THE DISAPPOINTMENT OF THE RIEMANN HYPOTHESIS

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Abstract

The opinions of this work are revising, stalking and proving in details the derivation of Riemann Zeta Function and Riemann Hypothesis, which Riemann did roughly for more than 150 years ago without proof, and correcting all mistakes about the boundaries of the integrals that was found and those undefined (and/or multiplied by zero) functional equations which caused very big problems to this Riemann Hypothesis.

Proof or disproof of Riemann Hypothesis's derivation will be very useful for many mathematicians and physicists nowadays because the Hypothesis is widely used in many subjects and works, unaware of risks, thought it is not officially proved right or wrong.

Introduction

1. Riemann Zeta Function and Riemann Hypothesis

Riemann Zeta Function is a function of a complex variable $s = (\sigma + it)$ that can be written as the summation of infinite series

$$\zeta(s) = \sum_{n=1}^{+\infty} \left(\frac{1}{n^s}\right)$$

$$= 1+ \frac{1}{2^s} + \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} + \ldots \text{ ; } n = 1, 2, 3, \ldots, +\infty \text{ (all natural numbers)}$$

For $(n)^s = (e)^{sLogn}$; $\zeta(s) = \frac{1}{(e)^{sLog1}} + \frac{1}{(e)^{sLog2}} + \frac{1}{(e)^{sLog3}} + \ldots$

$$\zeta(s) = \frac{1}{(e)^{\sigma Log1}(e)^{itLog1}} + \frac{1}{(e)^{\sigma Log2}(e)^{itLog2}} + \frac{1}{(e)^{\sigma Log3}(e)^{itLog3}} + \ldots$$

$$= r_1(e)^{-itLog1} + r_2(e)^{-itLog2} + r_3(e)^{-itLog3} + \ldots$$

$$= r_1(costLog1 - isintLog1) + r_2(costLog2 - isintLog2)$$

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$+ \mathrm{r}_3(\mathrm{costLog3} - i\mathrm{sintLog3}) + \ldots$

$= \sum_{n=1}^{+\infty} (\mathrm{r_n}(\mathrm{e})^{-\mathrm{itLogn}})$; $\mathrm{r_n} = \frac{1}{(\mathrm{e})^{\sigma\mathrm{Logn}}} = \frac{1}{(\mathrm{n})^{\sigma}}$ = modulus or amplitude and $(\mathrm{e})^{-\mathrm{itLogn}} = (\mathrm{costLogn} - i\mathrm{sintLogn})$ = argument or phasor of ζ(s) on each complex plane of each value of n.

1.1 Riemann Zeta Function of natural numbers on the positive real line while $\mathrm{s} = (\sigma + i\mathrm{t})$ or any complex numbers.

This is the original function of $\zeta(\mathrm{s}) = \sum_{n=1}^{+\infty} (\frac{1}{\mathrm{n^s}})$. The value of real part, σ of complex number $\mathrm{s} = (\sigma + i\mathrm{t})$(or totally the value of the modulus or amplitude of ζ(s)) points out that ζ(s) will converge when real part of s (or σ) is more than 1 and will diverge when real part of s (or σ) is equal or less than 1(with σ = zero and negative numbers too).

While the value of imaginary part, it of complex number s (or totally the phasor or argument (tLogn) of ζ(s)), where Log(n) stands for natural logarithm of n) will show the rotation of modulus or amplitude $\left(\frac{1}{(\mathrm{n})^{\sigma}}\right)$ on the complex plane. The amplitude $\frac{1}{(\mathrm{n})^{\sigma}}$ of ζ(s) will lay on real line (axis) when $i\mathrm{tLogn} = 0, \pm\pi, \pm 2\pi, \ldots$ and will lay on imaginary axis when $\mathrm{tLogn} = \pm\frac{\pi}{2}, \pm\frac{3\pi}{4}, \ldots$. With other angles or arguments (– tLogn), each of all the amplitudes, $\frac{1}{(\mathrm{n})^{\sigma}}$ will rotate around each complex plane of each positive value of n in space.

1.1.1 Case $\sigma > 1$, t = 0

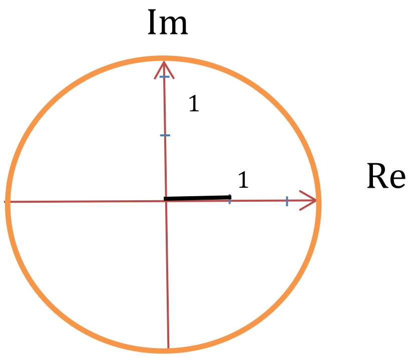
For $\sigma = 2$, t = 0 ; $\zeta(2 + i0) = \sum_{n=1}^{+\infty} (\mathrm{r_n}(\mathrm{e})^{-i\mathrm{tLogn}})$

$$= \mathrm{r}_1(\mathrm{e})^{-i(0)\mathrm{Log1}} + \mathrm{r}_2(\mathrm{e})^{-i(0)\mathrm{Log2}} + \mathrm{r}_3(\mathrm{e})^{-i(0)\mathrm{Log3}} + \ldots$$

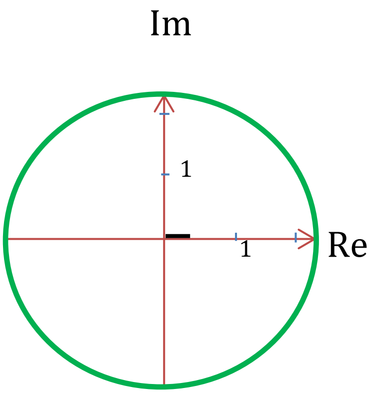
$$= \frac{1}{1^2}[\cos(0) + i\sin(0)] + \frac{1}{2^2}[\cos(0) + i\sin(0)] +$$

$$+ \frac{1}{3^2}[\cos(0) + i\sin(0)] + \ldots$$

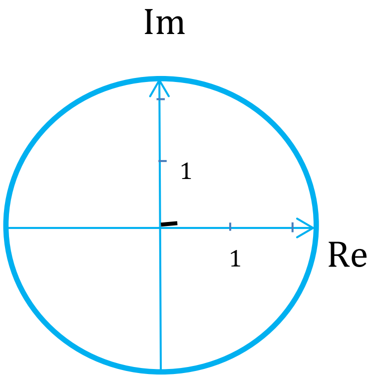
$$= \frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$$



$r_1 = 1$

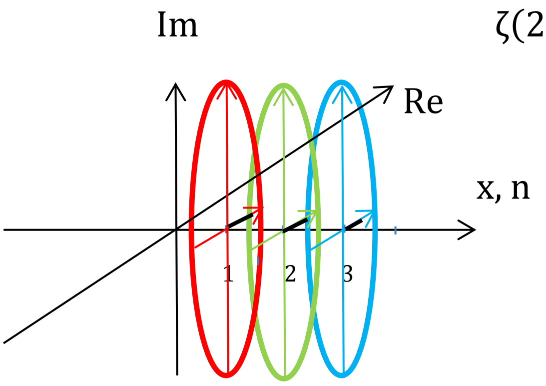


$r_2 = \frac{1}{2^2}$



$r_3 = \frac{1}{3^2}$

………..



$$\zeta(2 + i0) = \sum_{n=1}^{+\infty} \left(r_n (e)^{-it\mathrm{Log}n}\right)$$

$$= r_1 + r_2 + r_3 + \ldots$$

$$= \frac{(\pi)^{(2)}}{6}$$

≈ 1.646 converges while

each amplitude, r_n lays on the real axis of each complex plane of each positive value of n in space.

1.1.2 Case $\sigma > 1$, $\mathrm{t} \neq 0$

For $\sigma = 2$, $\mathrm{t} \neq 0$; $\zeta(2+it) = \sum_{\mathrm{n}=1}^{+\infty} \left(\mathrm{r_n}(\mathrm{e})^{-it\mathrm{Logn}}\right)$

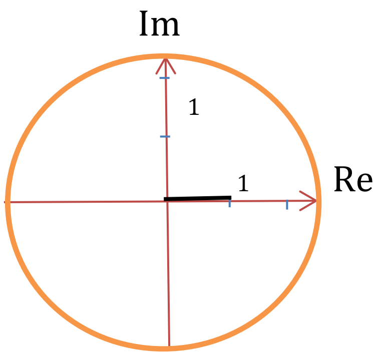
$$= \mathrm{r_1}(\mathrm{e})^{-it\mathrm{Log1}} + \mathrm{r_2}(\mathrm{e})^{-it\mathrm{Log2}} + \mathrm{r_3}(\mathrm{e})^{-it\mathrm{Log3}} + \ldots$$

$$= \frac{1}{1^2}(\mathrm{e})^{-it(0)} + \frac{1}{2^2}(\mathrm{e})^{-it\mathrm{Log2}} + \frac{1}{3^2}(\mathrm{e})^{-it\mathrm{Log3}} + \ldots$$

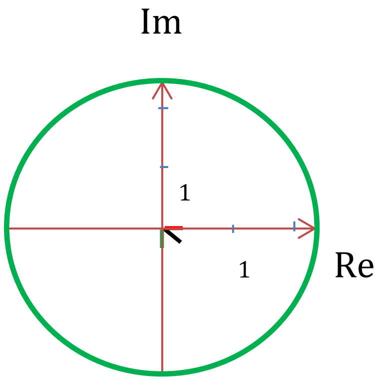
$$= \frac{1}{1^2}[\cos(\mathrm{tLog1}) - i\sin(\mathrm{tLog1})] + \frac{1}{2^2}[\cos(\mathrm{tLog2}) - i\sin(\mathrm{tLog2})] + \frac{1}{3^2}[\cos(\mathrm{tLog3}) - i\sin(\mathrm{tLog3})] + \ldots$$

$$= \sum_{\mathrm{n}=1}^{+\infty} [\mathrm{r_n}\cos(\mathrm{tLogn}) + i\mathrm{r_n}\sin(\mathrm{tLogn})]$$

Then $\zeta(2+it)$ are series of amplitudes (moduli) $\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2}, \ldots = (\mathrm{r_n})$, those will rotate around each of all the complex planes of each value of n with arguments or phasors or angles of $-\mathrm{tLog1}, -\mathrm{tLog2}, -\mathrm{tLog3}, \ldots$ and $\zeta(2+it)$ will always converge up to the value of $\mathrm{r_n}$ only. The value of $\cos(\mathrm{tLogn})$ or the value of $\sin(\mathrm{tLogn})$ (maximum or minimum or between) will not change the convergence of $\zeta(2+it)$.



$\mathrm{r_1} = 1$, $(\mathrm{e})^{-it\mathrm{Log1}} = [\mathrm{costLog1} - i\mathrm{sintLog1}]$

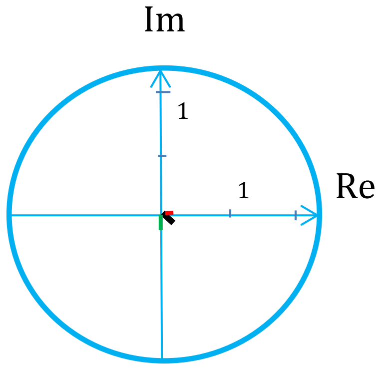


$\mathrm{r_2} = \frac{1}{2^2}$, $(\mathrm{e})^{-it\mathrm{Log2}} = [\mathrm{costLog2} - i\mathrm{sintLog2}]$

$= \mathrm{r_2}(\mathrm{e})^{-it\mathrm{Log2}}$

$= \mathrm{r_2costLog2}$

$= i\mathrm{r_2sintLog2}$

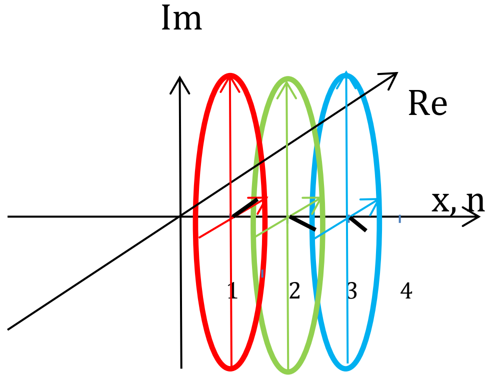


$r_3 = \frac{1}{3^2}$, $(e)^{-it\text{Log}3} = [\cos t\text{Log}3 - i\sin t\text{Log}3]$

$= r_3(e)^{-it\text{Log}3}$

$= r_3\cos t\text{Log}3$

$= i r_3 \sin t\text{Log}3$



$\zeta(2+it) = \sum_{n=1}^{+\infty} \left(r_n(e)^{-it\text{Log}n}\right)$

$= \frac{1}{1^2}(e)^{-it\text{Log}1} + \frac{1}{2^2}(e)^{-it\text{Log}2}$

$+ \frac{1}{3^2}(e)^{-it\text{Log}3} + \cdots$

$= \sum_{n=1}^{+\infty} [r_n\cos t\text{Log}n + i\, r_n \sin t\text{Log}n]$

$\zeta(2+it)$ always converges while each amplitude lays on each complex plane of each positive value of n in space.

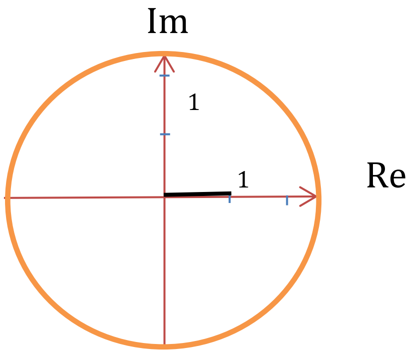
1.1.3 Case $\sigma = 1, t = 0$

For $\sigma = 1, t = 0$; $\zeta(1+i0) = \sum_{n=1}^{+\infty} \left(r_n(e)^{-it\text{Log}n}\right)$

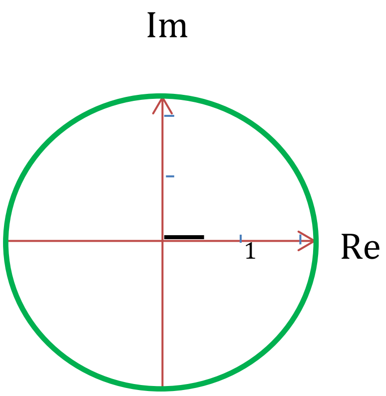
$= r_1(e)^{-i(0)\text{Log}1} + r_2(e)^{-i(0)\text{Log}2} + r_3(e)^{-i(0)\text{Log}3} + \ldots$

$= \frac{1}{1^1} + \frac{1}{2^1} + \frac{1}{3^1} + \ldots$ (Harmonic series)

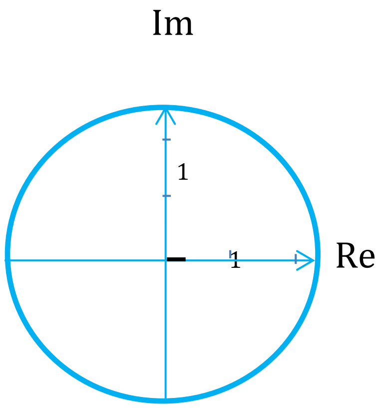
$= +\infty$



$r_1 = 1$

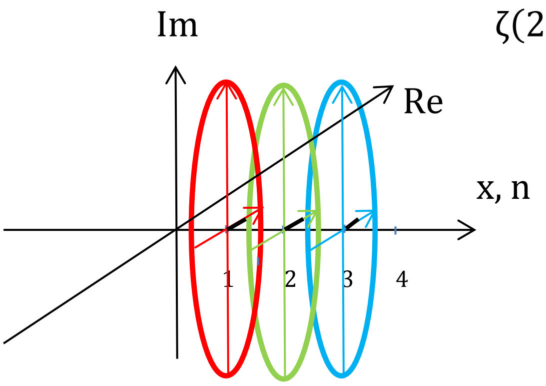


$r_2 = \frac{1}{2}$



$r_3 = \frac{1}{3}$

....



$$\zeta(2 + i0) = \sum_{n=1}^{+\infty} \left(r_n (e)^{-itLogn}\right)$$

$$= r_1 + r_2 + r_3 + \ldots$$

$$= \frac{1}{1^1} + \frac{1}{2^1} + \frac{1}{3^1} + \ldots$$

$= \infty$ diverges while each amplitude lays on the real axis of each complex plane of each positive value of n in space.

