FLORENTIN SMARANDACHE

Some Periodical Sequences

In Florentin Smarandache: "Collected Papers", vol. II. Chisinau (Moldova): Universitatea de Stat din Moldova, 1997.

SOME PERIODICAL SEQUENCES

1) Let N be a positive integer with not all digits the same, and N' its digital reverse.

Then, let N_1 =abs($N - N'$), and N_1' its digital reverse. Again, let N_2 =abs($N_1 - N_1'$), N_2' its digital reverse, and so on.

After a finite number of steps one finds an N_j which is equal to a previous N_i, therefore the sequence is periodical [because if N has, say, n digits, all other integers following it will have n digits or less, hence their number is limited, and one applies the Dirichlet's box principle].

Foe examples:

a) If one starts with $N = 27$, then $N' = 72$;

abs$(27 - 72) = 45$; its reverse is 54;

abs$(45 - 54) = 09, \ldots$

thus one gets: $27, 45, 09, 81, 63,\ 27, 45, \ldots$;

the Lentgh of the Period $LP = 5$ numbers $(27, 45, 09, 91, 63)$, and Length of the Sequence 'till the first repetition occurs $LS = 5$ numbers either.

b) If one starts with 52, then one gets:

$52, 27, 45, 09, 81, 63,\ 27, 45, \ldots$;

then $LP = 5$ numbers, while $LS = 6$.

c) If one starts with 42, then one gets:

$42, 18, 63, 27, 45, 09, 81,\ 63, 27, \ldots$;

then $LP = 5$ numbers, while $LS = 7$.

For the sequences of integers of two digits, it seems like: $LP = 5$ numbers $(27, 45, 09, 81, 63)$; or circular permutation of them), and $5 \leq LS \leq 7$.

Question 1: To find the Length of the Period (with its corresponding numbers), and the Length of the Sequence'till the firs repetition ocurrs for: the integers of three digits, and integers of four digits. (It's easier to write a computer programm in these cases to check the LP and LS.)

An example for three digits: $321, 198, 693, 297, 495, 099, 891,\ 693, \ldots$;

(similar to the previous period, just inserting 9 in the middle of each number).

Generalization for the sequences of numbers of n digits.

2) Let N be a positive integer, and N' its digital reverse. For a given positive integer C,

let N_1 =abs($N' - C$) and N_1' its digital reverse. Again, let N_2 =abs($N_1 - C$), N_2' its digital reverse, and so on.

After a finite number of steps one finds an an N_j wich is equal to a previous N_i, therefore the sequence is periodical [same proof].

For example:

If $N = 52$, and $c = 1$, than one gets:

$52, 24, 41, 13, 30, 02, 19, 90, 08, 79, 96, 68, 85, 57, 74, 46, 63, 35, \ 52, \ldots;$

thus $LP = 18, LS = 18$.

Question 2: To find the Length of the Period (with its corresponding numbers), and the Length of the Sequence'till the first repetition occurs (with a given non-null c) for: integers of two digits, and the integers of three digits.

(It's easier to write a computer program in these cases to check the LP and LS.)

Generalization for sequences of numbers of n digits.

3) Let N be a positive integer with n digits $a_1 a_2 \ldots a_n$, and c a given integer > 1.

Multiply each digit a_i of N by c, and replace a_i with the last digit of the product $a_i x c$, say it is b_i. Note $N_1 = b_1 b_2 \ldots b_n$, do the same procedure for N_1, and so on.

After a finite number of steps one finds an N_j which is equal to a previous N_i, therefore the sequence is periodical [same proof].

For exemple:

If $N = 68$ and $c = 7$:

$68, 26, 42, 84, \ 68, \ldots$

thus $LP = 4, LS = 4$.

Question 3: To find the Length of the Period (with its corresponding numbers), and the Length of the Sequence'till the first repetition occurs (with a given c) for: integers of two digits, and the integers of three digits.

(It's easier to write a computer program in these cases to check the LP and LS.)

Generalization for sequences of numbers of n digits.

4.1) Generalized periodical sequence:

Let N be a positive integer with n digits $a_1 a_2 \ldots a_n$. If f is a function defined on the set of integers with n digits or less, and the values of f are also in the same set, then: there exist two natural numbers $i < j$ such that

$$f(f(\dots f(s)\dots)) = f(f(f(\dots f(s)\dots))),$$

where f occurs i times in the left side, and j times in the right side of the previuos equality.

Particularizing f, one obtaines many periodical sequences.

Say: If N has two digits a_1a_2, then: add'em (if the sum is greater than 10, add the resulted digits again), and substruct'em (take the absolute value) - they will be the first, and second digit respectively of N_1. And same procedure for N_1.

Example: 75, 32, 51, 64, 12, 31, 42, 62, 84, 34, 71, 86, 52, 73, 14, 53, 82, 16, 75, ...

4.2) More General:

Let S be a finite set, and $f : S \to S$ a function. Then: for any element s belonging to S, there exist two natural numbers $i < j$ such that

$$f(f(\dots f(s)\dots)) = f(f(f(\dots f(s)\dots))),$$

where f occurs i times in the left side, and j times in the right side of the previuos equality.