The Problem of Slow-witted Aliens, or
The Transverse Doppler Effect Simply Explained

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Abstract
The subject of this article is the dependence of observation results of the transverse Doppler effect on relative speed ascribed by an observer to himself.

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The popular online Wikipedia has an article dealing with the relativistic Doppler effect. The article contains a section on the transverse Doppler effect. It states the following:

The transverse Doppler effect is the nominal redshift or blueshift predicted by special relativity that occurs when the emitter and receiver are at the point of closest approach. Light emitted at this instant will be redshifted. Light received at this instant will be blueshifted.

Classical theory does not make a specific prediction for either of these two cases, as the shift depends on the motions relative to the medium.

Having read this note, one may raise a question as to which of these observations is true or how different results should be correctly interpreted. To all appearances, this question may likewise be raised by experts as evidenced by a series of relatively recent publications [1] – [4].

As an example, we can cite the problem of Joe and aliens, posted on a popular website.

http://spiff.rit.edu/classes/phys200/lectures/doppler/doppler.html

Q: Joe has an ordinary red laser pointer which emits red light, with a wavelength of about 650 nm. He points the laser directly up into the night sky.

Aliens flying past the Earth at \( v = 0.9c \) look straight down at Joe and his laser.

What wavelength do they measure?
Can their captive human servants see this light?

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Apparently, as figured out by the author, the aliens should see a shift of radiation emitted by Joe's laser pointer into the longwave range of the spectrum (redshift), which indicates a slowdown of Joe's clock. So, measured wavelength should be about 1492 nm.

The correct solution, however, is that the aliens in no way will be able to measure the frequency because both Joe and the aliens simultaneously ascribe the state of rest to themselves.

The aliens may have apparently had their own Einstein if they failed to take aberration into account and turn their eyes forward in the direction of motion at an angle corresponding to the relative speed ascribed to themselves in this problem.

In the present paper we give a review of the observation results of the transverse Doppler effect (Transverse Doppler Shift) and maintain that the relativistic observer and the source may ascribe any speeds to themselves (within an arbitrary inertial reference frame), so that the relativistic sum of these speeds is equal to their speed relative to each other.

For example, a relativistic observer can ascribe to himself the state of his proper rest ($V_0=0$). Here a radiation source moves within the observer’s reference frame, and the observer registers a radiation shift into the longwave range of the spectrum (redshift), which corresponds to the slowdown of the pace of a clock running within his reference frame [5].

The relativistic observer can, however, also ascribe to himself the state of motion relative to the source at rest. In this case, when observing the transverse Doppler effect, the observer who has accepted the fact of his motion will register a shift of frequency received from the radiation source at rest not into the longwave range of the spectrum, but into the shortwave range (blueshift). The observer in motion will explain the results of these observations by the slowdown of his own clock, which, in its turn, brings about visible acceleration of the processes observed by himself.

The relativistic observer can point his observation tool forward in the direction of motion at any arbitrarily chosen angle. By registering the radiation frequency of the source moving relative to himself, he may give a two-fold interpretation of the observation results. If he ascribes the state of rest to himself, he may consider a change of the measured frequency as resulting from the longitudinal relativistic Doppler effect. If he ascribes to himself a relative speed that corresponds to the tilt of the measuring tool considering aberration, the observer may conclude that the photon has arrived by the shortest route, and he is observing the transverse Doppler effect only.

Consider Observers $A$ and $B$ moving relative to each other at speed $V$. Observer $A$ is equipped with a green bulb emitting green monochromatic radiation, and Observer $B$ has an observation tool fitted with a detector, let us call it a tube.

Let the green bulb of Observer $A$ emit dispersed light. Assume Observer $B$ intends to measure the transverse Doppler effect only, without its longitudinal component. Thus he will be able to determine the slowdown degree of the clock of the source moving relative to himself. To do so, he will have to perform observation at time $t$, when the signal from Observer $A$ comes to him by the shortest route.

Special relativity postulates that any observer can ascribe to himself the state of rest since it is no different from the state of motion. Following this condition, Observer $B$ points the tube transversely to the direction of his motion (Fig. 1) and sees the red colour of the bulb, the frequency of which conforms to the formula for the transverse Doppler effect:
Observer $B$ decides that since he is at rest, observer $A$ is moving relative to him, which brings about a slowdown of his (Observer's $A$) clock and a shift of frequency of the emitted light into the longwave range of the spectrum.

$$f_b = \frac{f_a}{\gamma} \quad (1)$$

Fig. 1. The source of light is moving in the reference frame of the observer. The observer ascribes to himself the state of rest and points his tube transversely to the direction of motion of the source.

But suppose Observer $B$ is bored with considering himself at rest, and he makes up his mind to ascribe to himself the state of motion. Assume he knows what his and Observer's $A$ relative speed of motion is and decides that now Observer $A$ is at rest, and he himself (Observer $B$) is in motion relative to Observer $A$. Then Observer $B$ has to take aberration into account and tilt the tube at an angle corresponding to his relative speed. He tilts the tube forward (Fig. 2), and what does he see? He sees that the radiation frequency has shifted into the blue, shortwave range. Observer $B$ thinks over this phenomenon and comes to the conclusion that because of his own motion he himself is subjected to the slowdown of time and sees the acceleration of the pace of Observer's $A$ clock with the observed radiation frequency increasing accordingly:

$$f_b = \gamma f_a \quad (2)$$
Fig. 2. The observer moves within the reference frame of the source, i.e. he ascribes to himself the state of relative motion and takes aberration into account.