1.1.4 Case $\sigma = 1, t \neq 0$

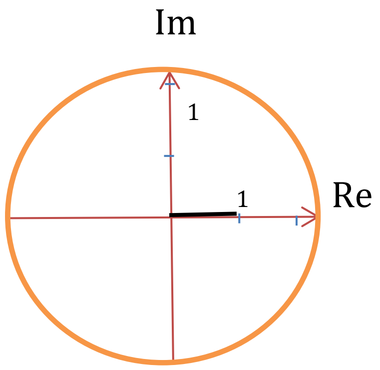
For $\sigma = 1, t \neq 0$; $\zeta(1 + it) = \sum_{n=1}^{+\infty} \left(r_n (e)^{-itLogn}\right)$

$$= r_1 (e)^{-itLog1} + r_2 (e)^{-itLog2} + r_3 (e)^{-itLog3} + \ldots$$

$$= \frac{1}{(1)^1} (e)^{-it(0)} + \frac{1}{(2)^1} (e)^{-itLog2} + \frac{1}{(3)^1} (e)^{-itLog3} + \ldots$$

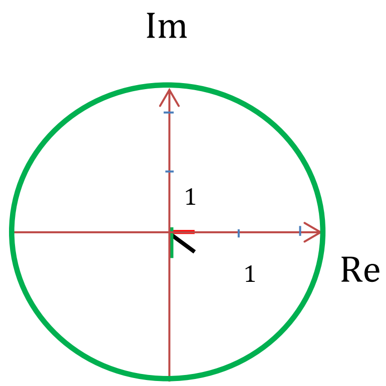
The $\zeta(1+it)$ are series of the amplitudes (moduli) $1, \frac{1}{2}, \frac{1}{3}, \ldots = (r_n)$, those are rotating around the complex plane with arguments or angles *of* $-$ tLog1, $-$tLog2, $-$tLog3, ...

$\zeta(1+it)$ may converge on real axis of the complex plane if all of the values of cos(tLogn) are nearly zero or all of the values of (tLogn) are nearly $\frac{\pi}{2}, \frac{3\pi}{4}$, ...). But, on imaginary axis, it still diverges. This is impossible case because the value of n always changes from 1 to$+\infty$ and causes the argument or angle to change too. So $\zeta(1+it)$ always diverges up to the value of infinite sum of all the amplitudes $\left(\sum_{n=1}^{+\infty} \frac{1}{(n)^{\frac{1}{2}}} = +\infty\right)$ with no effect from the arguments, $(e)^{-it\mathrm{Logn}}$ (note that $\infty \times k = \infty$) .



$r_1 = \frac{1}{1^1}$, $(e)^{-it\mathrm{Log1}} = [\mathrm{costLog1}$

$-i\mathrm{sintLog1}]$



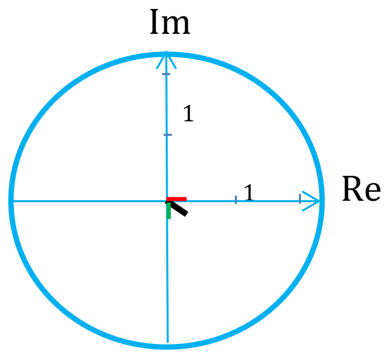
$r_2 = \frac{1}{2^1}$, $(e)^{-it\mathrm{Log2}} = [\mathrm{costLog2}$

$-i\mathrm{sintLog2}]$

$= r_2(e)^{-it\mathrm{Log2}}$

$= r_2\mathrm{costLog2}$

$= ir_2\mathrm{sintLog2}$



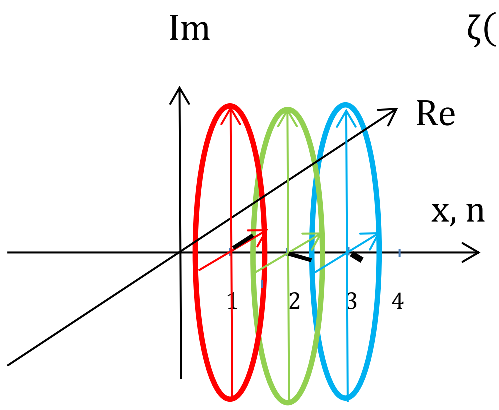
$r_3 = \frac{1}{3^1}$, $(e)^{-it\mathrm{Log3}} = [\mathrm{costLog3}$

$-i\mathrm{sintLog3}]$

$= r_3(e)^{-it\mathrm{Log3}}$

$= r_3\cos(\mathrm{tLog3})$

$= ir_3\sin(\mathrm{tLog3})$



$$\zeta(1+it) = \sum_{n=1}^{+\infty} \left(r_n(e)^{-it\mathrm{Log}n}\right)$$
$$= \frac{1}{1^1}(e)^{-it\mathrm{Log}1} + \frac{1}{2^1}(e)^{-it\mathrm{Log}2} + \frac{1}{3^1}(e)^{-it\mathrm{Log}3} + \cdots$$
$$= \sum_{n=1}^{+\infty} [r_n\cos(t\mathrm{Log}n) + ir_n\sin(t\mathrm{Log}n)]$$

$\zeta(1+it)$ always diverges up to the infinite sum of all the amplitudes, r_n of each positive value of n in space.

1.1.5 Case $\sigma < 1$, t = 0

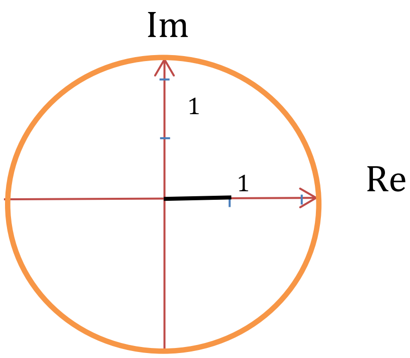
For $\sigma = \frac{1}{2}, t = 0$; $\zeta\left(\frac{1}{2} + i0\right) = \sum_{n=1}^{+\infty} \left(r_n(e)^{-it\mathrm{Log}n}\right)$

$$= r_1(e)^{-i(0)\mathrm{Log}1} + r_2(e)^{-i(0)\mathrm{Log}2} + r_3(e)^{-i(0)\mathrm{Log}3} + \ldots$$
$$= \frac{1}{(1)^{\frac{1}{2}}} + \frac{1}{(2)^{\frac{1}{2}}} + \frac{1}{(3)^{\frac{1}{2}}} + \ldots$$
$$= \frac{(1)^{\frac{1}{2}}}{1} + \frac{(2)^{\frac{1}{2}}}{2} + \frac{(3)^{\frac{1}{2}}}{3} + \ldots$$
$$= 1 + \frac{1.414}{2} + \frac{1.732}{3} + \ldots$$

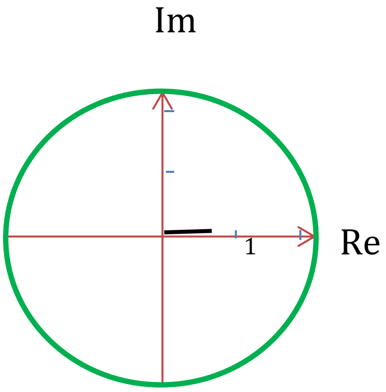
But from Harmonic series $1 + \frac{1}{2} + \frac{1}{3} + \ldots \quad = +\infty$

And from $1 + \frac{1.414}{2} + \frac{1.732}{3} + \ldots > 1 + \frac{1}{2} + \frac{1}{3} + \ldots$

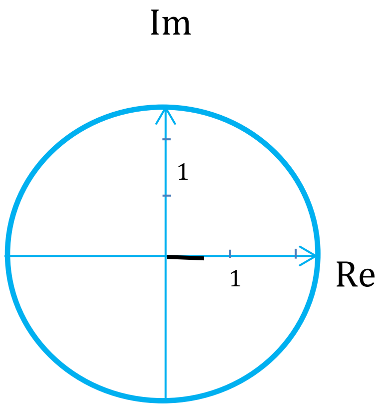
Then $\quad \zeta\left(\frac{1}{2} + i0\right) = 1 + \frac{1.414}{2} + \frac{1.732}{3} + \ldots \; = +\infty$



$r_1 = 1$

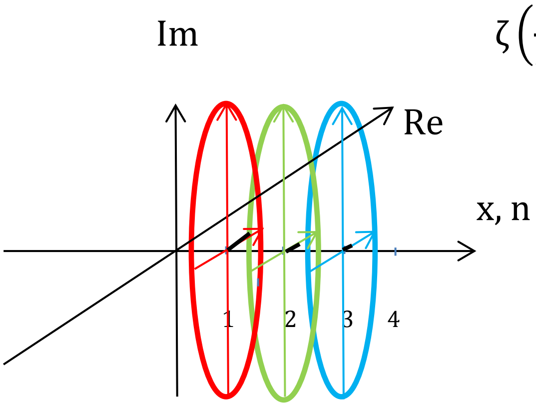


$$r_2 = \frac{1}{(2)^{\frac{1}{2}}}$$



$$r_3 = \frac{1}{(3)^{\frac{1}{2}}}$$

....



$$\zeta\left(\frac{1}{2} + i0\right) = \sum_{n=1}^{+\infty} \left(r_n (e)^{-it\text{Log}n}\right)$$

$$= r_1 + r_2 + r_3 + \ldots$$

$$= \frac{1}{(1)^{\frac{1}{2}}} + \frac{1}{(2)^{\frac{1}{2}}} + \frac{1}{(3)^{\frac{1}{2}}} + \ldots$$

$= \infty$ diverges while each Amplitude, r_n lays on the real axis of each complex plane of each positive value of n in space.

1.1.6 Case $\sigma < 1$, $t \neq 0$

For $\sigma = \frac{1}{2}, t \neq 0$; $\zeta\left(\frac{1}{2} + it\right) = \sum_{n=1}^{+\infty} \left(r_n (e)^{-it\text{Log}n}\right)$

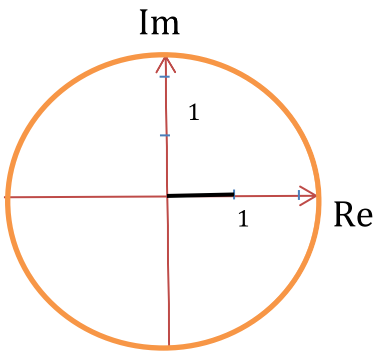
$$= r_1 (e)^{-it\text{Log}1} + r_2 (e)^{-it\text{Log}2} + r_3 (e)^{-it\text{Log}3} + \ldots$$

$$= \frac{1}{(1)^{\frac{1}{2}}}(e)^{-it(0)} + \frac{1}{(2)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}2} + \frac{1}{(3)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}3} + \ldots$$

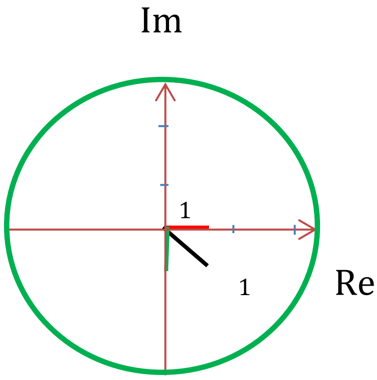
$$= 1 + \frac{(2)^{\frac{1}{2}}(e)^{-it\mathrm{Log}2}}{2} + \frac{(3)^{\frac{1}{2}}(e)^{-it\mathrm{Log}3}}{3} + \ldots$$

The $\zeta\left(\frac{1}{2}+it\right)$ are series of amplitudes (moduli) $1, \frac{1}{(2)^{\frac{1}{2}}}, \frac{1}{(3)^{\frac{1}{2}}}, \ldots = (r_n)$, those are rotating around the complex planes of each value of n with arguments or angles $-t\mathrm{Log}1, -t\mathrm{Log}2, -t\mathrm{Log}3, \ldots$

$\zeta\left(\frac{1}{2}+it\right)$ will converge on real axis of all of the complex planes if all of the values of cos(tLogn) are nearly 0 (or all of the values of (tLogn) are nearly $\frac{\pi}{2}, \frac{3\pi}{4}$, ...).But, on imaginary axis, it still diverges. This is impossible case because the value of n always changes from 1 to$+\infty$ and causes the argument or angle to change too. So $\zeta\left(\frac{1}{2}+it\right)$ always diverges up to the value of infinite sum of all the amplitudes $\left(\sum_{n=1}^{+\infty} \frac{1}{(n)^{\frac{1}{2}}} = +\infty\right)$ with no effect from the arguments $(e)^{-it\mathrm{Log}n}$ (note that $\infty \times k = \infty$)



$r_1 = \frac{1}{(1)^{\frac{1}{2}}}$, $(e)^{-it\mathrm{Log}1} = [\cos t\mathrm{Log}1 - i\sin t\mathrm{Log}1]$

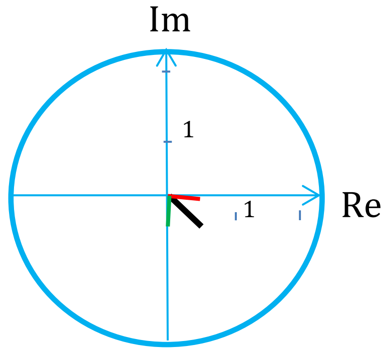


$r_2 = \frac{1}{(2)^{\frac{1}{2}}}$, $(e)^{-it\mathrm{Log}2} = [\cos t\mathrm{Log}2 - i\sin t\mathrm{Log}2]$

$= r_2(e)^{-it\mathrm{Log}2}$

$= r_2\cos t\mathrm{Log}2$

$= ir_2\sin t\mathrm{Log}2$



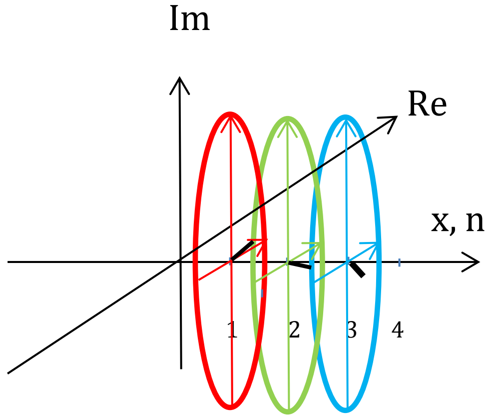
$r_3 = \frac{1}{(3)^{\frac{1}{2}}}$, $(e)^{-it\mathrm{Log}3} = [\mathrm{cos}t\mathrm{Log}3$

$-i\mathrm{sin}t\mathrm{Log}3]$

$= r_3(e)^{-it\mathrm{Log}3}$

$= r_3\cos(t\mathrm{Log}3)$

$= ir_3\sin(t\mathrm{Log}3)$



$\zeta(1+it) = \sum_{n=1}^{+\infty} (r_n(e)^{-it\mathrm{Log}n})$

$= \frac{1}{(1)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}1} + \frac{1}{(2)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}2}$

$+ \frac{1}{(3)^{\frac{1}{2}}}(e)^{-it\mathrm{Log}3} + \cdots$

$= \sum_{n=1}^{+\infty} [r_n\cos(t\mathrm{Log}n)$

$+ ir_n\sin(t\mathrm{Log}n)]$

$\zeta(1+it)$ always diverges up to the infinite sum of all the amplitudes, r_n of each positive value of n in space.

1.2 Riemann Zeta Function of natural numbers on the negative real Line (actually on the whole real line) while $s = (\sigma + it)$ or any complex numbers.

Riemann tried to find the Riemann Zeta Function of natural numbers on the negative real Line as well as on the positive real line while s $= (\sigma + it)$ = any complex numbers by applying analytic continuation on an equation. He started from using Pi function, $\prod(s-1)$ = Gamma function, $\Gamma(s) = \int_0^{+\infty}(e)^{(-u)}(u)^{(s-1)}\,du$ as source of his new derived equation $\prod(s-1)\zeta(s) = \int_0^{+\infty}\frac{(x)^{(s-1)}}{(e^x-1)}dx$. Next he applied analytic continuation technique to get functional equation $2\sin(\pi s)\prod(s-1)\zeta(s) = i\int_{+\infty}^{+\infty}\frac{(-x)^{(s-1)}}{(e^x-1)}dx$ of x on the whole real line and then he gave out trivial zeroes or Res $= \sigma =$

$-2, -4, -6, \ldots$ which he thought that they caused the functional equation
$2\sin(\pi s)\prod(\mathrm{s}-1)\zeta(\mathrm{s}) = i\int_{+\infty}^{+\infty}\frac{(-\mathrm{x})^{(s-1)}}{(\mathrm{e}^{\mathrm{x}}-1)}\mathrm{dx} = 0.$

The connection between the Riemann Zeta Function and Prime

numbers was discovered by Euler as

$$\sum_{n=1}^{+\infty}\left(\frac{1}{n^s}\right) = \prod_{\text{p prime}}\left[\frac{1}{(1-\frac{1}{p^s})}\right]$$

p = 2, 3, 5, … (all prime numbers)

While $\prod_{p\ prime}\left[\frac{1}{(1-\frac{1}{p^s})}\right] = \frac{1}{\left(1-\frac{1}{2^s}\right)\left(1-\frac{1}{3^s}\right)\left(1-\frac{1}{5^s}\right)\left(1-\frac{1}{7^s}\right)\left(1-\frac{1}{11^s}\right)\ldots}$ is called the Euler Product Formula

In Riemann's 1859 article "On the Number of Primes Less Than a Given Magnitude", he extended the Euler Product Formula to a complex variable, presented its meromorphic continuation and functional equation, and established a relation between its zeroes (if existed) and the distribution of prime numbers.

