	0.5	0.7

g	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
T_g	0.1	0.2	0
I_g	0.2	0.3	0
F_g	0.5	0.7	1

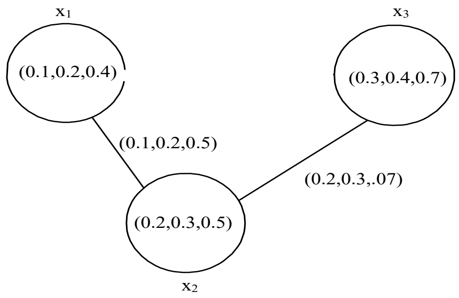

Figure 9

4.3 Definition Let $G = (G^*, f, g)$ be a strong neutrosophic graph. The complement $\overline{G} = (G^*, \overline{f}, \overline{g})$ of strong neutrosophic graph $G = (G^*, f, g)$ is neutrosophic graph where

(i) $T_f(x) = \overline{T}_f(x), I_f(x) = \overline{I}_f(x), F_f(x) = \overline{F}_f(x),$ *for* all $x \in V.$

and

$$(ii) \quad \overline{T}_g(x,y) = \begin{cases} \min\{T_f(x), T_f(y)\} & \text{if } T_g(x,y) = 0 \\ 0 & otherwise \end{cases}$$

$$\overline{I}_g(x,y) = \begin{cases} \min\{I_f(x), I_f(y)\} & \text{if } I_g(x,y) = 0 \\ 0 & otherwise \end{cases}$$

$$\overline{F}_g(x,y) = \begin{cases} \max\{F_f(x), F_f(y)\} & \text{if } F_g(x,y) = 1 \\ 1 & otherwise \end{cases}$$

4.4 Example For the strong neutrosophic graph in previous example, i.e. The complement of

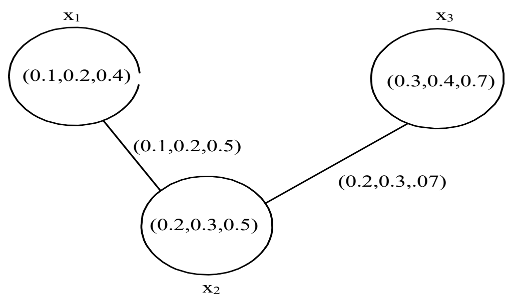

Figure 10

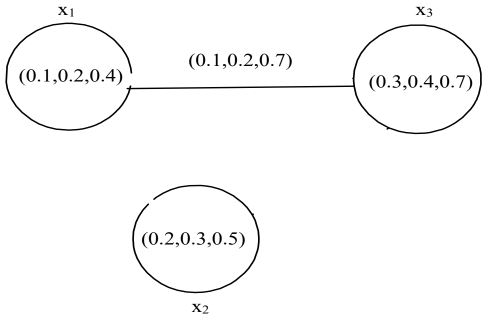

Figure 11

Similarly the complement of neutrosophic graph

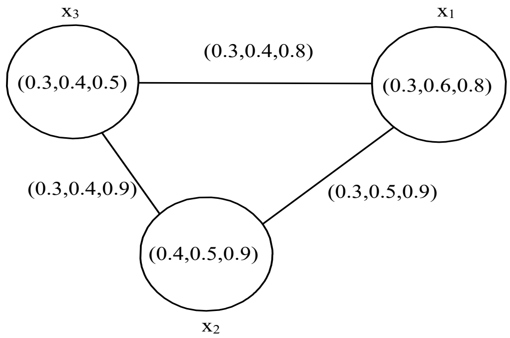

Figure 12

is given by

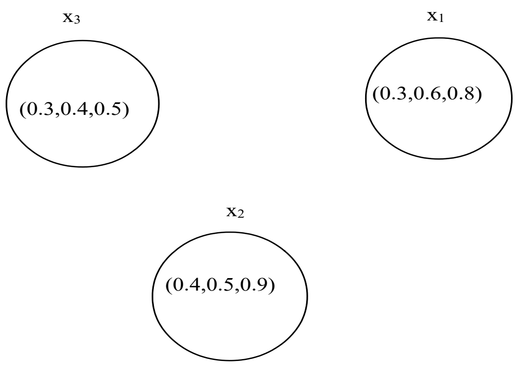

Figure 13

5 Homomorphism Of Neutrosophic Graphs

In this section we introduced and discussed the notion of homomorphisms of neutrosophic graphs. We have also discussed weak isomorphism, co- weak isomorphism and isomorphism here.

