New spherical static solution in Gravity field

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ABSTRACT
In the general relativity theory, we discover new solution in gravity field by Einstein’s gravity field equation with cosmological constant term. We treats curvature tensor in new solution.

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1. Introduction

We solve new solution in gravity field by gravity field equation with cosmological constant term.

Gravity field equation with cosmological constant term is in vacuum

$$R_{\mu\nu} = \Lambda g_{\mu\nu}$$  \hspace{1cm} (1)

The spherical coordinate is

$$dr^2 = W(r,t)dt^2 - \frac{1}{c^2} [U(t,r)dr^2 + V(t,r)\{d\theta^2 + \sin^2 \theta d\phi^2\}]$$  \hspace{1cm} (2)

In this time, Einstein’s gravity equation is

$$R_{tt} = \frac{1}{2} \frac{\ddot{U}U - \dot{U}^2}{U^2} + \frac{1}{2} \frac{W'W - W^2}{U^2} + \frac{1}{4} \frac{W'^2}{U^2} - \frac{1}{4} \frac{W'W}{U^2} - \frac{1}{4} \frac{U'W}{U^2} + \frac{1}{2} \frac{V^2}{V^2} + \frac{1}{2} \frac{\dot{W}W - \ddot{W}}{2UW} - \frac{1}{2} \frac{W'V}{2UV} = -\Lambda W$$  \hspace{1cm} (3)

$$R_{\theta\theta} = \frac{1}{2} \frac{W'^2}{W^2} - \frac{1}{4} \frac{W'W}{4UW} + \frac{1}{4} \frac{W'V}{4UW} + \frac{1}{4} \frac{V'U - VU}{2U^2} - \frac{1}{4} \frac{U'V}{4UW} + \frac{1}{4} \frac{U'V}{4U^2} - 1$$  \hspace{1cm} (4)

$$= \Lambda V$$  \hspace{1cm} (5)

$$R_{\phi\phi} = \sin^2 \theta R_{\theta\theta}$$  \hspace{1cm} (6)

$$R_{r\theta} = \frac{V}{V} - \frac{\dot{V}}{2V^2} - \frac{\dot{U}}{2U} - \frac{W}{2U} = 0$$  \hspace{1cm} (7)

In this time,  \hspace{1cm} $\dot{A} = \frac{\partial A}{\partial r}, \overset{\cdot}{A} = \frac{1}{c} \frac{\partial A}{\partial t}$  \hspace{1cm} (8)

2. New spherical static solution in Gravity field

We think

$$W(r,t) = g(r), \quad U(t,r) = 1, \quad V(t,r) = h(r)$$  \hspace{1cm} (9)

In vacuum, Eq(7) is

$$R_{r\theta} = \frac{V}{V} - \frac{\dot{V}}{2V^2} - \frac{\dot{U}}{2U} - \frac{W}{2U} = 0$$  \hspace{1cm} (10)

In vacuum, Eq(3)-(5) is

$$R_{tt} = \frac{1}{2} \frac{1}{U} - \frac{1}{4} \frac{W^2}{W^2} - \frac{WV}{2UV} = -\Lambda W$$
\[
R'' = -\frac{g''}{2} + \frac{g'^2}{4g} - \frac{g'h}{2h} = -\Lambda g \\
R' = \frac{1}{2} W' - \frac{W'H}{4W^2} + \frac{V'}{V} - \frac{1}{2} \frac{V'^2}{V^2} = \Lambda U \\
R'' = \frac{1}{2} \frac{g''}{g} - \frac{g'^2}{4g^2} + \frac{h''}{h} - \frac{1}{2} \frac{h'^2}{h^2} = \Lambda \\
R_{\theta\theta} = \frac{W'V}{4WW} + \frac{V'}{2U} - 1 = \Lambda V \\
R_{\theta\theta} = \frac{g'h}{4g} + \frac{h'}{2} - 1 = \Lambda h
\]

Therefore, Eq(11)-Eq(13) is
\[
-\frac{g''}{2g} + \frac{g'^2}{4g^2} - \frac{g'h}{2hg} = -\Lambda \\
\frac{g''}{2g} - \frac{g'^2}{4g^2} + \frac{h'}{h} - \frac{1}{2} \frac{h'^2}{h^2} = \Lambda \\
\frac{g'h}{4g} + \frac{h'}{2} - 1 = \Lambda h
\]

In Eq(16), if \( h \) is constant, the equation (14)-(16) solved.
\[
h = -\frac{1}{\Lambda}
\]

Hence, Eq(14)-(15) is
\[
\frac{g''}{2g} - \frac{g'^2}{4g^2} = \Lambda
\]

The solution of Eq(18) is
\[
g = \exp(2\sqrt{\Lambda}r)
\]

Therefore, new solution is in vacuum in gravity field
\[
dr^2 = -c^2 dt^2 = -c^2 e \times p\partial(\sqrt{\Lambda}r)dt^2 + dr^2 - \frac{1}{\Lambda}(d\theta^2 + ri\theta d\phi^2)
\]

We treat curvature tensor of new solution.
\[
g_{\mu} = -e \times p\partial(\sqrt{\Lambda}r) , \ g_{\mu} = 1 , \ g_{\theta\theta} = -\frac{1}{\Lambda} \\
\Gamma'_{\mu} = \frac{1}{2} g'' \left(-\frac{\partial g_{\mu}}{\partial r}\right) = \sqrt{\Lambda} e \times p\partial(\sqrt{\Lambda}r) \\
R'' = \frac{\partial \Gamma'_{\mu}}{\partial r} = 2\Lambda \exp(2\sqrt{\Lambda}r) , \ R_{\mu\nu} = g_{\mu}R'_{\mu\nu} = 2\Lambda \exp(2\sqrt{\Lambda}r)
\]
3. Conclusion

Therefore, new spherical solution in gravity field is

\[\text{d}t^2 = \exp(2\sqrt{\Lambda}r)\text{d}t^2 - \frac{1}{c^2}[\text{d}r^2 - \frac{1}{\Lambda}(\text{d}\theta^2 + \sin^2 \theta\text{d}\phi^2)]\]

(21)

According to the variable \(r\), the observer’s light speed is over light velocity \(c\) in vacuum.

Reference