The Riemann Zeta Function was hoped to satisfy the Riemann Functional Equation below

$$\zeta(\mathrm{s}) = (2)^{(\mathrm{s})}\pi^{(\mathrm{s}-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-\mathrm{s})\zeta(1-\mathrm{s}).$$

This equation related values of Riemann Zeta Function at point s = (1− s).The functional equation implied that ζ(s) had zeroes at each negative even integer s =−2, −4,…(which has to be proved whether it is true or false).

Riemann also defined a function

$$\prod\left(\frac{s}{2}\right)\left(\frac{s}{2}-1\right)(\pi)^{\left(-\frac{s}{2}\right)}\zeta(\mathrm{s}) = \frac{1}{(\mathrm{s})(\mathrm{s}-1)} + \int_1^{+\infty}\psi(\mathrm{x})\left[(\mathrm{x})^{\left(\frac{s}{2}-1\right)}+(\mathrm{x})^{-\left(\frac{1+s}{2}\right)}\right]\mathrm{dx}$$

And set $\mathrm{s} = \frac{1}{2} + \mathrm{it}$

So, $\prod\left(\frac{s}{2}\right)\left(\frac{s}{2}-1\right)(\pi)^{\left(-\frac{s}{2}\right)}\zeta(\mathrm{s}) = \frac{1}{2} + \frac{\left(tt+\frac{1}{4}\right)}{2}\int_1^{+\infty}\psi(\mathrm{x})(\mathrm{x})^{-\left(\frac{3}{4}\right)}\cos\left(\frac{1}{2}\mathrm{tLogx}\right)\mathrm{dx}$

$$= \xi(\mathrm{t})$$

(Log x = natural logarithm of x)

Riemann Hypothesis states that all nontrivial zeroes of the Riemann Zeta Function have their real parts equal to $\frac{1}{2}$ or all nontrivial zeroes are in the open strip { s ϵ C : 0 < Res < 1} which is called the critical strip, and the line Res = $\frac{1}{2}$ which all of the nontrivial zeroes lie on is called the critical line.

2. Gamma function

Gamma function is an extension of the factorial function n! (which is the product of all positive integers less than or equal to n), with its argument shifted by 1, to real line and complex plane.

$\Gamma(n) = (n\text{-}1)!$, for n = 1, 2, 3, ... (positive integers)

$\Gamma(s+1) = s\,\Gamma(s)$

And $\Gamma(s) = \int_0^{+\infty} (e)^{(-x)}\,(x)^{(s-1)}dx$, for s = complex numbers $(\sigma + it)$ with a positive real part, Res $= \sigma = 1, 2, 3, \ldots$

This improper integral can be extended by analytic continuation technique to all real and complex numbers except the non-positive integers, (where Γ(s) has simple poles), yielding the meromorphic function.

3. Gamma function on the complex plane

Gamma function, which defines in positive half- complex plane, has a unique analytic continuation to the negative half- complex plane.

$$\Gamma(s) = \frac{\Gamma(s+k)}{s(s+1)(s+2)\ldots(s+k-1)}$$

k =0, 1, 2, 3, ... and $s+k > 0$, or $s > -k$, $s \neq 0, -1, -2, \ldots, -(k-1)$.

The product of the denominator is zero when s equals any integer 0, 1, 2, 3,... Then the gamma function must be undefined at those points. It is a meromorphic function with simple poles at those non-positive integers.

The gamma function is non-zero everywhere along the real line, although it comes nearly close to zero as $s \to \infty$. There is no complex number s for which $\Gamma(s) = 0$ and hence the reciprocal gamma function $\frac{1}{\Gamma(s)}$ is an entire function with zeroes at $s = 0, -1, -2, -3, \ldots$

4. Analytic function and analytic continuation technique

Analytic function is a function that is locally given by a convergent power series. There are both real and complex analytic functions. Analytic functions of each type are infinitely differentiable. A function is analytic if and only if its Taylor series about x_0 converges to the function in some neighborhood for every x_0 in its domain.

Any real analytic function on some open set on the real line can be extended to a complex analytic function on some open set of the complex plane. However, not every real analytic function defined on the whole real line can be extended to a complex analytic function defined on the whole complex plane. For example, the function $f(x) = \frac{1}{x^2+1}$ is not defined for $x = \pm i$.

Analytic continuation is a technique to extend the domain of a given analytic function. Analytic continuation often succeeds in defining further values of a function, for example in a new region where an infinite series represent in terms of which it is initially defined to be divergent.

Suppose f is an analytic function defined on a non-empty open subset U of the complex plane C. If V is a larger open subset than U of C (U is contained in V), and F is an analytic function defined on V such that $F(z) = f(z)$, then F is called an analytic continuation of f. The restriction of F to U is the function f we started with.

5. Pi function

Pi function is an alternative notation which was originally introduced by Gauss which in term of the gamma function is

$$\begin{aligned} \prod(s) &= \Gamma(s+1) \\ &= s\Gamma(s) \\ &= \int_0^{+\infty} (e)^{(-x)} (x)^{(s)} dx \end{aligned}$$

Or $\prod(n) = n!$, for $n = 1, 2, 3, \ldots$ (positive integers)

6. Functional equation

A functional equation is any equation that cannot be reduced to algebraic equation easily. The equation relates the values of a function (or functions) at some points with its values at the other points, for example

$$f(x+y) = f(x) + f(y)$$

A main method to solve elementary functional equation is substitution.

7. Cauchy's Integral Theorem

Cauchy's Integral Theorem implies that the line integral of every holomorphic function along a loop vanishes.

$$\oint_{\gamma} f(z)\,dz = 0$$

Here γ is a rectifiable path in a simply connected open subset U of the complex plane C whose starting point is equal to its end point, and f: U→ C is a holomorphic function.

<u>Disproof of Riemann Zeta Function's derivation and Riemann Hypothesis</u>

1. Proof that $\zeta(s)\prod(s-1) \neq \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e^x-1)} dx$, $s = \sigma + it$, $\mathrm{Re}s = \sigma > 1$, x = zero and positive real numbers

From $\prod(s-1) = \Gamma(s)$

$$= \int_0^{+\infty} (e)^{(-x)}\,(x)^{(s-1)}\,dx \quad , s = \sigma + it \quad , \mathrm{Re}s = \sigma > 0$$

When defined $\Gamma(s)$ in term of an improper integral, s = all complex numbers ($\sigma + it$) with positive real part, and x = zero and positive real numbers, or in term of a limit

$$\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_a^b (e)^{(-x)}\,(x)^{(s-1)} dx \ , s = \sigma + it \quad , \mathrm{Re}s = \sigma > 0$$

Riemann tried to substitute nx for x as the starting variable in the integrands, he derived his equation as

$$\prod(\mathrm{s}-1) = \int_0^{+\infty} (\mathrm{e})^{(-\mathrm{nx})}\,(\mathrm{nx})^{(\mathrm{s}-1)}\mathrm{dnx} \quad , s = \sigma + it \quad , \mathrm{Re}s = \sigma > 0$$

$n = 1, 2, 3, \ldots, +\infty$

Or in term of a limit,

$$\prod(\mathrm{s}-1) = \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{nx})}\,(\mathrm{nx})^{(\mathrm{s}-1)}\mathrm{dnx}$$

But he forgot to change the boundaries of the integral (or the value of nx) after he separated out the variable of the integrands from nx to n and x, and moved term $(\mathrm{n})^{(\mathrm{s})}$ to the left hand side as he might have done below

$$\prod(\mathrm{s}-1) = \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{nx})}\,(\mathrm{n})^{(\mathrm{s})}(\mathrm{x})^{(\mathrm{s}-1)}\mathrm{dx}$$

Multiply by $\left(\frac{1}{\mathrm{n}^{\mathrm{s}}}\right)$ both sides

$$\left(\tfrac{1}{\mathrm{n}^{\mathrm{s}}}\right)\prod(\mathrm{s}-1) = \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{x})(\mathrm{n})}\,(\mathrm{x})^{(\mathrm{s}-1)}\mathrm{dx}$$

$$\left(\tfrac{1}{1^{\mathrm{s}}}\right)\prod(\mathrm{s}-1) = \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{x})(1)}\,(\mathrm{x})^{(\mathrm{s}-1)}\mathrm{dx}$$

$$\left(\tfrac{1}{2^{\mathrm{s}}}\right)\prod(\mathrm{s}-1) = \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{x})(2)}(\mathrm{x})^{(\mathrm{s}-1)}\mathrm{dx}$$

…

$$\left(\tfrac{1}{\infty^{\mathrm{s}}}\right)\prod(\mathrm{s}-1) = \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{x})(\mathrm{b})}\,(\mathrm{x})^{(\mathrm{s}-1)}\mathrm{dx}$$

Then Riemann took summation of the above equations from $\left(\frac{1}{1^{\mathrm{s}}}\right)\prod(\mathrm{s}-1)$ to $\left(\frac{1}{\infty^{\mathrm{s}}}\right)\prod(\mathrm{s}-1)$ and he thought that terms $(\mathrm{e})^{(-\mathrm{x})(\mathrm{b})}$ of all the right hand sides could be sum to infinite or $= \lim_{\mathrm{b}\to+\infty} \sum_{n=1}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{x})(\mathrm{n})}$. So he made the summation.

$$\left[\left(\tfrac{1}{1^{\mathrm{s}}}\right) + \left(\tfrac{1}{2^{\mathrm{s}}}\right) + \ldots + \left(\tfrac{1}{\infty^{\mathrm{s}}}\right)\right]\prod(\mathrm{s}-1)$$

$$= \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} \left[(\mathrm{e})^{(-\mathrm{x})(1)} + (\mathrm{e})^{(-\mathrm{x})(2)} + \cdots + (\mathrm{e})^{(-\mathrm{x})(\mathrm{b})}\right](\mathrm{x})^{(\mathrm{s}-1)}\,\mathrm{dx}$$

$$\sum_{n=1}^{+\infty} \left(\tfrac{1}{n^{s}}\right)\prod(\mathrm{s}-1) = \lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty} \int_{\mathrm{a}}^{\mathrm{b}} \left[\sum_{n=1}^{\mathrm{b}} (\mathrm{e})^{(-\mathrm{x})(\mathrm{n})}\right](\mathrm{x})^{(\mathrm{s}-1)}\,\mathrm{dx}$$

From Geometric series $\sum_{n=1}^{+\infty} (e)^{(-x)(n)} = (\frac{(e)^{(-x)}}{1-(e)^{(-x)}}) = (\frac{1}{(e)^{(x)}-1})$

Then $\zeta(s)\prod(s-1) = \lim_{a\to 0,b\to+\infty} \int_a^b \frac{(x)^{(s-1)}}{(e)^{(x)}-1} dx$

Or $\zeta(s)\prod(s-1) = \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e)^{(x)}-1} dx \quad , s = \sigma + it \ , \operatorname{Re}s = \sigma > 1 \ \ldots \text{(A)}$

As you can see, the above equation (integral) is wrong because the boundaries of the integral were still from a=0 to b=+∞. Actually the boundaries had to be changed after separating variable of the integrands from nx_n to x_n and n, preparing for moving $(n)^{(s)}$ to the left hand side.

From $\prod(s-1) = \lim_{a\to 0,b\to+\infty} \int_a^b (e)^{(-x_n)(n)} (nx_n)^{(s-1)} dnx_n \ , s = \sigma + it$

$, \operatorname{Re}s = \sigma > 0$

$n = 1, 2, 3, \ldots, +\infty$

Then $\prod(s-1) = \lim_{a\to 0,b\to+\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (n)^{(s)} (x_n)^{(s-1)} dx_n$

Term $(n)^{(s)}$ is separated out from the integral and moved to the left hand side of the equation and then n is increased in value from 1 to +∞ in each equation and finally combined to become $\sum_{n=1}^{+\infty} (\frac{1}{n^s})$ or $\zeta(s)$ at the left hand side. The values of nx_n change from 0 to +∞ and x_n is also changing upto increasing value of n of those integrals at the right hand side. The lower boundaries of those integrals vary from a/1 to a/+∞ and the upper boundaries vary from b/1 to b/+∞, (remember that nx had boundaries from 0 to +∞ in the original integral), as below.

Multiply by $\frac{1}{n^s}$ both sides

$$\left(\frac{1}{n^s}\right) \prod(s-1) = \lim_{a\to 0,b\to+\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (x_n)^{(s-1)} dx_n$$

Substitute $s = (\sigma + it)$ and $(x_n)^{(s-1)} = (e)^{(\sigma+it-1)\mathrm{Log}(x_n)}$

$$\left(\frac{1}{n^s}\right) \prod(s-1) = \left(\frac{1}{(e)^{(\sigma+it)\mathrm{Log}(n)}}\right) \prod(\sigma + it - 1)$$

$$= \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (e)^{(\sigma+it-1)\mathrm{Log}(x_n)}\, dx_n$$

$$= \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)+(\sigma+it-1)\mathrm{Log}(x_n)}\, dx_n$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{1}{-(n)+\frac{(\sigma-1)}{(x_n)}+\frac{(it)}{(x_n)}}][(e)^{(-x_n)(n)+(\sigma+it-1)\mathrm{Log}(x_n)}]\,]_{a/n}^{b/n}$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(x_n)^{(\sigma-1)}}{(e)^{(x_n)(n)}}][\frac{(x_n)^{(it)}}{[-(n)+\frac{(\sigma-1)}{(x_n)}+\frac{(it)}{(x_n)}]}]\,]_{a/n}^{b/n}$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(x_n)^{(\sigma-1)}(x_n)^{(it)}}{-(n)(e)^{(x_n)(n)}+\frac{(\sigma-1)}{(x_n)}(e)^{(x_n)(n)}+\frac{(it)}{(x_n)}(e)^{(x_n)(n)}}]\,]_{a/n}^{b/n}$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(b/n)^{(\sigma-1)}(b/n)^{(it)}}{-(n)e^{(b/n)n}+\frac{(\sigma-1)}{(b/n)}e^{(b/n)(n)}+\frac{(it)}{(b/n)}e^{(b/n)(n)}}]$$

$$-[\frac{(a/n)^{(\sigma-1)}(a/n)^{(it)}}{-(n)(e)^{(a/n)n}+\frac{(\sigma-1)}{(a/n)}e^{(a/n)(n)}+\frac{(it)}{(a/n)}e^{(a/n)(n)}}]$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(b)^{(\sigma-1)}(b)^{(it)}(n)^{(-it)}}{-(n)^{(\sigma)}(e)^{(b)}+(n)^{(\sigma)}\frac{(\sigma-1)}{(b)}e^{(b)}+(n)^{(\sigma)}\frac{(it)}{(b)}e^{(b)}}]$$

$$-[\frac{(a)^{(\sigma-1)}(a)^{(it)}(n)^{(-it)}}{-(n)^{(\sigma)}(e)^{(a)}+(n)^{(\sigma)}\frac{(\sigma-1)}{(a)}e^{(a)}+(n)^{(\sigma)}\frac{(it)}{(a)}e^{(a)}}]$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(b)^{(\sigma-1)}(e)^{(it\mathrm{Log}b)}(n)^{(-it)}}{-(n)^{(\sigma)}(e)^{(b)}+(n)^{(\sigma)}\frac{(\sigma-1)}{(b)}e^{(b)}+(n)^{(\sigma)}\frac{(it)}{(b)}e^{(b)}}]$$

$$-[\frac{(a)^{(\sigma-1)}(e)^{(it\mathrm{Log}a)}(n)^{(-it)}}{-(n)^{(\sigma)}(e)^{(a)}+(n)^{(\sigma)}\frac{(\sigma-1)}{(a)}e^{(a)}+(n)^{(\sigma)}\frac{(it)}{(a)}e^{(a)}}]$$

$$= \lim_{a\to 0, b\to +\infty} [\frac{(b)^{(\sigma-1)}[\cos(t\mathrm{Log}b)+i\sin(t\mathrm{Log}b)](n)^{(-it)}}{-(n)^{(\sigma)}(e)^{(b)}+(n)^{(\sigma)}\frac{(\sigma-1)}{(b)}e^{(b)}+(n)^{(\sigma)}\frac{(it)}{(b)}e^{(b)}}]$$

$$-[\frac{(a)^{(\sigma-1)}[\cos(t\mathrm{Log}a)+i\sin(t\mathrm{Log}a)](n)^{(-it)}(n)^{(-it)}}{-(n)^{(\sigma)}(e)^{(a)}+(n)^{(\sigma)}\frac{(\sigma-1)}{(a)}e^{(a)}+(n)^{(\sigma)}\frac{(it)}{(a)}e^{(a)}}]$$