5.1 Definition A Homomorphism $h: G_1 \to G_2$ between two neutrosophic graphs $G_1 = (G_1^*, f^1, g^1)$ and $G_2 = (G_2^*, f^2, g^2)$ is a mapping $h: V_1 \to V_2$ which satisfies

(i) $T_{f^1}(x) \le T_{f^2}(h(x)),\ I_{f^1}(x) \le I_{f^2}(h(x)),$

$F_{f^1}(x) \ge F_{f^2}(h(x)),$ for all $x \in V_1$.

(ii) $T_{g^1}(x,y) \le T_{g^2}(h(x),h(y)),\ I_{g^1}(x,y) \le I_{g^2}(h(x),h(y)),$

$F_{g^1}(x,y) \ge F_{g^2}(h(x),h(y)),$ for all $x, y \in V_1$.

5.2 Definition A weak isomorphism $h: G_1 \to G_2$ between two neutrosophic graphs $G_1 = (G_1^*, f^1, g^1)$ and $G_2 = (G_2^*, f^2, g^2)$ is a mapping $h: V_1 \to V_2$ which is a bijective homomorphism such that $T_{f^1}(x) = T_{f^2}(h(x)),\ I_{f^1}(x) = I_{f^2}(h(x)),\ F_{f^1}(x) = F_{f^2}(h(x))$, for all $x, y \in V$.

5.3 Example Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two simple graphs with $V_1 = \{x_1, x_2, x_3\}$, $E_1 = \{$ $\{(x_1,x_2),\ (x_2,x_3),\ (x_3,x_1)\}$

$V_2 = \{x_1', x_2', x_3'\}$. $E_2 = \{(x_1', x_2'), (x_2', x_3'), (x_1', x_3')\}$. Two neutrosophic graphs $G_1 = (G_1^*, f^1, g^1)$ and $G_2 = (G_2^*, f^2, g^2)$ are given in table 10 and Table 11 below and $T_g(x_i,x_j) = 0, I_g(x_i,x_j) = 0$ and $F_g(x_i,x_j) = 1$ for all $(x_i,x_j) \in V \times V \setminus \{\{(x_1,x_2),\ (x_2,x_3),\ (x_1,x_3)\}$.

Also

$T_g(x_i', x_j') = 0, I_g(x_i', x_j') = 0$ and $F_g(x_i', x_j') = 1$

for all $(x_i', x_j') \in V \times V \setminus \{(x_1', x_2'), (x_2', x_3'), (x_1', x_3')\}$.

Table 10

f^1	x_1	x_2	x_3
T_{f^1}	0.2	0.1	0.1
I_{f^1}	0.3	0.2	0.5
F_{f^1}	0.5	0.4	0.7

g^1	(x_1, x_2)	(x_2, x_3)	(x_1, x_3)
T_{g^1}	0.1	0.1	0.1
I_{g^1}	0.1	0.2	0.3
F_{g^1}	0.9	0.7	0.9

Table 11

f^2	x_1'	x_2'	x_3'
T_{f^2}	0.2	0.1	0.1
I_{f^2}	0.3	0.2	0.5
F_{f^2}	0.5	0.4	0.7

g^2	(x_1', x_2')	(x_2', x_3')	(x_1', x_3')
T_{g^2}	0.1	0.1	0.1
I_{g^2}	0.2	0.2	0.3
F_{g^2}	0.8	0.7	0.8

