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A Property For A Counterexample
To Carmichaël’s Conjecture


[Published in “Gamma”, XXV, Year VIII, No. 3, June 1986, pp. 4-5.]
A PROPERTY FOR A COUNTEREXAMPLE TO CARMICHAËL’S CONJECTURE

Carmichaël has conjectured that:
\[(\forall) \ n \in \mathbb{N}, \ (\exists) \ m \in \mathbb{N}, \text{ with } m \neq n, \text{ for which } \varphi(n) = \varphi(m), \text{ where } \varphi \text{ is Euler’s totient function.} \]

There are many papers on this subject, but the author cites the papers which have influenced him, especially Klee’s papers.

Let \( n \) be a counterexample to Carmichaël’s conjecture.

Grosswald has proved that \( n \) is a multiple of 32, Donnelly has pushed the result to a multiple of \( 2^{14} \), and Klee to a multiple of \( 2^{42} \cdot 3^{47} \). Smarandache has shown that \( n \) is a multiple of \( 2^2 \cdot 3^2 \cdot 7^2 \cdot 43^2 \). Masai & Valette have bounded \( n > 10^{10000} \).

In this note we will extend these results to: \( n \) is a multiple of a product of a very large number of primes.

We construct a recurrent set \( M \) such that:

a) the elements \( 2, 3 \in M \);

b) if the distinct elements \( 2, 3, q_1,..., q_r \in M \) and \( p = 1 + 2^a \cdot 3^b \cdot q_1 \cdots q_r \) is a prime, where \( a \in \{0,1,2,...,41\} \) and \( b \in \{0,1,2,...,46\} \), then \( p \in M; \ r \geq 0 \);

c) any element belonging to \( M \) is obtained only by the utilization (a finite number of times) of the rules a) or b).

Of course, all elements from \( M \) are primes.

Let \( n \) be a multiple of \( 2^{42} \cdot 3^{47} \);

if \( 5 \mid n \) then there exists \( m = 5n/4 \neq n \) such that \( \varphi(n) = \varphi(m) \); hence \( 5 \mid n \); whence \( 5 \in M \);

if \( 5^2 \mid n \) then there exists \( m = 4n/5 \neq n \) with our property; hence \( 5^2 \mid n \);

analogously, if \( 7 \mid n \) we can take \( m = 7n/6 \neq n \), hence \( 7 \mid n \); if \( 7^2 \mid n \) we can take \( m = 6n/7 \neq n \); whence \( 7 \in M \) and \( 7^2 \mid n \); etc.

The method continues until it isn’t possible to add any other prime to \( M \), by its construction.

For example, from the 168 primes smaller than 1000, only 17 of them do not belong to \( M \) (namely: 101, 151, 197, 251, 401, 491, 503, 601, 607, 677, 701, 727, 751, 809, 883, 907, 983); all other 151 primes belong to \( M \).

Note \( M = \{2,3,p_1,...,p_s,...\} \), then \( n \) is a multiple of \( 2^{42} \cdot 3^{47} \cdot p_1^2 \cdot p_2^2 \cdots p_s^2 \cdots \)

From our example, it results that \( M \) contains at least 151 elements, hence \( s \geq 149 \).

If \( M \) is infinite then there is no counterexample \( n \), whence Carmichaël’s conjecture is solved.

(The author conjectures \( M \) is infinite.)

Using a computer it is possible to find a very large number of primes, which divide \( n \), using the construction method of \( M \), and trying to find a new prime \( p \) if \( p - 1 \) is a product of primes only from \( M \).
REFERENCES