[$\lim_{a\to 0} \mathrm{Log}\,(a) = z$, or $\mathrm{Log}\,(\approx 0) = z$,

But $(e)^{(z)} \approx 0$, or $(e)^{(-\infty)} = \frac{1}{(e)^{(\infty)}} \approx 0$, So $\lim_{a\to 0}$ Log (a) or $\lim_{a\to 0}$ Log $(\approx 0) \approx -\infty$]

$$= [\frac{(\approx+\infty)^{(\sigma-1)}[\cos(\mathrm{tLog}(\approx+\infty))+i\sin(\mathrm{tLog}(\approx+\infty))](\mathrm{n})^{(-i\mathrm{t})}}{-(\mathrm{n})^{(\sigma)}(\mathrm{e})^{(\approx+\infty)}+(\mathrm{n})^{(\sigma)}\frac{(\sigma-1)}{(\approx+\infty)}\mathrm{e}^{(\approx+\infty)}+(\mathrm{n})^{(\sigma)}\frac{(i\mathrm{t})}{(\approx+\infty)}\mathrm{e}^{(\approx+\infty)}}]$$

$$-[\frac{(\approx 0)^{(\sigma-1)}[\cos(\mathrm{tLog}(\approx-\infty))+i\sin(\mathrm{t}(\approx-\infty))](\mathrm{n})^{(-i\mathrm{t})}(\mathrm{n})^{(-i\mathrm{t})}}{-(\mathrm{n})^{(\sigma)}(\mathrm{e})^{(\approx 0)}+(\mathrm{n})^{(\sigma)}\frac{(\sigma-1)}{(\approx 0)}\mathrm{e}^{(\approx 0)}+(\mathrm{n})^{(\sigma)}\frac{(i\mathrm{t})}{(\approx 0)}\mathrm{e}^{(\approx 0)}}]$$

$$= [\frac{(\approx+\infty)^{(\sigma-1)}(\approx 1)(\mathrm{n})^{(-i\mathrm{t})}}{-(\mathrm{n})^{(\sigma)}(\mathrm{e})^{(\approx+\infty)}+(\mathrm{n})^{(\sigma)}\frac{(\sigma-1)}{(\approx+\infty)}\mathrm{e}^{(\approx+\infty)}+(\mathrm{n})^{(\sigma)}\frac{(i\mathrm{t})}{(\approx+\infty)}\mathrm{e}^{(\approx+\infty)}}]$$

$$-[\frac{(\approx 0)^{(\sigma-1)}(\approx 1)(\mathrm{n})^{(-i\mathrm{t})}}{-(\mathrm{n})^{(\sigma)}(\mathrm{e})^{(\approx 0)}+(\mathrm{n})^{(\sigma)}\frac{(\sigma-1)}{(\approx 0)}\mathrm{e}^{(\approx 0)}+(\mathrm{n})^{(\sigma)}\frac{(i\mathrm{t})}{(\approx 0)}\mathrm{e}^{(\approx 0)}}]$$

$$= [\frac{(\approx+\infty)^{(\sigma-1)}(\approx 1)(\mathrm{n})^{(-i\mathrm{t})}}{-(\mathrm{n})^{(\sigma)}(\mathrm{e})^{(\approx+\infty)}+(\mathrm{n})^{(\sigma)}(\sigma-1)(\approx 1)+(\mathrm{n})^{(\sigma)}(i\mathrm{t})(\approx 1)}]$$

$$-[\frac{(\approx 0)^{(\sigma-1)}(\approx 1)(\mathrm{n})^{(-i\mathrm{t})}}{-(\mathrm{n})^{(\sigma)}(\approx 1)+(\mathrm{n})^{(\sigma)}(\sigma-1)(\approx+\infty)+(\mathrm{n})^{(\sigma)}(i\mathrm{t})(\approx+\infty)}]$$

$$= -(\frac{+\infty}{+\infty}-0)(\text{indeterminate form})$$

And From $\frac{\mathrm{d}(\mathrm{x_n})^{(\sigma-1)}(\mathrm{x_n})^{(i\mathrm{t})}}{\mathrm{dx_n}}$

$$= \frac{\mathrm{d}(\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}}{\mathrm{dx_n}}$$

$$= (\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}\frac{\mathrm{d}(\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}}{\mathrm{dx_n}} + (\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}\frac{\mathrm{d}(\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}}{\mathrm{dx_n}}$$

$$= (\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}\frac{(i\mathrm{t})}{\mathrm{x_n}} + (\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}\frac{(\sigma-1)}{\mathrm{x_n}}$$

$$= (\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{-\mathrm{Log}(\mathrm{x_n})}(i\mathrm{t})$$

$$+(\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{-\mathrm{Log}(\mathrm{x_n})}(\sigma-1)$$

$$= [(i\mathrm{t})+(\sigma-1)][\,(\mathrm{e})^{(\sigma-1)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{-\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}]$$

$$= (\sigma+i\mathrm{t}-1)\,[\,(\mathrm{e})^{[(\sigma-1)-1]\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{(i\mathrm{t})\mathrm{Log}(\mathrm{x_n})}]$$

$$= (\sigma+i\mathrm{t}-1)\,[\,(\mathrm{x_n})^{[(\sigma-2)]}(\mathrm{x_n})^{(i\mathrm{t})}]$$

Next $\frac{\mathrm{d}(\sigma+i\mathrm{t}-1)(\mathrm{x_n})^{(\sigma-2)}(\mathrm{x_n})^{(i\mathrm{t})}}{\mathrm{dx_n}}$

$$= (\sigma + it - 1)\frac{\mathrm{d(e)^{(\sigma-2)Log(x_n)}(e)^{(\mathit{i}t)Log(x_n)}}}{\mathrm{dx_n}}$$

$$= (\sigma + it - 1)[\mathrm{(e)^{(\sigma-2)Log(x_n)}}\frac{\mathrm{d(e)^{(\mathit{i}t)Log(x_n)}}}{\mathrm{dx_n}}$$

$$+ \ \mathrm{(e)^{(\mathit{i}t)Log(x_n)}}\frac{\mathrm{d(e)^{(\sigma-2)Log(x_n)}}}{\mathrm{dx_n}}]$$

$$= (\sigma + it - 1)[\mathrm{(e)^{(\sigma-2)Log(x_n)}(e)^{(\mathit{i}t)Log(x_n)}}\frac{(it)}{\mathrm{x_n}}$$

$$+\mathrm{(e)^{(\mathit{i}t)Log(x_n)}(e)^{(\sigma-2)Log(x_n)}}\frac{(\sigma-2)}{\mathrm{x_n}}]$$

$$= (\sigma + it - 1)[\mathrm{(e)^{(\sigma-2)Log(x_n)}(e)^{(\mathit{i}t)Log(x_n)}(e)^{-Log(x_n)}}(it)$$

$$+\mathrm{(e)^{(\mathit{i}t)Log(x_n)}(e)^{(\sigma-2)Log(x_n)}(e)^{-Log(x_n)}}(\sigma - 2)]$$

$$= (\sigma + it - 1)[(it)+(\sigma - 2)][\ \mathrm{(e)^{(\sigma-2)Log(x_n)}(e)^{-Log(x_n)}(e)^{(\mathit{i}t)Log(x_n)}}]$$

$$= (\sigma + it - 1)[(\sigma + it - 2)][\ \mathrm{(e)^{[(\sigma-2)-1]Log(x_n)}(e)^{(\mathit{i}t)Log(x_n)}}]$$

$$=(\sigma + it - 1)\ [(\sigma + it - 2)][\ \mathrm{(x_n)^{[(\sigma-3)]}(x_n)^{(\mathit{i}t)}}]$$

So $\frac{\mathrm{d^{(\sigma-1)}[(x_n)^{(\sigma-1)}(x_n)^{(\mathit{i}t)}]}}{\mathrm{d(x_n)^{(\sigma-1)}}}$

$$= [(\sigma + it - 1)\ (\sigma + it - 2)\ \dots\ ]\ \mathrm{(x_n)^{(0)}(x_n)^{(\mathit{i}t)}}$$

$$= \textstyle\sum_{k=1}^{\sigma-1}(\sigma + it - \mathrm{k})[\ \mathrm{(x_n)^{(0)}(x_n)^{(\mathit{i}t)}}]$$

And $\frac{\mathrm{d^{(\sigma)}[(x_n)^{(\sigma)}(x_n)^{(\mathit{i}t)}]}}{\mathrm{d(x_n)^{(\sigma)}}}$

$$=[(\sigma + it)\ (\sigma + it - 1)\ \dots\ ]\mathrm{(x_n)^{(0)}(x_n)^{(\mathit{i}t)}}$$

$$= \textstyle\sum_{k=1}^{\sigma}(\sigma + it - \mathrm{k}+1)[\ \mathrm{(x_n)^{(0)}(x_n)^{(\mathit{i}t)}}]$$

Apply L' Hospital's Rule (σ) times to

$$\lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty}[\frac{\mathrm{(x_n)^{(\sigma-1)}(x_n)^{(\mathit{i}t)}}}{-\mathrm{(n)(e)^{(x_n)(n)}}+\frac{(\sigma-1)}{\mathrm{(x_n)}}\mathrm{(e)^{(x_n)(n)}}+\frac{(it)}{\mathrm{(x_n)}}\mathrm{(e)^{(x_n)(n)}}}]\ ]_{\mathrm{a/n}}^{\mathrm{b/n}}$$

Or $$\lim_{\mathrm{a}\to 0,\mathrm{b}\to+\infty}[\frac{\mathrm{(x_n)^{(\sigma-1)}(x_n)(x_n)^{(\mathit{i}t)}}}{-\mathrm{(n)(x_n)(e)^{(x_n)(n)}}+(\sigma-1)\mathrm{(e)^{(x_n)(n)}}+(it)\mathrm{(e)^{(x_n)(n)}}}]\ ]_{\mathrm{a/n}}^{\mathrm{b/n}}$$

Or $\lim\limits_{a\to 0,b\to+\infty}\left[\frac{(x_n)^{(\sigma)}(x_n)^{(it)}}{-(n)(x_n)(e)^{(x_n)(n)}+(\sigma-1)(e)^{(x_n)(n)}+(it)(e)^{(x_n)(n)}}\right]\Big]_{a/n}^{b/n}$

until $(x_n)^{(\sigma)} = (x_n)^{(0)} = 1$

Then $\lim\limits_{a\to 0,b\to+\infty}\left[\frac{(x_n)^{(\sigma)}(x_n)^{(it)}}{-(n)(x_n)(e)^{(x_n)(n)}+(\sigma-1)(e)^{(x_n)(n)}+(it)(e)^{(x_n)(n)}}\right]\Big]_{a/n}^{b/n}$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{\sum_{k=1}^{\sigma}(\sigma+it-k+1)[(x_n)^{(0)}(x_n)^{(it)}]}{(e)^{(x_n)(n)}[-(n)^{(\sigma+1)}(x_n)-\sigma(n)^{(\sigma)}]+(\sigma-1)(n)^{(\sigma)}+(it)(n)^{(\sigma)}}\right]\Big]_{a/n}^{b/n}$$

$$=\lim_{a\to 0,b\to+\infty}\Bigg[\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](1)(b/n)^{(it)}}{(e)^{(b/n)(n)}[-(n)^{(\sigma+1)}(b/n)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}$$

$$-\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](1)(a/n)^{(it)}}{(e)^{(a/n)(n)}[-(n)^{(\sigma+1)}(a/n)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}\Bigg]$$

$$=\lim_{a\to 0,b\to+\infty}\Bigg[\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](1)(b)^{(it)}(n)^{(-it)}}{(e)^{(b)}[-(n)^{(\sigma)}(b)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}$$

$$-\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](1)(a)^{(it)}(n)^{(-it)}}{(e)^{(a)}[-(n)^{(\sigma)}(a)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}\Bigg]$$

$$=\lim_{a\to 0,b\to+\infty}\Bigg[\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](1)(e)^{(it\mathrm{Log}b)}(n)^{(-it)}}{(e)^{(b)}[-(n)^{(\sigma)}(b)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}$$

$$-\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](1)(e)^{(it\mathrm{Log}a)}(n)^{(-it)}}{(e)^{(a)}[-(n)^{(\sigma)}(a)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}\Bigg],$$

[$\lim\limits_{a\to 0}\mathrm{Log}\,(a) = z$, or $\mathrm{Log}\,(\approx 0) = z$,

But $(e)^{(z)} \approx 0$, or $(e)^{(-\infty)} = \frac{1}{(e)^{(\infty)}} \approx 0$,

So $\lim\limits_{a\to 0}\mathrm{Log}\,(a)$ or $\mathrm{Log}\,(\approx 0) \approx -\infty$]

$$=\lim_{a\to 0,b\to+\infty}\Bigg[\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)][\cos(t\mathrm{Log}(b))+i\sin(t\mathrm{Log}(b))](n)^{(-it)}}{(e)^{(b)}[-(n)^{(\sigma)}(b)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}$$

$$-\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)][\cos(t\mathrm{Log}(a))+i\sin(t\mathrm{Log}(a))](n)^{(-it)}}{(e)^{(a)}[-(n)^{(\sigma)}(a)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}\Bigg]$$

$$=\lim_{a\to 0,b\to+\infty}\Bigg[\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)][\cos(\approx+\infty)+i\sin(\approx+\infty)](n)^{(-it)}}{(e)^{(\approx+\infty)}[-(n)^{(\sigma)}(\approx+\infty)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}$$

$$-\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)][\cos(\approx-\infty)+i\sin(\approx-\infty)](n)^{(-it)}}{(e)^{(\approx 0)}[-(n)^{(\sigma)}(\approx 0)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}\Bigg]$$

$$= \ \Big[\ \frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](\approx 1+i\approx 0)(n)^{(-it)}}{(\approx +\infty)[\,-(n)^{(\sigma)}(\approx +\infty)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}$$

$$-\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](\approx 1+i\approx 0)(n)^{(-it)}}{(\approx 1)[\,-(n)^{(\sigma)}(\approx 0)-(n)^{(\sigma)}+(it)(n)^{(\sigma)}]}\ \Big]$$

$$= \ 0-\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)](n)^{(-it)}}{[(it-1)(n)^{(\sigma)}]}$$

$$= \qquad \frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(n)^{(\sigma+it)}}$$

For $n=1$ $\qquad \frac{1}{(1)^{s}}\prod(s-1) = \lim_{a\to 0,b\to +\infty}\int_{a/1}^{b/1}(e)^{(-x_1)(1)}(x_1)^{(s-1)}dx_1$

$$= \frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(1)^{(\sigma+it)}}$$

For $n=2$ $\qquad \frac{1}{(2)^{s}}\prod(s-1) = \lim_{a\to 0,b\to +\infty}\int_{a/2}^{b/2}(e)^{(-x_2)(2)}(x_2)^{(s-1)}dx_2$

$$= \frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(2)^{(\sigma+it)}}$$

...