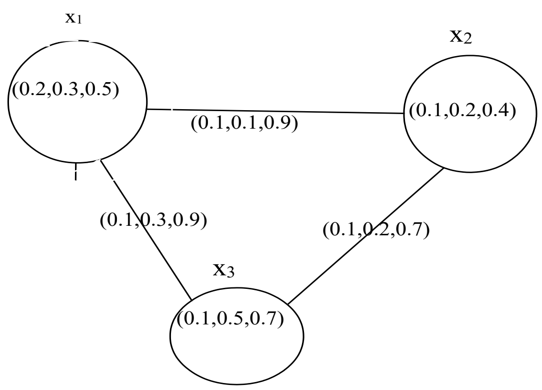

Figure 14

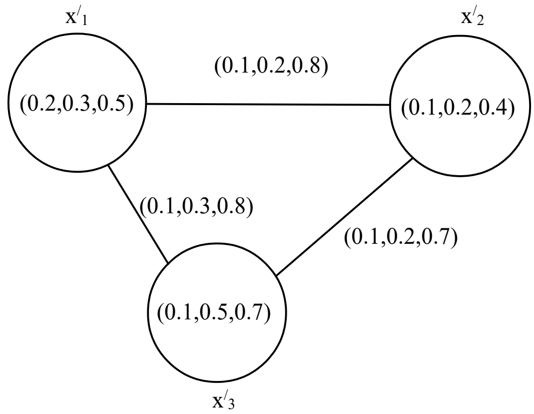

Figure 15

Now we define $h: V_1 \to V_2$ by $h(x_1) = x_1'$, $h(x_2) = x_2'$, $h(x_3) = x_3'$, then $T_{f^1}(x) = T_{f^2}(h(x)),\ I_{f^1}(x) = I_{f^2}(h(x)),\ F_{f^1}(x) = F_{f^2}(h(x))$, for all $x \in V_1$ ▷

By easy calculation, it can be seen that h is a weak isomorphism.

5.4 Proposition

Weak isomorphism between neutrosophic graphs satisfies the partial order relation.

Proof

Let $G_1 = (G_1^*, f^1, g^1)$, $G_2 = (G_2^*, f^2, g^2)$ and $G_3 = (G_3^*, f^3, g^3)$ be three neutrosophic graphs with sets of vertices V_1, V_2 and V_3 respectively. Then

1) The relation is reflexive. Let $h : V_1 \to V_2$ be an identity mapping such that $h(x_1) = x_1$, for all $x \in V_1$. That is $Tf1\ (x) = Tf2\ (h(x)), If1\ (x) = If2\ (h(x)), Ff1\ (x) = Ff2\ (h(x))$, for all $x \in V_1$ and $Tg1\ (x, y) \le Tg2\ (h(x), h(y)), Ig1\ (x, y) \le Ig2\ (h(x), h(y)), F_{g1}(x, y) \ge F_{g2}(h(x), h(y))$,for all $x, y \in V_1$. So $h : V_1 \to V_1$ is a weak isomorphism of the neutrosophic graph G_1 onto itself.

2) The relation is anti-symmetric. Let h be a weak isomorphism between the neutrosophic graphs G_1 and G_2, that is $h : V_1 \to V_2$ is a bijective mapping. Therefore $h(x_1) = x_2$, for all $x_1 \in V_1$ satisfying $T_{f1}(x_2) = T_{f2}(h(x_1)), I_{f1}(x_2) = I_{f2}(h(x_1)), F_{f1}(x_2) = F_{f2}(h(x_1))$, for all $x_1 \in V_1$ and $T_{g1}(x_1, y_1) \le T_{g2}(h(x_1), h(y_1)), I_{g1}(x_1, y_1) \le I_{g2}(h(x_1), h(y_1)))$, $F_{g1}(x_1, y_1) \ge F_{g2}(h(x_1), h(y_1))$....(1) , for all $x_1, y_1 \in V_1$.

Let k be a weak isomorphism between the neutrosophic graph G_2 and G_1 so the relation is anti-symmetric that is $k : V_2 \to V_1$ is a bijective map with $T_{f2}(x_2) = T_{f1}(k(x_2)), I_{f2}(x_2) = I_{f1}(k(x_2))$,.$F_{f2}(x_2) = F_{f1}(k(x_2))$ for all $x_2 \in V_2$ and $T_{g2}(x_2, y2) \le T_{g1}(k(x_2), k(y_2)), I_{g2}(x_2, y_2) \le I_{g1}(k(x_2), k(y_2))$, $F_{g2}(x_2, y_2) \le F_{g1}(k(x_2), k(y_2))$ for all $(x_2, y_2) \in (V_2 \times V_2)$….(2), for all $x_2, y_2 \in V_2$. The subset relation (1) and (2) hold good on the finite sets V_1 , V_2 when the neutrosophic graphs G_1 and G_2 have the same no. of edges and the corresponding edges are identical. Hence G_1 and G_2 are identical.

