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Sequences of Sub-Sequences

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SEQUENCES OF SUB-SEQUENCES

For all of the following sequences:

a) Crescendo Sub-sequences:

1, 1,2, 1,2,3, 1,2,3,4, 1,2,3,4,5, 1,2,3,4,5,6, 1,2,3,4,5,6,7, 1,2,3,4,5,6,7,8, ...

b) Descrescendo Sub-sequences:

1, 2,1, 3,2,1, 4,3,2,1, 5,4,3,2,1, 6,5,4,3,2,1, 7,6,5,4,3,2,1, 8,7,6,5,4,3,2,1, ...

c) Crescenco Pyramidal Sub-sequences:

1, 1,2,1, 1,2,3,2,1, 1,2,3,4,3,2,1, 1,2,3,4,5,4,3,2,1, 1,2,3,4,5,6,5,4,3,2,1,...

d) Descrescenco Pyramidal Sub-sequences:

1, 2,1,2, 3,2,1,2,3, 4,3,2,1,2,3,4, 5,4,3,2,1,2,3,4,5, 6,5,4,3,2,1,2,3,4,5,6,...

e) Crescendo Symmetric Sub-sequences:

1,1, 1,2,2,1, 1,2,3,3,2,1, 1,2,3,4,4,3,2,1, 1,2,3,4,5,5,4,3,2,1,

1,2,3,4,5,6,6,5,4,3,2,1,...

f) Descrescenco Symmetric Sub-sequences:

1,1, 2,1,1,2, 3,2,1,1,2,3, 4,3,2,1,1,2,3,4, 5,4,3,2,1,1,2,3,4,5,

6,5,4,3,2,1,1,2,3,4,5,6,...

g) Permutation Sub-sequences:

1,2, 1,3,4,2, 1,3,5,6,4,2, 1,3,5,7,8,6,4,2, 1,3,5,7,9,10,8,6,4,2,...

find a formula for the general term of the sequence.

Solutions:

For purposes of notatipn in all problems, let $a(n)$ denote the n-th term in the complete sequence and $b(n)$ the n-th subsequence. Therefore, $a(n)$ will be a number and $b(n)$ a sub-sequence.

a) Clearly, $b(n)$ contains n terms. Using a well-known summation formula, at the end of $b(n)$ there would be a total of $\frac{n(n+1)}{2}$ terms. Therefore, since the last number of $b(n)$ is n, $a((n(n+1))/2) = n$. Finally, since this would be the terminal number in the sub-sequence $b(n) = 1, 2, 3, \ldots, n$ the general formula is $a(((n(n+1)/2)-i) = n-i$ for $n \geq 1$ and $0 \leq i \leq n-i$.

b) With modifications for decreasing rather than increasing, the proof is essentialy the same. The final formula is $a(((n(n+1)/2)-i) = 1+i$ for $n \geq 1$ and $0 \leq i \leq n-1$.

c) Clearly, $b(n)$ has $2n-1$ terms. Using the well-known formula of summation $1+3+5+$

$\ldots + (2n-1) = n^2$, the last term of $b(n)$ is n, so counting back $n-1$ positions, they increase in value by one each step until n is reached.

$$a(n^2 - i) = 1 + i, \text{ for } 0 \le i \le n-1.$$

After the maximum value at $n-1$ position back from n^2, the values descreases by one. So at the n-th position back, the value is $n-1$, at the $(n-1)$-st position back the value is $n-2$ and so forth.

$$a(n^2 - n - i) = n - i - 1 \text{ for } 0 \le i \le n-2.$$

d) Using similar reasoning $a(n^2) = n$ for $n \ge 1$ and

$$a(n_2^2 - i) = n - i, \text{ for } 0 \le i \le n-1$$

$$a(n^2 - n - i) = 2 + i, \text{ for } 0 \le i \le n-2.$$

e) Clearly, $b(n)$ contains $2n$ terms. Applying another well-known summation formula $2 + 4 + 6 + \ldots + 2n = n(n+1)$, for $n \ge 1$. Therefore, $a(n(n+1)) = 1$. Counting backwards $n-1$ positions, each term descreases by 1 up to a maximum of n.

$$a((n(n+1)) - i) = 1 + i, \text{ for } 0 \le i \le n-1.$$

The value n psitions down is also n and then the terms descrease by one back down to one.

$$a((n(n+1)) - n - i) = n - i, \text{ for } 0 \le i \le n-1.$$

f) The number of terms in $b(n)$ is the same as that for (e). The only difference is that now the direction of increase/decrease is reversed.

$$a((n(n+1)) - i) = n - i, \text{ for } 0 \le i \le n-1.$$

$$a((n(n+1)) - n - i) = 1 + i, \text{ for } 0 \le i \le n-1.$$

g) Given the following circular permutation on the first n integers.

$$\varphi_n = \begin{vmatrix} 1 & 2 & 3 & 4 & \ldots & n-2 & n-1 & n \\ 1 & 3 & 5 & 7 & \ldots & 6 & 4 & 2 \end{vmatrix}$$

Once again, $b(n)$ has $2n$ terms. Therefore, $a(n(n+1)) = 2$. Counting backwards $n - 1$ positions, each term is two larger than the successor

$$a((n(n+1)) - i) = 2 + 2i, \text{ for } 0 \leq i \leq n-1.$$

The next position down is one less than the previuos and after that, each term is again two less the successor.

$$a((n(n+1)) - n - i) = 2n - 1 - 2i, \text{ for } 0 \leq i \leq n-1.$$

As a single formula using the permutation

$$a((n(n+1)) - i) = \varphi_n(2n - i), \text{ for } 0 \leq i \leq 2n-1.$$

References

[1] F.Smarandache, "Numerical Sequences", University of Craiova, 1975; [See Arizona State University, Special Collection, Tempe, AZ, USA].