For $n=\infty$ $\qquad \frac{1}{(+\infty)^{s}}\prod(s-1) = \lim_{a\to 0,b\to +\infty}\int_{a/+\infty}^{b/+\infty}(e)^{(-x_\infty)(+\infty)}(x_\infty)^{(s-1)}dx_\infty$

$$= \frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(\infty)^{(\sigma+it)}}$$

Actually the correct method of integration of the above equations should be.

$$[\left(\frac{1}{1^{s}}\right)+\left(\frac{1}{2^{s}}\right)+\ldots+\left(\frac{1}{\infty^{s}}\right)]\prod(s-1)$$

$$= \lim_{a\to 0,b\to +\infty}[\int_{a/(1)}^{b/(1)}(e)^{(-x_1)(1)}(x_1)^{(s-1)}dx_1$$

$$+\int_{a/(2)}^{b/(2)}(e)^{(-x_2)(2)}(x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{a/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(x_\infty)^{(s-1)}dx_\infty\ ]$$

$$\sum_{n=1}^{+\infty} \left(\frac{1}{n^s}\right) \prod(s-1) = \frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(1)^{(\sigma+it)}}$$

$$+\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(2)^{(\sigma+it)}}$$

$$+\ldots$$

$$+\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(\infty)^{(\sigma+it)}}$$

Or $\zeta(s) \prod(s-1) = \sum_{n=1}^{+\infty} \left[\frac{[\sum_{k=1}^{\sigma}(\sigma+it-k+1)]}{(1-it)(n)^{(\sigma+it)}}\right]$

$\zeta(s)\prod(s-1) \neq \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e)^{(x)}-1} dx,$ because infinite summation (Geometric series), $\sum_{n=1}^{+\infty} (e)^{(-x)n} = \frac{1}{(e)^{(x)}-1}$ of one of the integrand can be taken only when values of another integrand $(x_n)^{(s-1)}$ do not change when n changes. But in this case the different boundaries tell us that $x_1, x_2, \ldots, x_\infty$ are not the same, then

$$\int_{a/(1)}^{b/(1)} (x_1)^{(s-1)} dx_1 \neq \int_{a/(2)}^{b/(2)} (x_2)^{(s-1)} dx_2$$

$$\neq \cdots$$

$$\neq \int_{a/(\infty)}^{b/(\infty)} (x_\infty)^{(s-1)}\, dx_\infty$$

, and infinite summation of the integrands $(e)^{(-x)n}$ of the above integrals cannot be done as Riemann did.

2. Proof that $2\sin(\pi s)\prod(s-1)\zeta(s) \neq i\int_{+\infty}^{+\infty} \frac{(-x)^{(s-1)}}{(e^x-1)} dx$, $s = \sigma + it$, $\mathrm{Re}s = \sigma > 1$. And $\zeta(s)$ of this functional equation has no trivial zeroes $(-2, -4, -6, \ldots)$.

Start from finding $\zeta(s)$ for x on the whole real line (negative and positive integers) using analytic continuation while s = complex numbers $(\sigma + it)$

From Riemann's equation

$$\zeta(s)\prod(s-1) = \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e)^{(x)}-1} dx \quad , s = \sigma + it, \quad \mathrm{Re}s = \sigma > 1 \qquad \ldots (A)$$

$$= \lim_{a\to 0, b\to +\infty} \int_a^b \frac{(x)^{(s-1)}}{(e)^{(x)}-1} dx$$

Apply analytic continuation technique (by stalking Riemann's work, and to do this we have to assume that equation … A is correct) then

$$\zeta(s)\prod(s-1) = \lim_{b\to +\infty} \int_{-b}^b \frac{(-1)^{(s-1)}(-x)^{(s-1)}}{(e)^{(x)}-1} dx \qquad \ldots (B)$$

$$= (-1)^{(s-1)} \lim_{b\to +\infty} \int_{-b}^b \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

From $(\cos\pi - i\sin\pi) = -1$; $(\cos\pi = -1, \sin\pi = 0)$

$$\zeta(s)\prod(s-1) = \frac{(\cos\pi - i\sin\pi)^{(s)}}{(-1)} \lim_{b\to +\infty} \int_{-b}^b \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

From $(\cos\pi - i\sin\pi) = (e)^{(-\pi i s)}$

$$\zeta(s)\prod(s-1) = -(e)^{(-\pi i s)} \lim_{b\to +\infty} \int_{-b}^b \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

Multiply by $-(e)^{(\pi i s)}$ both sides

$$-(e)^{(\pi i s)}\zeta(s)\prod(s-1) = \lim_{b\to +\infty} \int_{-b}^b \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx \qquad \ldots (C)$$

From $(\cos\pi + i\sin\pi) = -1$; $(\cos\pi = -1, \sin\pi = 0)$

$$\zeta(s)\prod(s-1) = \frac{(\cos\pi + i\sin\pi)^{(s)}}{(-1)} \lim_{b\to +\infty} \int_{-b}^b \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

$$= -(e)^{(\pi i s)} \lim_{b\to +\infty} \int_{-b}^b \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx$$

Multiply by $-(e)^{(-\pi i s)}$ both sides

$$-(e)^{(-\pi i s)}\zeta(s)\prod(s-1) = \lim_{b\to +\infty} \int_{-b}^b \frac{(-x)^{(s-1)}}{(e)^{(x)}-1} dx \qquad \ldots (D)$$

(C) $-$(D) ; $[-(e)^{(\pi i s)} + (e)^{(-\pi i s)}]\zeta(s)\prod(s-1)$

$$= \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx - \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx$$

$$[-\cos\pi s - i\sin\pi s + \cos\pi s - i\sin\pi s]\zeta(s)\prod(s-1)$$

$$= \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx - \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx$$

$$-2i\sin\pi s\zeta(s)\prod(s-1) = \lim_{b\to+\infty} \int_{-b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx + \lim_{b\to+\infty} \int_{b}^{-b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx$$

$$= \lim_{b\to+\infty} \int_{b}^{b} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx$$

$$= \int_{+\infty}^{+\infty} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx$$

Multiply by i both sides

$$2\sin\pi s\zeta(s)\prod(s-1) = i\int_{+\infty}^{+\infty} \frac{(-x)^{(s-1)}}{(e)^{(x)}-1}dx \qquad \text{... (E)}$$

$= 0$ (by Cauchy's Integral Formula and by method which was used to derive the equation above)

But the above derivation (all red characters) is wrong because it starts from wrong equation

$$\zeta(s)\prod(s-1) = \int_0^{+\infty} \frac{(x)^{(s-1)}}{(e^x-1)} dx \qquad \text{... (A)}$$

The right derivation in this case should start from equation

$$\prod(s-1) = \lim_{a\to0, b\to+\infty} \int_a^b (e)^{(-nx_n)} (nx_n)^{(s-1)} dnx_n \qquad , s = \sigma + it$$

, $\mathrm{Re}s = \sigma > 0$, $x =$ zero and positive real numbers

Then change the boundaries because of separation of the variable from nx_n to n and x_n, preparing for moving $(n)^{(s)}$ to the left hand side.

$$\prod(s-1) = \lim_{a\to0, b\to+\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)} (n)^{(s)} (x_n)^{(s-1)} dx_n$$

Multiply by $\frac{1}{(n)^s}$ both sides

$$\frac{1}{(n)^s}\prod(s-1) = \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)}(x_n)^{(s-1)}dx_n$$

Set $n = 1$ to $+\infty$ and take summation as above, then

$$\zeta(s)\prod(s-1) = \sum_{n=1}^{+\infty} \lim_{a\to 0, b\to +\infty} \int_{a/n}^{b/n} (e)^{(-x_n)(n)}(x_n)^{(s-1)}dx_n$$

Apply analytic continuation technique for x on the whole real line, then

$$\zeta(s)\prod(s-1) = \lim_{a\to -\infty, b\to +\infty} [\int_{a/(1)}^{b/(1)} (e)^{(-x_1)(1)}(-1)^{(s-1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{a/(2)}^{b/(2)} (e)^{(-x_2)(2)}(-1)^{(s-1)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{a/(+\infty)}^{b/(+\infty)} (e)^{(-x_\infty)(+\infty)}(-1)^{(s-1)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$=(-1)^{(s-1)} \lim_{b\to +\infty} [\int_{-b/(1)}^{b/(1)} (e)^{(-x_1)(1)}(-x_1)^{(s-1)}\ dx_1$$

$$+\int_{-b/(2)}^{b/(2)} (e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)} (e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

From $(\cos\pi - i\sin\pi) = -1$; $(\cos\pi = -1,\ \sin\pi = 0)$

$$\zeta(s)\prod(s-1) = \frac{(\cos\pi - i\sin\pi)^{(s)}}{(-1)} \lim_{b\to +\infty} [\int_{-b/(1)}^{b/(1)} (e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)} (e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)} (e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

From $(\cos\pi - i\sin\pi) = (e)^{(-\pi i)}$

$$\zeta(s)\prod(s-1) = -(e)^{(-\pi is)} \lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(\infty)}^{b/(\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

Multiply by $-(e)^{(\pi is)}$ both sides

$$-(e)^{(\pi is)}\zeta(s)\prod(s-1) = \lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty \quad \ldots (C')$$

From $(\cos\pi + i\sin\pi) = -1$; $(\cos\pi = -1, \sin\pi = 0)$

$$\zeta(s)\prod(s-1) = \frac{(\cos\pi + i\sin\pi)^{(s)}}{(-1)} \lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

From $(\cos\pi + i\sin\pi) = (e)^{(\pi i)}$

$$\zeta(s)\prod(s-1) = -(e)^{(\pi is)} \lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

Multiply by $-(e)^{(-\pi is)}$ both sides

$$-(e)^{(-\pi is)}\zeta(s)\prod(s-1)=\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]\quad\ldots(D')$$

(C’) −(D’); $[-(e)^{(\pi is)}+(e)^{(-\pi is)}]\zeta(s)\prod(s-1)$

$$=\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$-\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$[-\cos\pi s-i\sin\pi s+\cos\pi s-i\sin\pi s]\zeta(s)\prod(s-1)$$

$$=\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$-\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$-2i\sin\pi s\zeta(s)\prod(s-1)\ =\lim_{b\to+\infty}[\int_{-b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{-b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{-b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$+\lim_{b\to+\infty}[\int_{b/(1)}^{-b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{b/(2)}^{-b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{b/(+\infty)}^{-b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$=\lim_{b\to+\infty}[\int_{b/(1)}^{b/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{b/(2)}^{b/(2)}(e)^{(-x_2)(2)}(-x_2)^{(s-1)}dx_2$$

$$+\ldots$$

$$+\int_{b/(+\infty)}^{b/(+\infty)}(e)^{(-x_\infty)(+\infty)}(-x_\infty)^{(s-1)}dx_\infty\ ]$$

$$=\ [\int_{+\infty/(1)}^{+\infty/(1)}(e)^{(-x_1)(1)}(-x_1)^{(s-1)}dx_1$$

$$+\int_{+\infty/(2)}^{+\infty/(2)}(\mathrm{e})^{(-\mathrm{x}_2)(2)}(-\mathrm{x}_2)^{(\mathrm{s}-1)}\mathrm{dx}_2$$

$$+\ldots$$

$$+\int_{+\infty/+(\infty)}^{+\infty/+(\infty)}(\mathrm{e})^{(-\mathrm{x}_\infty)(+\infty)}(-\mathrm{x}_\infty)^{(\mathrm{s}-1)}\mathrm{dx}_\infty\,]$$

Multiply by i both sides

$$2\sin\pi s\zeta(\mathrm{s})\prod(\mathrm{s}-1)=i[\int_{+\infty/(1)}^{+\infty/(1)}(\mathrm{e})^{(-\mathrm{x}_1)(1)}(-\mathrm{x}_1)^{(\mathrm{s}-1)}\mathrm{dx}_1$$

$$+\int_{+\infty/(2)}^{+\infty/(2)}(\mathrm{e})^{(-\mathrm{x}_2)(2)}(-\mathrm{x}_2)^{(\mathrm{s}-1)}\mathrm{dx}_2$$

$$+\ldots$$

$$+\int_{+\infty/+(\infty)}^{+\infty/+(\infty)}(\mathrm{e})^{(-\mathrm{x}_\infty)(+\infty)}(-\mathrm{x}_\infty)^{(\mathrm{s}-1)}\mathrm{dx}_\infty\,]$$

$$=i\sum_{n=1}^{+\infty}\int_{+\infty/(\mathrm{n})}^{+\infty/(\mathrm{n})}(\mathrm{e})^{(-\mathrm{x}_\mathrm{n})(\mathrm{n})}(-\mathrm{x}_\mathrm{n})^{(\mathrm{s}-1)}\mathrm{dx}_\mathrm{n}$$

$= 0$ (by Cauchy's Integral Formula and by method which was used to derive the equation above)

Consider $\prod(\mathrm{s}-1)=\Gamma(\mathrm{s})=\frac{\Gamma(\mathrm{s+k})}{\mathrm{s(s+1)(s+2)\ldots(s+k-1)}}$

$\mathrm{k}=0, 1, 2, 3,\ldots$ and $\mathrm{s}+\mathrm{k}>0$, or $\mathrm{s}-\mathrm{k}$, $\mathrm{s}\neq 0,-1,-2,\ldots,-(\mathrm{k}-1)$

$\Gamma(\mathrm{s})$ has poles at $\mathrm{s}=0,-1,-2,-3,\ldots$ (or undefined) and $\Gamma(\mathrm{s})$ never becomes zero for any values of s.

$\mathrm{Sin}\pi s=0$ all the time by the method that was used to derive $2\sin\pi s\,\zeta(\mathrm{s})\prod(\mathrm{s}-1)$ as mentioned above (or πs must be $0,\pm\pi,\pm 2\pi,\pm 3\pi,\pm 4\pi,\ldots$, and s must be $0,\pm 1,\pm 2,\pm 3,\pm 4,\ldots$).

Note that $0\times\mathrm{a}=0$, a can be any numbers or $\mathrm{a}=\frac{0}{0}$ (undefined).In this case $\mathrm{a}=\zeta(\mathrm{s})\prod(\mathrm{s}-1)$ while $\prod(\mathrm{s}-1)\neq 0$ and $\zeta(\mathrm{s})\neq 0$ but $\zeta(\mathrm{s})=\sum_{\mathrm{n}=1}^{+\infty}[\frac{[\sum_{\mathrm{k}=1}^{\sigma}(\sigma+it-\mathrm{k}+1)]}{(1-it)(\mathrm{n})^{(\sigma+it)}}]/\prod(\mathrm{s}-1)$.

In this case, it could not be said that $\zeta(\mathrm{s})=0$ when $\sin\pi\mathrm{s}=0$ or $\mathrm{s}=-2,-4,-6,\ldots$ which are called trivial zeroes. Actually $\zeta(\mathrm{s})\neq 0$, or there are no trivial zeroes that make $\zeta(s) = 0$. And $2\sin\pi s\,\zeta(s)\prod(s\text{–}1) =$ $i\sum_{n=1}^{+\infty}\int_{+\infty/(n)}^{+\infty/(n)}(e)^{(-x_n)(n)}(-x_n)^{(s-1)}dx_n = 0$ because $\sin\pi s$ of the left hand side and the whole $i\sum_{n=1}^{+\infty}\int_{+\infty/(n)}^{+\infty/(n)}(e)^{(-x_n)(n)}(-x_n)^{(s-1)}dx_n$ of the right hand side, both $= 0$ by the method that was used to derive the equation not by $\zeta(s)$.

The above derivation gived disproof that there are no trivial zeroes of Riemann Zeta Function $2\sin\pi s\,\zeta(s)\prod(s\text{–}1) = 0$ for x on the whole real line (zero and negative and positive integers).