3) The relation is transitive. Let $h : V_1 \to V_2$ and $k : V_2 \to V_3$ be weak isomorphisms of the neutrosophic graphs $G_1 = (G_1^*, f^1, g^1)$ onto $G_2 = (G_2^*, f^2, g^2)$ and $G_2 = (G_2^*, f^2, g^2)$ onto $G_3 = (G_3^*, f^3, g^3)$ respectively. Then kof is a bijective mapping from V_1 to V_3 and defined as $(koh)(x_1) = k[h(x_1)]$, for all $x_1 \in V_1$. As h is a weak isomorphism, so $h(x_1) = x_2$, for all $x_1 \in V_1$ and $T_{f^1}(x) = T_{f^2}(h(x), I_{f^1}(x) = I_{f^2}(h(x), F_{f^1}(x) = F_{f^2}(h(x)$ for all $x_1 \in V_1$. Also $T_{g1}(x_1, y_1) \le T_{g2}(h(x_1), h(y_1)), I_{g1}(x_1, y_1) \le I_{g2}(h(x_1), h(y_1)))$, $F_{g1}(x_1, y_1) \ge F_{g2}(h(x_1), h(y_1))$, for all $x_1, y_1 \in V_1$.

As k is a weak isomorphism, so $k(x_2) = x_3$, for all $x_2 \in V_2$ and $Tf2\ (x_2) = T_{f3}(k(x_2))$,
$I_{f2}(x_2) = I_{f3}(k(x_2))$,
$F_{f2}(x_2) = F_{f3}(k(x_2))$ and
$T_{g2}(x_2, y_2) \le T_{g3}(k(x_2), k(y_2))$,
$I_{g2}(x_2, y_2) \le I_{g3}(k(x_2), k(y_2))$,
$F_{g2}(x_2, y_2) \le F_{g3}(k(x_2), k(y_2))$,
for all $x_2, y_2 \in V_2$.

As $T_{f1}(x) = T_{f2}(h(x)), I_{f1}(x_1) = I_{f2}(h(x_1))$, $F_{f1}(x_1) = F_{f2}(h(x_1))$, for all $x_1 \in V_1$ and $T_{f2}(x_2) = T_{f3}(k(x_2)), I_{f2}(x_2) = I_{f3}(k(x_2))$, $F_{f2}(x_2) = F_{f3}(k(x_2))$ for all $x_2 \in V_2$. so

$T_{f^1}(x_2) = T_{f^3}(k((h(x_1))), I_f(x_2) = I_{f_e^3}(k((h(x_1))), F_{f_e^1}(x_2) = F_{f_e^3}(k((h(x_1)))$
for all $x_1 \in V_1$. As $T_{g1}(x_1, y_1) \le T_{g2}(h(x_1), h(y_1)) = T_{g2}(x_2, y_2)$, $I_{g1}(x_1, y_1) \le I_{g2}(h(x_1), h(y_1)) = I_{g2}(x_2, y_2), F_{g1}(x_1, y_1) \ge F_{g2}(h(x_1), h(y1)) = F_{g2}(x_2, y_2)$ for all $x_1, y_1 \in V_1$, so

$T_{g1}(x_1, y_1) \le T_{g2}(x_2, y_2)$,
$I_{g1}(x_1, y1) \le I_{g2}(x_2, y_2), F_{g1}(x_1, y_1) \ge F_{g2}(x_2, y_2)$
for all $x_1, y_1 \in V_1$.

But $T_{g2}(x_2, y_2) \le T_{g3}(k(x_2), k(y_2))$,
$I_{g2}(x_2, y_2) \le I_{g3}(k(x_2), k(y_2))$,
$F_{g2}(x_2, y_2) \ge F_{g3}(k(x_2), k(y_2))$,
Therefore
$T_{g1}(x1, y_1) \le T_{g3}(k(x_2), k(y_2))$,
$I_{g1}(x_1, y_1) \le I_{g3}(k(x_2), k(y_2))$,
$F_{g1}(x_1, y_1) \ge F_{g3}(k(x_2), k(y_2))$ for all $x_1, y_1 \in V_1$.

So koh is a weak isomorphism between G_1 and G_3. that is, the relation is transitive. Hence the theorem.

5.5 Definition A co-weak isomorphism $h : G_1 \to G_2$ is a map $h : V_1 \to V_2$ between two neutrosophic graphs

$G_1=(G_1^*, f^1, g^1)$ and $G_2=(G_2^*, f^2, g^2)$ which is a bijective homomorphism that satisfies the condition
$T_{g1}(x_1, y_1) = T_{g2}(h(x_1), h(y_1))$,
$I_{g1}(x_1, y_1) = I_{g2}(h(x_1), h(y_1))$,
$F_{g1}(x_1, y_1) = F_{g2}(h(x_1), h(y_1))$
for all $x, y \in V_1$.