3. Proof that $\zeta(s)\pi^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) \neq \frac{1}{s(s-1)} + \int_1^{+\infty}\psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]\,dx$

From $\prod(s\text{–}1) = \Gamma(s)$

$$= \int_0^{+\infty}(e)^{(-x)}(x)^{(s-1)}dx \quad , s = \sigma + it \ , \mathrm{Re}s = \sigma=>0,$$

x = zero and positive real numbers, or in limit form

$$\prod(s\text{–}1) = \lim_{a\to 0,b\to+\infty}\int_a^b(e)^{(-x)}(x)^{(s-1)}\,dx$$

Riemann tried to start from denoted $s = \frac{s}{2}$ or $\frac{(\sigma+it)}{2}$ and $x = nn\pi x$ to the above equation

$$\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0,b\to+\infty}\int_a^b(e)^{(-nn\pi x)}\,(nn\pi x)^{\left(\frac{s}{2}-1\right)}\,dnn\pi x$$

But he forgot to change the boundaries of the integral after separating the variable from $nn\pi x$ to nn , π, and x, as shown below

$$\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0,b\to+\infty}\int_a^b(e)^{(-nn\pi x)}(n)^{(s)}\,(\pi)^{\left(\frac{s}{2}\right)}(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

Multiply by $\frac{1}{(n)^s}\,(\pi)^{\left(-\frac{s}{2}\right)}$ both sides.

$$\frac{1}{(n)^s}\,(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0,b\to+\infty}\int_a^b(e)^{(-nn\pi x)}\,(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

$$\frac{1}{(1)^s}\,(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0,b\to+\infty}\int_a^b(e)^{-(1)(1)(\pi)(x)}\,(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

$$\frac{1}{(2)^s}(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b (e)^{-(2)(2)(\pi)(x)}\,(x)^{(\frac{s}{2}-1)}\,dx$$

...

$$\frac{1}{(\infty)^s}(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b (e)^{(-bb\pi x)}\,(x)^{(\frac{s}{2}-1)}\,dx$$

Then Riemann took summation of the above equations from $(\frac{1}{1^s})\,(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)$ to $\left(\frac{1}{\infty^s}\right)(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)$ and he thought that term $(e)^{(-x)(b)}$ of all the right hand sides could be sum to infinite or $=\sum_{n=1}^{\infty}\ (e)^{(-nn\pi x)}$.

$$[\left(\frac{1}{1^s}\right)+\left(\frac{1}{2^s}\right)+...+\left(\frac{1}{\infty^s}\right)]\,(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)$$

$$=\lim_{a\to 0,b\to+\infty}\int_a^b[(e)^{-(1)(1)(\pi)(x)}+(e)^{-(2)(2)(\pi)(x)}+...$$

$$+(e)^{(-bb\pi x)}](x)^{(\frac{s}{2}-1)}dx$$

$$=\lim_{a\to 0,b\to+\infty}\int_a^b[\sum_{n=1}^{b}\ (e)^{(-nn\pi x)}]\,(x)^{(\frac{s}{2}-1)}dx$$

Or $\sum_{n=1}^{+\infty}\ (\frac{1}{n^s})\,(\pi)^{-(\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b[\sum_{n=1}^{b}\ (e)^{(-nn\pi x)}]\,(x)^{(\frac{s}{2}-1)}dx$

Or $\zeta(s)(\pi)^{-(\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b[\sum_{n=1}^{b}\ (e)^{(-nn\pi x)}]\,(x)^{(\frac{s}{2}-1)}dx$

$$=\int_0^{+\infty}[\sum_{n=1}^{+\infty}\ (e)^{(-nn\pi x)}]\,(x)^{(\frac{s}{2}-1)}dx$$

Then Riemann denoted $\sum_{n=1}^{+\infty}\ (e)^{(-nn\pi x)}\ =\ \psi(x)$

So $\zeta(s)(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b\psi(x)\,(x)^{(\frac{s}{2}-1)}dx$

From $(2\psi(x)+1)\ =\ (x)^{(-\frac{1}{2})}(2\psi(\frac{1}{x})+1)$ (Jacobi, Fund S.184)

$$\zeta(s)(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}-1\right)\ =\ \lim_{a\to 0,b\to+\infty}\int_a^1\psi(x)(x)^{(\frac{s}{2}-1)}\,dx+\int_1^b\psi(x)(x)^{(\frac{s}{2}-1)}\,dx$$

$$=\lim_{a\to 0,b\to+\infty}\int_a^1\psi\left(\frac{1}{x}\right)(x)^{(\frac{s-3}{2})}\,dx$$

$$+\frac{1}{2}\int_a^1 [(x)^{\left(\frac{s-3}{2}\right)} - (x)^{\left(\frac{s}{2}-1\right)}]\,dx + \int_1^b \psi(x)(x)^{\left(\frac{s}{2}-1\right)}\,dx$$

Consider $\lim_{a\to 0}\int_a^1 \psi\left(\frac{1}{x}\right)(x)^{\left(\frac{s-3}{2}\right)}\,dx$

Let $u = \frac{1}{x}$ then $du = (-1)\,(x)^{(-2)}dx$

And $dx = (-1)\,(u)^{(-2)}du$

Then

$$\lim_{a\to 0}\int_a^1 \psi\left(\frac{1}{x}\right)(x)^{\left(\frac{s-3}{2}\right)}\,dx = \lim_{a\to 0}\int_{1/a}^{1/1} \psi(u)\left(\frac{1}{u}\right)^{\left(\frac{s-3}{2}\right)}(-1)\,(u)^{(-2)}du$$

$$= -\lim_{b\to+\infty}\int_b^1 \psi(u)(u)^{-\left(\frac{1+s}{2}\right)}\,du$$

$$= \lim_{b\to+\infty}\int_1^b \psi(u)(u)^{-\left(\frac{1+s}{2}\right)}\,du$$

But $$\lim_{b\to+\infty}\int_1^b \psi(u)(u)^{-\left(\frac{1+s}{2}\right)}\,du = \lim_{b\to+\infty}\int_1^b \psi(x)(x)^{-\left(\frac{1+s}{2}\right)}\,dx$$

And $$\lim_{a\to 0}\frac{1}{2}\int_a^1 [(x)^{\left(\frac{s-3}{2}\right)} - (x)^{\left(\frac{s}{2}-1\right)}]\,dx = \lim_{a\to 0}\left[\left(\frac{1}{2}\right)\frac{(x)^{\left(\frac{s-1}{2}\right)}}{\left(\frac{s-1}{2}\right)}\right]_a^1 - \left(\frac{1}{2}\right)\frac{(x)^{\left(\frac{s}{2}\right)}}{\left(\frac{s}{2}\right)}\Big]_a^1\Big]$$

$$= +\frac{1}{2}\left[\frac{(1-0)}{\left(\frac{s-1}{2}\right)}\right] - \frac{1}{2}\left[\frac{(1-0)}{\left(\frac{s}{2}\right)}\right]$$

$$= \frac{1}{(s)(s-1)}$$

So $$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \lim_{a\to 0,b\to+\infty}\frac{1}{2}\int_a^1 [(x)^{\left(\frac{s-3}{2}\right)} - (x)^{\left(\frac{s}{2}-1\right)}]\,dx$$

$$+ \int_1^b \psi(x)(x)^{\left(\frac{s}{2}-1\right)}\,dx + \int_1^b \psi(x)(x)^{-\left(\frac{1+s}{2}\right)}\,dx$$

$$= \frac{1}{(s)(s-1)} + \lim_{b\to+\infty}\int_1^b \psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]dx$$

$$= \frac{1}{(s)(s-1)} + \int_1^{+\infty} \psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]dx \quad \dots (F)$$

As you know, the above equation (integral)… (F) is wrong because the boundaries of the integral were still from a=0 to b=+∞. Actually, the boundaries must be changed after separating variable of the integrands from $nn\pi x$ to nn, π, and x, as below

From $\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_a^b (e)^{(-nn\pi x_n)}(nn\pi x_n)^{\left(\frac{s}{2}-1\right)}dnn\pi x_n$, s= $(\sigma+it)$, Res $=\sigma>0$, x = zero and positive real numbers

Then $\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_{a/nn\pi}^{b/nn\pi}(e)^{(-nn\pi x_n)}\ (n)^{(s)}(\pi)^{\left(\frac{s}{2}\right)}(x_n)^{\left(\frac{s}{2}-1\right)}dx_n$

Term $(n)^{(s)}$ is separated out from the integral and moved to the left hand side of the equation and then n is increased in value from 1 to $+\infty$ in each equation and finally combined to become $\sum_{n=1}^{+\infty}\left(\frac{1}{n^s}\right)$ or $\zeta(s)$ at the left hand side. The values of x_n at each n value of each integral at the right hand side are changed too (the lower boundaries vary from $a/(1)(1)\pi$ to $a/(\infty)(\infty)\pi$ and the upper boundaries vary from $b/(1)(1)\pi$ to $b/\ (\infty)(\infty)\pi$. (Remember that $nn\pi x_n$ had boundaries from 0 to $+\infty$ in the original integral).

Multiply by$(\pi)^{-\left(\frac{s}{2}\right)}\ (\frac{1}{n^s})$ both sides

$$(\pi)^{-\left(\frac{s}{2}\right)}\left(\frac{1}{n^s}\right)\prod\left(\frac{s}{2}-1\right)=\lim_{a\to 0,b\to+\infty}\int_{a/nn\pi}^{b/nn\pi}(e)^{(-nn\pi x_n)}\ (x_n)^{\left(\frac{s}{2}-1\right)}dx_n$$

Substitute $s=(\sigma+it)$ and $(x_n)^{\left(\frac{s}{2}-1\right)}=(e)^{\left(\frac{(\sigma+it)}{2}-1\right)\text{Log}(x_n)}$

$$(\pi)^{-\left(\frac{s}{2}\right)}\left(\frac{1}{n^s}\right)\prod\left(\frac{s}{2}-1\right)=(\pi)^{-\frac{(\sigma+it)}{2}}\left(\frac{1}{(e)^{\frac{(\sigma+it)}{2}\text{Log}(n)}}\right)\prod\left(\frac{(\sigma+it)}{2}-1\right)$$

$$=\lim_{a\to 0,b\to+\infty}\int_{a/nn\pi}^{b/nn\pi}(e)^{(-x_n)(nn\pi)}(e)^{\left(\frac{(\sigma+it)}{2}-1\right)\text{Log}(x_n)}\ dx_n$$

$$=\lim_{a\to 0,b\to+\infty}\int_{a/nn\pi}^{b/nn\pi}(e)^{(-x_n)(nn\pi)+\left(\frac{(\sigma+it)}{2}-1\right)\text{Log}(x_n)}\ dx_n$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{1}{-(nn\pi)+\frac{\left(\frac{(\sigma+it)}{2}-1\right)}{(x_n)}}\right]\left[(e)^{[(-x_n)(nn\pi)+\left(\frac{(\sigma+it)}{2}-1\right)\text{Log}(x_n)]}\right]\Big]_{a/nn\pi}^{b/nn\pi}$$

$$=\lim_{a\to 0,b\to+\infty}\left[\frac{(x_n)^{\left(\frac{\sigma}{2}-1\right)}}{(e)^{(x_n)(nn\pi)}}\right]\left[\frac{(x_n)^{\left(\frac{it}{2}\right)}}{\left[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(x_n)}+\frac{\left(\frac{it}{2}\right)}{(x_n)}\right]}\ \right]\Big]_{a/nn\pi}^{b/nn\pi}$$

$$= \lim_{a\to 0, b\to+\infty} \left[\frac{(x_n)^{(\frac{\sigma}{2}-1)}(x_n)^{\left(\frac{it}{2}\right)}}{(e)^{(x_n)(nn\pi)}[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(x_n)}+\frac{\left(\frac{it}{2}\right)}{(x_n)}]} \right]_{a/nn\pi}^{b/nn\pi}$$

$$= \lim_{a\to 0, b\to+\infty} \left(\left[\frac{(b/nn\pi)^{\left(\frac{\sigma}{2}-1\right)}(b/nn\pi)^{\left(\frac{it}{2}\right)}}{-(e)^{(b/nn\pi)(nn\pi)}[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(b/nn\pi)}+\frac{\left(\frac{it}{2}\right)}{(b/nn\pi)}]} \right] \right.$$

$$\left. -\left[\frac{(a/nn\pi)^{(\frac{\sigma}{2}-1)}(a/nn\pi)^{\left(\frac{it}{2}\right)}}{(e)^{(a/nn\pi)(nn\pi)}[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(a/nn\pi)}+\frac{\left(\frac{it}{2}\right)}{(a/nn\pi)}]} \right] \right)$$

$$= \lim_{a\to 0, b\to+\infty} \left(\left[\frac{(b)^{\left(\frac{\sigma}{2}-1\right)}(b)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}[-(nn\pi)^{(\frac{\sigma}{2})}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{\sigma}{2}-1\right)}{(b)}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{it}{2}\right)}{(b)}]} \right] \right.$$

$$\left. -\left[\frac{(a)^{\left(\frac{\sigma}{2}-1\right)}(a)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}[-(nn\pi)^{(\frac{\sigma}{2})}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{\sigma}{2}-1\right)}{(a)}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{it}{2}\right)}{(a)}]} \right] \right)$$

$$= \lim_{a\to 0, b\to+\infty} \left(\left[\frac{(b)^{\left(\frac{\sigma}{2}-1\right)}(e)^{\left(\frac{itLogb}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}[-(nn\pi)^{(\frac{\sigma}{2})}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{\sigma}{2}-1\right)}{(b)}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{it}{2}\right)}{(b)}]} \right] \right.$$

$$\left. -\left[\frac{(a)^{\left(\frac{\sigma}{2}-1\right)}(e)^{\left(\frac{itLoga}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}[-(nn\pi)^{(\frac{\sigma}{2})}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{\sigma}{2}-1\right)}{(a)}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{it}{2}\right)}{(a)}]} \right] \right)$$

$$= \lim_{a\to 0, b\to+\infty} \left(\left[\frac{(b)^{\left(\frac{\sigma}{2}-1\right)}[\cos(tLogb)+i\sin(tLogb)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}[-(nn\pi)^{(\frac{\sigma}{2})}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{\sigma}{2}-1\right)}{(b)}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{it}{2}\right)}{(b)}]} \right] \right.$$

$$\left. -\left[\frac{(a)^{\left(\frac{\sigma}{2}-1\right)}[\cos(tLoga)+i\sin(tLoga)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}[-(nn\pi)^{(\frac{\sigma}{2})}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{\sigma}{2}-1\right)}{(a)}+\frac{(nn\pi)^{(\frac{\sigma}{2})}\left(\frac{it}{2}\right)}{(a)}]} \right] \right)$$

[$\lim_{a\to 0} \mathrm{Log}\,(a) = z$, or $\mathrm{Log}\,(\approx 0) = z$,

But $(e)^{(z)} \approx 0$, or $(e)^{(-\infty)} = \frac{1}{(e)^{(\infty)}} \approx 0$,

So $\lim_{a\to 0} \mathrm{Log}\,(a)$ or $\mathrm{Log}\,(\approx 0) \approx -\infty$]

$$= \left(\left[\frac{(\approx+\infty)^{\left(\frac{\sigma}{2}-1\right)}[\cos(\approx+\infty)+i\sin(\approx+\infty)](\mathrm{nn}\pi)^{\left(-\frac{it}{2}\right)}}{(\mathrm{e})^{(\approx+\infty)}\left[-(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{\sigma}{2}-1\right)}{(\approx+\infty)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{it}{2}\right)}{(\approx+\infty)}\right]}\right]\right.$$

$$\left.-\left[\frac{(\approx 0)^{\left(\frac{\sigma}{2}-1\right)}[\cos(\approx-\infty)+i\sin(\approx-\infty)](\mathrm{nn}\pi)^{\left(-\frac{it}{2}\right)}}{(\mathrm{e})^{(\approx 0)}\left[-(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{\sigma}{2}-1\right)}{(\approx 0)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{it}{2}\right)}{(\approx 0)}\right]}\right]\right)$$

$$= \left(\left[\frac{(\approx+\infty)^{\left(\frac{\sigma}{2}-1\right)}[\approx 1+i(\approx 0)](\mathrm{nn}\pi)^{\left(-\frac{it}{2}\right)}}{(\mathrm{e})^{(\approx+\infty)}\left[-(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{\sigma}{2}-1\right)}{(\approx+\infty)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{it}{2}\right)}{(\approx+\infty)}\right]}\right]\right.$$

$$\left.-\left[\frac{(\approx 0)^{\left(\frac{\sigma}{2}-1\right)}[\approx 1+i(\approx 0)](\mathrm{nn}\pi)^{\left(-\frac{it}{2}\right)}}{(\mathrm{e})^{(\approx 0)}\left[-(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{\sigma}{2}-1\right)}{(\approx 0)}+\frac{(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}\left(\frac{it}{2}\right)}{(\approx 0)}\right]}\right]\right)$$

$$= \left(\left[\frac{(\approx+\infty)(\approx 1)(\mathrm{nn}\pi)^{\left(-\frac{it}{2}\right)}}{(\mathrm{e})^{(\approx+\infty)}\left[-(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}+\approx 0+\approx 0\right]}\right]\right.$$

$$\left.-\left[\frac{(\approx 0)(\approx 1)(\mathrm{nn}\pi)^{\left(-\frac{it}{2}\right)}}{(\mathrm{e})^{(\approx 0)}\left[-(\mathrm{nn}\pi)^{\left(\frac{\sigma}{2}\right)}+\approx\infty+\approx\infty\right]}\right]\right)$$

$$= -\left(\frac{+\infty}{+\infty}-0\right) \text{ (indeterminate form)}$$

And From $\frac{d(\mathrm{x_n})^{\left(\frac{\sigma}{2}-1\right)}(\mathrm{x_n})^{\left(\frac{it}{2}\right)}}{d\mathrm{x_n}}$

$$= \frac{d(\mathrm{e})^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{\left(\frac{it}{2}\right)\mathrm{Log}(\mathrm{x_n})}}{d\mathrm{x_n}}$$

$$= (\mathrm{e})^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(\mathrm{x_n})}\frac{d(\mathrm{e})^{\left(\frac{it}{2}\right)\mathrm{Log}(\mathrm{x_n})}}{d\mathrm{x_n}} + (\mathrm{e})^{\left(\frac{it}{2}\right)\mathrm{Log}(\mathrm{x_n})}\frac{d(\mathrm{e})^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(\mathrm{x_n})}}{d\mathrm{x_n}}$$

$$=(\mathrm{e})^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(\mathrm{x_n})}\frac{(it)(\mathrm{e})^{\left(\frac{it}{2}\right)\mathrm{Log}(\mathrm{x_n})}}{\mathrm{x_n}} +(\mathrm{e})^{\left(\frac{it}{2}\right)\mathrm{Log}(\mathrm{x_n})}\frac{\left(\frac{\sigma}{2}-1\right)(\mathrm{e})^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(\mathrm{x_n})}}{\mathrm{x_n}}$$

$$= (\mathrm{e})^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(\mathrm{x_n})}\left(\frac{it}{2}\right)\left[(\mathrm{e})^{\left(\frac{it}{2}\right)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{-\mathrm{Log}(\mathrm{x_n})}\right]$$

$$+(\mathrm{e})^{\left(\frac{it}{2}\right)\mathrm{Log}(\mathrm{x_n})}\left(\frac{\sigma}{2}-1\right)\left[(\mathrm{e})^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(\mathrm{x_n})}(\mathrm{e})^{-\mathrm{Log}(\mathrm{x_n})}\right]$$

$$= [\left(\frac{it}{2}\right)+\left(\frac{\sigma}{2}-1\right)][\ (e)^{\left(\frac{\sigma}{2}-1\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$= [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\ (e)^{[\left(\frac{\sigma}{2}-1\right)-1]\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$= [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\ (x_n)^{[\left(\frac{\sigma}{2}-1\right)-1]}(x_n)^{\left(\frac{it}{2}\right)}]$$

$$= [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\ (x_n)^{\left(\frac{\sigma}{2}-2\right)}(x_n)^{\left(\frac{it}{2}\right)}]$$

And $$\frac{d[\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)][\ (x_n)^{\left(\frac{\sigma}{2}-2\right)}(x_n)^{\left(\frac{it}{2}\right)}]}{dx_n}$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)\frac{d(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}}{dx_n}$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}\frac{d(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}}{dx_n}$$

$$+\ (e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}\frac{d(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}}{dx_n}]$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}\frac{(it)}{x_n}$$

$$+(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}(e)^{(\sigma-2)\mathrm{Log}(x_n)}\frac{(\sigma-2)}{x_n}]$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}\left(\frac{it}{2}\right)$$

$$+(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}(e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}\left(\frac{\sigma}{2}-2\right)]$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[\left(\frac{it}{2}\right)+\left(\frac{\sigma}{2}-2\right)][\ (e)^{\left(\frac{\sigma}{2}-2\right)\mathrm{Log}(x_n)}(e)^{-\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$= \left(\frac{\sigma}{2}+\frac{it}{2}-1\right)[\left(\frac{\sigma}{2}+\frac{it}{2}-2\right)][\ (e)^{[\left(\frac{\sigma}{2}-2\right)-1]\mathrm{Log}(x_n)}(e)^{\left(\frac{it}{2}\right)\mathrm{Log}(x_n)}]$$

$$=\left(\frac{\sigma}{2}+it-1\right)[\left(\frac{\sigma}{2}+it-2\right)][\ (x_n)^{[\left(\frac{\sigma}{2}-3\right)]}(x_n)^{\left(\frac{it}{2}\right)}]$$

So $$\frac{d^{\left(\frac{\sigma}{2}-1\right)}[(x_n)^{\left(\frac{\sigma}{2}-1\right)}(x_n)^{\left(\frac{it}{2}\right)}]}{d(x_n)^{\left(\frac{\sigma}{2}-1\right)}}$$

$$= [\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)\left(\frac{\sigma}{2}+\frac{it}{2}-2\right)\dots\ ]\ (x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}$$

$$= \sum_{k=1}^{\frac{\sigma}{2}-1}\left(\frac{\sigma}{2}+\frac{it}{2}-k\right)[\ (x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}]$$

And $\frac{d^{\left(\frac{\sigma}{2}\right)}[(x_n)^{\left(\frac{\sigma}{2}\right)}(x_n)^{(it)}]}{d(x_n)^{(\sigma)}}$

$$=[\left(\frac{\sigma}{2}+\frac{it}{2}\right)\left(\frac{\sigma}{2}+\frac{it}{2}-1\right)\dots\ ](x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}$$

$$= \sum_{k=1}^{\frac{\sigma}{2}}\left(\frac{\sigma}{2}+\frac{it}{2}-k+1\right)[\ (x_n)^{(0)}(x_n)^{\left(\frac{it}{2}\right)}]$$

Apply L' Hospital's Rule $\left(\frac{\sigma}{2}\right)$ times to

$$\lim_{a\to 0,b\to +\infty}\left[\frac{(x_n)^{\left(\frac{\sigma}{2}-1\right)}(x_n)^{\left(\frac{it}{2}\right)}}{(e)^{(x_n)(nn\pi)}[-(nn\pi)+\frac{\left(\frac{\sigma}{2}-1\right)}{(x_n)}+\frac{\left(\frac{it}{2}\right)}{(x_n)}]}\right]_{a/nn\pi}^{b/nn\pi}$$

or $$\lim_{a\to 0,b\to +\infty}\left[\frac{(x_n)^{\left(\frac{(\sigma)}{2}-1\right)}(x_n)(x_n)^{\left(\frac{it}{2}\right)}}{(e)^{(x_n)(nn\pi)}(x_n)+[-(nn\pi)+\left(\frac{\sigma}{2}-1\right)+\left(\frac{it}{2}\right)]}\right]_{a/nn\pi}^{b/nn\pi}$$

or $$\lim_{a\to 0,b\to +\infty}\left[\frac{(x_n)^{\frac{(\sigma)}{2}}(x_n)^{\left(\frac{it}{2}\right)}}{(e)^{(x_n)(nn\pi)}+[-(nn\pi)(x_n)+\left(\frac{\sigma}{2}-1\right)+\left(\frac{it}{2}\right)]}\right]_{a/nn\pi}^{b/nn\pi}$$

until $(x_n)^{\left(\frac{\sigma}{2}\right)} = (x_n)^{(0)} = 1$

Then $\lim_{a\to 0,b\to +\infty}$

$$\left[\frac{(x_n)^{\left(\frac{(\sigma)}{2}\right)}(x_n)^{\left(\frac{it}{2}\right)}}{-(nn\pi)(x_n)(e)^{(x_n)(nn\pi)}+\left(\frac{\sigma}{2}-1\right)(nn\pi)(e)^{(x_n)(nn\pi)}+\left(\frac{it}{2}\right)(nn\pi)(e)^{(x_n)(nn\pi)}}\right]_{a/nn\pi}^{b/nn\pi}$$

$$= \lim_{a\to 0,b\to +\infty}\frac{\sum_{k=1}^{\frac{\sigma}{2}}\left(\frac{\sigma}{2}+\frac{it}{2}-k+1\right)[\ (x_n)^{(0)}(x_n)^{(it)}]}{(e)^{(x_n)(nn\pi)}[-(nn\pi)^{\left(\frac{\sigma}{2}+1\right)}(x_n)-\frac{\sigma}{2}(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{\sigma}{2}-1\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}+(it)(nn\pi)^{\left(\frac{\sigma}{2}\right)}]}\Bigg]_{a/nn\pi}^{b/nn\pi}$$

$$= \lim_{a\to 0,b\to +\infty}\left[\frac{\sum_{k=1}^{\frac{\sigma}{2}}\left(\frac{\sigma}{2}+\frac{it}{2}-k+1\right)[\ (1)(b/nn\pi)^{(it)}]}{(e)^{(b/nn\pi)(nn\pi)}[-(nn\pi)^{\left(\frac{\sigma}{2}+1\right)}(b/nn\pi)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+(it)(nn\pi)^{\left(\frac{\sigma}{2}\right)}]}\right.$$

$$\left.-\ \frac{\sum_{k=1}^{\frac{\sigma}{2}}\left(\frac{\sigma}{2}+\frac{it}{2}-k+1\right)[\ (1)(b/nn\pi)^{(it)}]}{(e)^{(b/nn\pi)(nn\pi)}[-(nn\pi)^{\left(\frac{\sigma}{2}+1\right)}(b/nn\pi)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+(it)(nn\pi)^{\left(\frac{\sigma}{2}\right)}]}\ \right]$$

$$= \lim_{a\to 0, b\to +\infty} \left[\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)](1)(b)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(n)^{\left(\frac{\sigma}{2}\right)}(b)-(n)^{\left(\frac{\sigma}{2}\right)}+(it)(n)^{\left(\frac{\sigma}{2}\right)}\right]} \right.$$

$$\left. - \frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)](1)(a)^{\left(\frac{it}{2}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(a)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right]$$

$$= \lim_{a\to 0, b\to +\infty} \left[\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)](1)(e)^{\left(\frac{it}{2}\mathrm{Log}b\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(b)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right.$$

$$\left. - \frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)](1)(e)^{\left(\frac{it}{2}\mathrm{Loga}\right)}(nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(a)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right]$$

[$\lim_{a\to 0} \mathrm{Log}\,(a) = z$, or $\mathrm{Log}\,(\approx 0) = z$,

But $(e)^{(z)} \approx 0$, or $(e)^{(-\infty)} = \frac{1}{(e)^{(\infty)}} \approx 0$,

So $\lim_{a\to 0} \mathrm{Log}\,(a) \approx -\infty$]

$$= \lim_{a\to 0, b\to +\infty} \left[\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)][\cos\left(\frac{t}{2}\mathrm{Log}(b)\right)+i\sin\left(\frac{t}{2}\mathrm{Log}(b)\right)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(b)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(b)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right.$$

$$\left. - \frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)][\cos\left(\frac{t}{2}\mathrm{Log}(a)\right)+i\sin\left(\frac{t}{2}\mathrm{Log}(a)\right)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(a)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(a)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right]$$

$$= \left[\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)][\cos(\approx+\infty)+i\sin(\approx+\infty)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(\approx+\infty)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(\approx+\infty)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right.$$

$$\left. - \frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)][\cos(\approx-\infty)+i\sin(\approx-\infty)](nn\pi)^{\left(-\frac{it}{2}\right)}}{(e)^{(\approx 0)}\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(\approx 0)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right]$$

$$= \left[\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)](\approx 1+\approx i0)(nn\pi)^{\left(-\frac{it}{2}\right)}}{(\approx+\infty)\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(\approx+\infty)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]} \right.$$

$$-\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)](\approx 1+\approx i0)(nn\pi)^{\left(-\frac{it}{2}\right)}}{(\approx 1)\left[-(nn\pi)^{\left(\frac{\sigma}{2}\right)}(\approx 0)-(nn\pi)^{\left(\frac{\sigma}{2}\right)}+\left(\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]}\ ]$$

$$=\ 0-\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)](nn\pi)^{\left(-\frac{it}{2}\right)}}{\left[\left(\frac{it}{2}-1\right)(nn\pi)^{\left(\frac{\sigma}{2}\right)}\right]}$$

$$=\quad\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)(nn\pi)^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

For $n=1$ $\quad \frac{1}{(1)^s}\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}$

$$=\lim_{a\to 0,b\to+\infty}\int_{a/(1)(1)\pi}^{b/(1)(1)\pi}(e)^{(-x_1)(1)(1)\pi}(x_1)^{(\frac{s}{2}-1)}dx_1$$

$$=\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[(1)(1)\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

For $n=2$ $\quad \frac{1}{(2)^s}\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}$

$$=\lim_{a\to 0,b\to+\infty}\int_{a/(2)(2)\pi}^{b/(2)(2)\pi}(e)^{(-x_2)(2)(2)\pi}(x_2)^{(\frac{s}{2}-1)}dx_2$$

$$=\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[(2)(2)\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

...

For $n=\infty$

$$\frac{1}{(+\infty)^s}\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}\quad=\lim_{a\to 0,b\to+\infty}\int_{a/(\infty)(\infty)\pi}^{b/(\infty)(\infty)\pi}(e)^{(-x_\infty)(+\infty)}(x_\infty)^{\left(\frac{s}{2}-1\right)}dx_\infty$$

$$=\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[(\infty)(\infty)\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

Actually the correct method of integration of the above equations should be.

$$[\left(\frac{1}{1^s}\right)+\left(\frac{1}{2^s}\right)+\ldots+\left(\frac{1}{\infty^s}\right)]\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}$$

$$=\lim_{a\to 0,b\to+\infty}[\int_{a/(1)(1)\pi}^{b/(1)(1)\pi}(e)^{(-x_1)(1)(1)\pi}(x_1)^{\left(\frac{s}{2}-1\right)}dx_1$$

$$+\int_{a/(2)(2)\pi}^{b/(2)(2)\pi}(e)^{(-x_2)(2)(2)\pi}(x_2)^{\left(\frac{s}{2}-1\right)}dx_2$$

$$+\ldots$$

$$+\int_{a/(\infty)(\infty)\pi}^{b/(\infty)(\infty)\pi}(e)^{(-x_\infty)(\infty)(\infty)\pi}(x_\infty)^{\left(\frac{s}{2}-1\right)}dx_\infty$$

$$\sum_{n=1}^{+\infty}\left(\frac{1}{n^s}\right)\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}=\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[(\infty)(\infty)\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

$$+\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[(\infty)(\infty)\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

$$+\ldots$$

$$+\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[(\infty)(\infty)\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

$$\text{Or }\ \zeta(s)\prod\left(\frac{s}{2}-1\right)(\pi)^{-\left(\frac{s}{2}\right)}=\sum_{n=1}^{+\infty}\frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[nn\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)\neq\frac{1}{(s)(s-1)}+\int_1^{+\infty}\psi(x)[(x)^{\left(\frac{s}{2}-1\right)}+(x)^{-\left(\frac{1+s}{2}\right)}]dx$$

because the boundaries are not the same as Riemann did before. But in this case the different boundaries tell us that $x_1, x_2, \ldots, x_\infty$ are not the same,

$$\text{so }\int_{a/(1)(1)(\pi)}^{b/(1)(1)(\pi)}(x_1)^{\left(\frac{s}{2}-1\right)}dx_1\neq\int_{a/(2)(2)(\pi)}^{b/(2)(2)(\pi)}(x_2)^{\left(\frac{s}{2}-1\right)}dx_2$$

$$\neq\cdots$$

$$\neq \int_{a/(\infty)(\infty)(\pi)}^{b/(\infty)(\infty)(\pi)} (x_\infty)^{\left(\frac{s}{2}-1\right)}\, dx_\infty$$

, and infinite summation of the integrands $(e)^{(-nn\pi x)}$ of the above integrals cannot be done as Riemann did because infinite summation, $\sum_{n=1}^{+\infty}\ (e)^{(-nn\pi x)}$ of one of the integrand can be taken only when values of another integrand $(x_n)^{\left(\frac{s}{2}-1\right)}$ do not change when n changes.