5.6 Definition An isomorphism $h : G_1 \to G_2$ is a mapping $h : V_1 \to V_2$ which is bijective that satisfies the following conditions

$$(i)\ T_{f^2}(x)=T_{f^3}(h(x)),\ \ I_{f^1}(x)=I_{f^2}(h(x)),$$
$$F_{f^1}(x)=F_{f^2}(h(x)),\ \text{ for all } x\in V_1$$
$$(ii)\ T_{g^1}(x,y)=T_{g^2}(h(x),h(y)),$$
$$I_{g^1}(x,y)=I_{g^2}(h(x),h(y)),$$
$$F_{g^1}(x,y)=F_{g^2}(h(x),h(y)),\ \text{ for all } x,y\in V_1.$$

If such h exists then we say G_1 is isomorphic to G_2 and we write $G_1 \cong G_2$.

5.7 Proposition
The isomorphism between neutrosophic graphs is an equivalence relation.

Proof
Let $G_1=(G_1^*, f^1, g^1)$, $G_2=(G_2^*, f^2, g^2)$ and $G_3=(G_3^*, f^3, g^3)$ be three neutrosophic graphs with sets of vertices V_1, V_2 and V_3 respectively then
i) The relation is reflexive. Consider the identity mapping $h : V_1 \to V_1$ such that $h(x_1) = x_1$, for all $x_1 \in V_1$. Then h is a bijective mapping satisfying
,
(i) $T_{f1}(x) = T_{f2}(h(x))$, $I_{f1}(x) = I_{f2}(h(x))$, $F_{f1}(x) = F_{f2}(h(x))$, for all $x \in V_1$.

ii) $T_{g1}(x, y) = T_{g2}(h(x), h(y))$,
$I_{g1}(x, y) = I_{g2}(h(x), h(y))$,
$F_{g1}(x, y) = F_{g2}(h(x), h(y))$, for all $x, y \in V_1$

showing that h is an isomorphism of the neutrosophic graph G_1 on to itself, that is $G_1 \cong G_1$.
i) The relation is symmetric. Let $h : V_1 \to V_2$ be an isomorphism of G_1 onto G_2 then h is bijective function. Therefore $h(x_1) = x_2$, for all $x_1 \in V$.

Also $T_{f1}(x) = T_{f2}(h(x))$, $I_{f1}(x) = I_{f2}(h(x))$, $F_{f1}(x) = F_{f2}(h(x))$, for all $x \in V_1$ and

$T_{g1}(x, y) = T_{g2}(h(x), h(y))$,
$I_g(x, y) = I_{g2}(h(x), h(y))$,
$F_{g1}(x, y) = F_{g2}(h(x), h(y))$, for all $x, y \in V_1$.

Since h is bijective, so it is invertible, that is, $h^{-1}: G_2 \to G_1$ will exist and $h^{-1}(x_2) = x_1$, for all $x_2 \in V_2$.

Since $T_{f1}(x_2) = T_{f2}(h(x_1))$,
$I_{f1}(x_2) = I_{f2}(h(x_1))$, $F_{f1}(x_2) = F_f(h(x_1))$ so
$T_{f1}(h^{-1}(x_2)) = T_{f2}(x_2)$ or
$T_{f2}(x_2) = T_{f1}(h^{-1}(x_2))$,
$I_{f2}(x_2) = I_{f1}(h^{-1}(x_2))$ *and*
$F_{f1}(h^{-1}(x_2)) = F_{f1}(h^{-1}(x_2))$ for all $x_2 \in V_2$. Also

$T_{g1}(x_1, y_2) = T_{g2}(h(x_1), h(y1))$ so
$T_{g1}(h^{-1}(x_2), (h^{-1}(y2))) = T_{g2}(x2, y2)$ or
$T_{g2}(x_2, y2) = T_{g1}(h^{-1}(x2), (h^{-1}(y2)))$.
Similarly $I_{g1}(x_1, y1) = I_{g2}(h(x_1), h(y1))$ so
$I_{g1}(h^{-1}(x_2), (h^{-1}(y2))) = I_{g2}(x_2, y2)$ or
$I_{g2}(x_2, y2) = I_{g1}(h^{-1}(x_2), (h^{-1}(y2)))$, and
$F_{g1}(x_1, y1) = F_{g2}(h(x_1), h(y1))$ implies
$F_{g1}(h^{-1}(x_2), (h^{-1}(y2))) = F_{g2}(x_2, y2)$ or
$F_{g2}(x_2, y2) = F_{g1}(h^{-1}(x_2), (h^{-1}(y2)))$.
Hence $h^{-1} : G_2 \to G_1$ or $h^{-1} : V_2 \to V_1$ (Both one to one & onto) is an isomorphism from G_2 to G_1, that is $G_2 \cong G_1$. So $G_1 \cong G_2 \Rightarrow G_2 \cong G_1$.