4. Proof that $\prod\left(\frac{s}{2}\right)\left(\frac{s}{2}-1\right)(\pi)^{\left(-\frac{s}{2}\right)}\zeta(s)$, for $s=\frac{1}{2}+it$

$$\neq \frac{1}{2}+\frac{\left(tt+\frac{1}{4}\right)}{2}\int_1^{+\infty}\psi(x)(x)^{-\left(\frac{3}{4}\right)}\cos\left(\frac{1}{2}t\mathrm{Log}x\right)dx$$

$$\neq \xi(t)$$

From Riemann's equation

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \frac{1}{(s)(s-1)}+\int_1^{+\infty}\psi(x)\left[(x)^{\left(\frac{s}{2}-1\right)}+(x)^{-\left(\frac{1+s}{2}\right)}\right]dx \quad \dots (F)$$

Multiply $\left(\frac{s}{2}\right)(s-1)$ both sides and set $s=\frac{1}{2}+it$

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right)\left(\frac{s}{2}\right)(s-1)$$

$$= \frac{\left(\frac{s}{2}\right)(s-1)}{(s)(s-1)}+\left(\frac{s}{2}\right)(s-1)\int_1^{+\infty}\psi(x)\left[(x)^{\left(\frac{s}{2}-1\right)}+(x)^{-\left(\frac{1+s}{2}\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{\frac{1}{2}+it}{2}\right)\left(\frac{1}{2}+it-1\right)\int_1^{+\infty}\psi(x)\left[(x)^{\left(\frac{1}{4}+\frac{it}{2}-1\right)}+(x)^{-\left(\frac{1+\frac{1}{2}+it}{2}\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{tt+\frac{1}{4}}{2}\right)\int_1^{+\infty}\psi(x)\left[(x)^{\left(-\frac{3}{4}\right)}(x)^{\left(\frac{it}{2}\right)}+(x)^{\left(-\frac{3}{4}\right)}(x)^{\left(-\frac{it}{2}\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{tt+\frac{1}{4}}{2}\right)\int_1^{+\infty}\psi(x)(x)^{\left(-\frac{3}{4}\right)}\left[(e)^{\left(\frac{1}{2}t\mathrm{Log}x\right)}+(e)^{\left(-\frac{1}{2}t\mathrm{Log}x\right)}\right]dx$$

$$= \frac{1}{2}+\left(\frac{tt+\frac{1}{4}}{2}\right)\int_1^{+\infty}\psi(x)(x)^{\left(-\frac{3}{4}\right)}\left[\cos\left(\frac{1}{2}t\mathrm{Log}x\right)+i\sin\left(\frac{1}{2}t\mathrm{Log}x\right)\right.$$

$$\left.\cos\left(\frac{1}{2}t\mathrm{Log}x\right)-i\sin\left(\frac{1}{2}t\mathrm{Log}x\right)\right]dx$$

$$= \frac{1}{2} + \left(\frac{tt+\frac{1}{4}}{2}\right) \int_1^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)} 2\cos(\frac{1}{2}tLogx)dx$$

$$= \frac{1}{2} + \left(tt + \frac{1}{4}\right) \int_1^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)} \cos(\frac{1}{2}tLogx)dx$$

$$= \xi(t)$$

Consider $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)} \prod\left(\frac{s}{2}-1\right)\left(\frac{s}{2}\right)(s-1)$

From $\Gamma(s) = \frac{\Gamma(s+1)}{s}$ for $s \neq 0$

And $\Gamma(s) = \frac{\Gamma(s+k)}{s(s+1)(s+2)\ldots(s+k-1)}$ for $k = 0, 1, 2, 3, \ldots$ and $s+k > 0$, or $s > -k$

$, s \neq 0, -1, -2, \ldots, -(k-1)$

And $\prod(s) = \Gamma(s+1) = s\Gamma(s) = s\prod(s-1)$

Then $\prod(\frac{s}{2}) = \Gamma(\frac{s}{2}+1) = \frac{s}{2}\Gamma(\frac{s}{2}) = \frac{s}{2}\prod\left(\frac{s}{2}-1\right)$

Thus $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\left(\frac{s}{2}\right)\prod\left(\frac{s}{2}-1\right)(s-1) = \zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}\right)(s-1)$

So $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}\right)(s-1) = \frac{1}{2} + \left(tt + \frac{1}{4}\right) \int_1^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)} \cos(\frac{1}{2}tLogx)\, dx$

$= \xi(t)$

However, the above equation is not true because it was derived from the wrong equation… (F)

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \frac{1}{(s)(s-1)} + \int_1^{+\infty} \psi(x)[(x)^{\left(\frac{s}{2}-1\right)} + (x)^{-\left(\frac{1+s}{2}\right)}]dx \quad \ldots (F)$$

While the right one is

$$\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\prod\left(\frac{s}{2}-1\right) = \sum_{n=1}^{+\infty} \int_{0/(n)(n)(\pi)}^{+\infty/(n)(n)(\pi)} (e)^{-(n)(n)(\pi)(x_n)}(x_n)^{\left(\frac{s}{2}-1\right)}dx_n$$

Multiply by $\left(\frac{s}{2}\right)$ both sides

Then $\zeta(s)(\pi)^{\left(-\frac{s}{2}\right)}\left(\frac{s}{2}\right)\prod\left(\frac{s}{2}-1\right)$

$$= \left(\frac{s}{2}\right)\sum_{n=1}^{+\infty}\int_{0/(n)(n)(\pi)}^{+\infty/(n)(n)(\pi)} (e)^{-(n)(n)(\pi)(x_n)}(x_n)^{\left(\frac{s}{2}-1\right)}dx_n$$

$$= \left(\frac{(\sigma+it)}{2}\right)\sum_{n=1}^{+\infty} \frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[nn\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

From $\prod(\frac{s}{2}) = \Gamma(\frac{s}{2}+1) = \frac{s}{2}\Gamma(\frac{s}{2}) = \frac{s}{2}\prod\left(\frac{s}{2}-1\right)$

So $\zeta(s)(\pi)^{(-\frac{s}{2})}\left(\frac{s}{2}\right)\prod\left(\frac{s}{2}-1\right) = \zeta(s)(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}\right)$

Hence $\zeta(s)(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}\right) = \left(\frac{(\sigma+it)}{2}\right)\sum_{n=1}^{+\infty} \frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[nn\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$

And if one set $s = \frac{1}{2} + it = \sigma + it$

Then $\zeta\left(\frac{1}{2}+it\right)(\pi)^{\left(-\frac{(\frac{1}{2}+it)}{2}\right)}\prod\left(\frac{\frac{1}{2}+it}{2}\right)$

$$= \xi(t)$$

$$= \left(\frac{(\frac{1}{2}+it)}{2}\right)\sum_{n=1}^{+\infty}\int_{0/(n)(n)(\pi)}^{+\infty/(n)(n)(\pi)} (e)^{-(n)(n)(\pi)(x_n)}(x_n)^{\left(\frac{(\frac{1}{2}+it)}{2}-1\right)}dx_n$$

$$= \left(\frac{1}{4}+\frac{it}{2}\right)\sum_{n=1}^{+\infty} \frac{[\sum_{k=1}^{\frac{1}{4}}(\frac{1}{4}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[nn\pi]^{\left(\frac{1}{4}+\frac{it}{2}\right)}}$$

Which is unreasonable because value of $\sigma = \frac{1}{2}$ will not satisfy the equation

$$\zeta(s)(\pi)^{(-\frac{s}{2})}\prod\left(\frac{s}{2}\right) = \left(\frac{(\sigma+it)}{2}\right)\sum_{n=1}^{+\infty} \frac{[\sum_{k=1}^{\frac{\sigma}{2}}(\frac{\sigma}{2}+\frac{it}{2}-k+1)]}{\left(1-\frac{it}{2}\right)[nn\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}}$$

which value of k start from 1 to $\frac{\sigma}{2}$, so σ must ≥ 2 [or s =($\geq 2+ it$)].

Hence $\xi(t) \neq \frac{1}{2} + \left(tt + \frac{1}{4}\right) \int_{1}^{+\infty} \psi(x)(x)^{\left(-\frac{3}{4}\right)} \cos\left(\frac{1}{2} tLogx\right) dx$ and $\xi(t)$ will not vanish ($= 0$) for any values of t. Actually, at $s = \frac{1}{2} + it$, $\xi(t) =$

$$\zeta\left(\frac{1}{2} + it\right)(\pi)^{\left(-\frac{(\frac{1}{2}+it)}{2}\right)} \prod\left(\frac{\frac{1}{2}+it}{2}\right) = \left(\frac{(\sigma+it)}{2}\right) \sum_{n=1}^{+\infty} \frac{[\sum_{k=1}^{\frac{\sigma}{2}} (\frac{\sigma}{2} + \frac{it}{2} - k+1)]}{\left(1-\frac{it}{2}\right)[nn\pi]^{\left(\frac{\sigma}{2}+\frac{it}{2}\right)}} \neq 0 \text{ but fail}$$

to exist, hence σ must ≥ 2 [or $s = (\geq 2 + it)$].

And this is a good proof that $(\pi)^{\left(-\frac{s}{2}\right)} \prod\left(\frac{s}{2}\right) \zeta(s)$ when $s = \left(\frac{1}{2} + it\right)$ or $\zeta\left(\frac{1}{2} + it\right)(\pi)^{\left(-\frac{(\frac{1}{2}+it)}{2}\right)} \prod\left(\frac{\frac{1}{2}+it}{2}\right) = \xi(t)$ is not equal to zero but unreasonable or there are no nontrivial zeroes of $\zeta(s)$ on the critical line $\text{Res} = \sigma = \frac{1}{2}$ within the critical strip $\{s \in C: 0 < \text{Res} < 1\}$.

5. Proof that $\zeta(s) = (2)^{(s)}(\pi)^{(s-1)} \sin(\frac{\pi s}{2})\Gamma(1-s)\,\zeta(1-s)$ only when $s = \sigma + it = \frac{1}{2} + i0 = \frac{1}{2}$.

Or in the other word, there are no trivial zeroes that can be found from this functional equation because $\zeta(s) = \zeta\left(\frac{1}{2}\right)$ will diverge ($= \infty$) not vanish ($= 0$).

From $\quad (\pi)^{\left(-\frac{s}{2}\right)} \prod\left(\frac{s}{2} - 1\right)\zeta(s)$

Or $\quad (\pi)^{\left(-\frac{s}{2}\right)} \Gamma\left(\frac{s}{2}\right)\zeta(s)$

Let $s = (1-s)$, and this will be true if and only if $s = (\sigma + it)$ $= \left(\frac{1}{2} + it\right) = (1-s) = 1 - \left(\frac{1}{2} + it\right) = \left(\frac{1}{2} - it\right)$ or $\left(\frac{1}{2} + it\right) = \left(\frac{1}{2} - it\right)$ and hence $t = 0$, $s = \frac{1}{2}$.

And $\quad (\pi)^{\left(-\frac{s}{2}\right)} \Gamma\left(\frac{s}{2}\right)\zeta(s) = (\pi)^{-\left(\frac{1-s}{2}\right)} \Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)$, for $s = (1-s) = \frac{1}{2}$

Multiply by $\Gamma\left(\frac{1+s}{2}\right)$ both sides

$$(\pi)^{\left(-\frac{s}{2}\right)} \Gamma\left(\frac{s}{2}\right)\zeta(s)\Gamma\left(\frac{1+s}{2}\right) = (\pi)^{-\left(\frac{1-s}{2}\right)} \Gamma\left(\frac{1-s}{2}\right)\zeta(1-s)\Gamma\left(\frac{1+s}{2}\right)$$

From Duplication Formula

$$\Gamma(z)\,\Gamma\left(z+\tfrac{1}{2}\right) = (2)^{(1-2z)}\sqrt{\pi}\,\Gamma(2z)$$

If $z = \frac{s}{2}$

Then $$\Gamma\left(\tfrac{s}{2}\right)\Gamma\left(\tfrac{1+s}{2}\right) = (2)^{(1-s)}\sqrt{\pi}\,\Gamma(s)$$

Thus $$(\pi)^{\left(-\frac{s}{2}\right)}(2)^{(1-s)}\sqrt{\pi}\Gamma(s)\zeta(s) = (\pi)^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\tfrac{1-s}{2}\right)\zeta(1-s)\Gamma\left(\tfrac{1+s}{2}\right)$$

From Euler's Reflection Formula

$$\Gamma(1-z)\,\Gamma(z) = \frac{\pi}{\sin(\pi z)}$$

Then $$(\pi)^{\left(-\frac{s}{2}\right)}(2)^{(1-s)}\sqrt{\pi}\frac{\pi}{\sin(\pi s)\Gamma(1-s)}\zeta(s)$$

$$= (\pi)^{-\left(\frac{1-s}{2}\right)}\Gamma\left(\tfrac{1-s}{2}\right)\zeta(1-s)\Gamma\left(\tfrac{1+s}{2}\right)$$

Or $$\zeta(s) = \frac{(2)^{(s)}}{(2)}(\pi)^{(s)}\sin(\pi s)\Gamma(1-s)\zeta(1-s)\Gamma\left(\tfrac{1-s}{2}\right)\Gamma\left(\tfrac{1+s}{2}\right)$$

From Euler's Reflection Formula again,

$$\Gamma(1-z)\,\Gamma(z) = \frac{\pi}{\sin(\pi z)}$$

$$Z = \left(\tfrac{1-s}{2}\right)$$

$$(1-z) = \left(1-\tfrac{(1-s)}{2}\right) = \left(\tfrac{1+s}{2}\right)$$

Then $$\Gamma\left(\tfrac{1-s}{2}\right)\Gamma\left(\tfrac{1+s}{2}\right) = \frac{\pi}{\sin(\pi\frac{(1-s)}{2})} = \frac{\pi}{\sin(\frac{\pi}{2}-\frac{\pi s}{2})} = \frac{\pi}{\cos(\frac{\pi s}{2})}$$

Hence $$\zeta(s) = \frac{(2)^{(s)}}{(2)}(\pi)^{(s)}\sin(\pi s)\Gamma(1-s)\zeta(1-s)\Gamma\left(\tfrac{1-s}{2}\right)\Gamma\left(\tfrac{1+s}{2}\right)$$

$$= \frac{(2)^{(s)}}{(2)}(\pi)^{(s)}2\sin(\tfrac{\pi s}{2})\cos(\tfrac{\pi s}{2})\Gamma(1-s)\zeta(1-s)\frac{\pi}{\cos(\frac{\pi s}{2})}$$

$$=(2)^{(s)}(\pi)^{(s-1)}\sin(\tfrac{\pi s}{2})\Gamma(1-s)\zeta(1-s)\text{ , for } s = (1-s) = \tfrac{1}{2}$$

So, there are no trivial zeroes that can be found from this functional equation $\zeta(s)=(2)^{(s)}(\pi)^{(s-1)}\sin\left(\frac{\pi s}{2}\right)\Gamma(1-s)\zeta(1-s)$, because the condition of deriving the equation was $s = (1 - s)$ or $(\frac{1}{2} + it) = (1 - (\frac{1}{2} + it))$, or $(\frac{1}{2} + it) = (\frac{1}{2} + it)$ or $t = 0, s = \frac{1}{2}$.

Then $\zeta(s) \neq 0$ but $\zeta(s) = \zeta\left(\frac{1}{2}\right)$ which will diverge $(=\infty)$.

[Note that $\zeta\left(\frac{1}{2}\right) = \sum_{n=1}^{+\infty} \left(\frac{1}{(n)^{(\frac{1}{2})}}\right)$

$$= 1 + \frac{1}{(2)^{(\frac{1}{2})}} + \frac{1}{(3)^{(\frac{1}{2})}} + \frac{1}{(4)^{(\frac{1}{2})}} + \frac{1}{(5)^{(\frac{1}{2})}} + \ldots$$

$$= 1 + \frac{(2)^{(\frac{1}{2})}}{2} + \frac{(3)^{(\frac{1}{2})}}{3} + \frac{(4)^{(\frac{1}{2})}}{4} + \frac{(5)^{(\frac{1}{2})}}{5} + \ldots$$

But $1 + \frac{(2)^{(\frac{1}{2})}}{2} + \frac{(3)^{(\frac{1}{2})}}{3} + \frac{(4)^{(\frac{1}{2})}}{4} + \frac{(5)^{(\frac{1}{2})}}{5} + \ldots > 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots$

And because $1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \ldots + \frac{1}{\infty} = \infty$ (harmonic series)

So $\zeta\left(\frac{1}{2}\right) = 1 + \frac{(2)^{(\frac{1}{2})}}{2} + \frac{(3)^{(\frac{1}{2})}}{3} + \frac{(4)^{(\frac{1}{2})}}{4} + \frac{(5)^{(\frac{1}{2})}}{5} + \ldots = \infty$]

Summary

I think it is no use to go on proving the rest of Riemann's paper. I hope that my paper is clear enough to point out all mistakes or give disproof of the original Riemann Zeta Function and Riemann Hypothesis. I feel good if my paper can give warning to people who are trying to apply the Riemann Hypothesis to explain physical phenomena which will be very dangerous in many cases.

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