iii) The relation is transitive.
Let $h: V_1 \to V_2$ and $k: V_2 \to V_3$ be the isomorphism of the neutrosophic graphs G_1 onto G_2 and G_2 onto G_3 respectively. Then $koh : V_1 \to V_3$ is also a bijective mapping from $V1$ to $V3$ defined as

$(koh)(x1) = k[h(x1)]$, for all $x1 \in V_1$. Since $h : V_1 \to V_2$ is an isomorphism therefore $h(x1) = x2$, for all $x1 \in V_1$. Also $T_{f1}(x_1) = T_{f2}(h(x_1))$,

$I_{f1}(x_1) = I_{f2}(h(x_1))$, $F_{f1}(x_1) = F_{f2}(h(x_1))$, for all $x_1 \in V_1$ and $T_{g1}(x_1, y_1) = T_{g2}(h(x_1), h(y_1))$,

$I_{g1}(x_1, y_1) = I_{g2}(h(x_1), h(y_1))$,
$F_{g1}(x_1, y_1) = F_{g2}(h(x_1), h(y_1))$, for all $x_1, y_1 \in V_1$.

Since $k : V_2 \to V_3$ is an isomorphism so

$k(x2) = x3$, $T_{f2}(x2) = T_{f3}(k(x2))$,
$I_{f2}(x2) = I_{f3}(k(x2))$, $F_{f2}(x2) = F_{f3}(k(x2))$ and
$T_{g2}(x2, y2) = T_{g3}(k(x2), k(y2))$, $I_{g2}(x2, y2) = I_{g3}(k(x2), k(y2))$, $F_{g2}(x2, y2) = F_{g3}(k(x2), k(y2))$, for all $x2, y2 \in V_2$. As $T_{f1}(x_1) = T_{f2}(h(x_1))$ and

$T_{f2}(x2) = T_{f3}(k(x2))$ so $T_{f1}(x_1) = T_{f2}(h(x_1)) =$

$T_{f_2}(x_2) = T_{f_3}(k(x_2)) = T_{f_3}(k(h(x_1))$, for all $x_1 \in V_1$ which shows $T_{f_1}(x_1) = T_{f_3}(k(h(x_1))$, for all $x_1 \in V_1$.

Similarly we can show $I_{f_1}(x_1) = I_{f_3}(k(h(x_1))$, $F_{f_1}(x_1) = F_{f_3}(k(h(x_1))$. Furthermore $T_{g_1}(x_1, y_1) = T_{g_2}(h(x_1), h(y_1))$ and $T_{g_2}(x_2, y_2) = T_{g_3}(k(x_2), k(y_2))$ so $T_{g_1}(x_1, y_1) = T_{g_2}(h(x_1), h(y_1)) = T_{g_2}(x_2, y_2) = T_{g_3}(k(x_2), k(y_2)) = T_{g_3}[(k(h(x_1)), (k(h(y_1))]$, so $T_{g_1}(x_1, y_1) = T_{g_3}[(k(h(x_1)), (k(h(y_1))]$ for all $x_1, y_1 \in V_1$.

Similarly we can show
$I_{g_1}(x_1, y_1) = I_{g_3}[(k(h(x_1)), (k(h(y_1))]$,
$F_{g_1}(x_1, y_1) = F_{g_3}[(k(h(x_1)), (k(h(y_1))]$.

So gof is isomorphism between G_1 and G_3.
Hence isomorphism between the neutrosophic graphs is an equivalence relation.

5.8 Remarks

1. If $G = G_1 = G_2$ then the homomorphism is called an endomorphism and the isomorphism is called an automorphism.
2. If $G_1 = G_2 = G$ then the co-weak and weak isomorphism become isomorphism.
3. A weak isomorphism preserves the equality of the of vertices but not necessarily the equality of edges.
4. A co-weak isomorphism preserves the equality of the edges but not necessarily the equality of vertices.
5. An isomorphism preserves the equality of edges and the equality of vertices.

Conclusion

In this paper we have described the neutrosophic graphs with the help of neutrosophic sets. Some operations on neutrosophic graphs are also presented in our work. We have proved that the isomorphism between neutrosophic graphs is an equivalence relation and weak isomorphism between neutrosophic graphs satisfies the partial order relation.

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Received: March 02, 2016. Accepted: June 15, 2016.