Let us recall that Observer $A$ emits green light. Can Observer $B$ see not the red, not the blue, but precisely the green light of the bulb? Yes, he can. At what angle should he tilt the tube to do so? If Observer $B$ turns the tube at an angle corresponding to about (considering the relativistic addition of speeds) half the speed of his motion relative to Observer $A$, then the green light is seen. Why the green one? Because having tilted the tube in this way, he has chosen the reference frame wherein both he and Observer $A$ move at identical speeds in the opposite direction. Accordingly, the slowdown of their clocks is the same.

Thus, Observer $B$ may look in any direction and see any colour of the source. Having ascribed to himself an appropriate speed, he may regard any observation as the observation of the transverse Doppler effect (Fig.3)
Fig. 3. The observer tilts the tube at some angle forward in the direction of motion. The observer will see the blue colour of the bulb. He can give a two-fold interpretation of the observation results. He may interpret them by either the longitudinal relativistic Doppler effect (if he considers himself at rest) or the transverse one (if he considers himself in motion).

Now let us consider the following example. Observer $A$ is holding not a bulb emitting dispersed light, but a laser 'machine gun' that is continuously firing directional very thin green pulses.

And again, Observer $A$ and Observer $B$ are in motion relative to each other at speed $V$. In so doing, both of them consider themselves at rest. Exactly as it is in the problem of Joe and aliens.

Let Observer $A$ (Joe) be located at the origin of the coordinate system in the reference frame $K$ and fire his 'machine gun' transversely to axis $x$. Alien $B$ is in motion within the reference frame $K$ at speed $V$ and at the moment of closest approach to $A$ while crossing axis $y$ looks through his tube. And yet he considers himself as being at rest too and holds the tube transversely to the direction of his motion. What does Observer $B$ see? Absolutely nothing, inasmuch as Observer $A$ is holding his
'machine gun' at 90 degrees too, and Observer $B$ has not taken aberration into account. If Observer $B$ accepts his motion relative to Observer $A$ and, considering aberration, turns his tube at an angle corresponding to his relative speed, he sees a laser beam, which will be blue though. As in the above case, Observer $B$ understands that because he has forcibly ascribed the speed to himself, the pace of his own processes and of the clock has slowed down, and he sees acceleration of Observer's $A$ clock and, accordingly, a higher frequency of the emitted light.

One can ask a question: Where's symmetry then? Doesn't Observer $B$ HAVE THE RIGHT to assume himself as being at rest?

Let now Observer $B$ assume himself at rest, and Observer $A$ with his 'machine gun' be flying by.

Observer $B$ is located at the origin of coordinates and holds his tube transversely to axis $x$. However, in order for Observer $B$ to see the pulses from the 'machine gun' because now Observer $A$ is in motion within the reference frame associated with Observer $B$, Observer $A$ has to admit the fact of his motion and tilt his 'machine' gun at an angle (crossing axis $y$) corresponding to aberration. Tilting the 'machine gun' backwards in the opposite direction is necessary for the pulses to take the shortest route into the tube of Observer $B$. Now Observer $B$ is at rest and sees the pulses of the red colour! The reason is clear: after all, Observer $A$ moves relative to Observer $B$, and his own clock (Observer's $A$) slows down, thus Observer $B$ can see the transverse Doppler effect and the radiation frequency shift into the longwave range.

Let's recall here that Observer $A$ 'fires' green pulses. Can Observer $B$ through his tube see the pulses not of the red and not of the blue, but of the green colour?

Yes, he can. To do so, Observer $A$ must tilt the 'machine gun' backward at an angle corresponding to roughly half their speed relative to each other. Observer $B$, in his turn, must tilt the tube forward at an angle corresponding to half their speed relative to each other (Fig. 4).
Fig. 4. The aliens and Joe can ascribe to themselves about half their relative speed each. Then the aliens must tilt the laser pointer backward, and Joe must tilt the tube forward in the direction of motion at equal angles. In this case the aliens will see that the radiation frequency of the source has not changed and the colour of the bulb has remained green.

Thus, any of them, Observer A and Observer B, may perfectly ascribe the state of rest to themselves. In doing so, they will fix the slowdown of time, the Lorentz contraction of longitudinal dimensions and the slowdown of the clock of the observer moving relative to them. However, in a purely physical sense, if one of them has ascribed to himself the state of rest, the other will have to ascribe to himself the state of motion.

After all, Observer A and Observer B actually move relative to each other! They move, meaning they are in motion but not at rest relative to each other. They move relative to each other within any their common reference frame.

Now, let us consider the following example. Assume Observer A is at rest in the centre of a circle, and Observer B is flying along the circumference around him. Are they moving relative to each other or not? After all, the distance between them does not change with time. What do we mean by motion?
It is of interest that if Observer $A$ has a green bulb, and Observer $B$ is looking at him through the tube, Observer $B$ will see the blue colour only. [6]-[8] Observer $B$ is unable to see any other colour. It is self-evident that in order to take aberration into account the tube has to be tilted at an angle corresponding to the linear velocity of the observer on the circumference (Fig. 5).

![Diagram showing observer and tube](image)

**Fig. 5.** The source is in the centre, and the observer rotates around it along its circumference.

If Observer $B$ has a green bulb, and Observer $A$ is looking at him through the tube, then Observer $A$ will see the red colour only (Fig. 6).
Fig. 6. The observer is in the centre of a circle with the source rotating around him.

Observer $A$ is unable to see any other colour.
Fig. 7. The observer on the opposite side of the circle will not detect any change in radiation frequency received from the source. This is because the clocks of the observer and of the source slow down equally. However, the observer in the centre of the circle will register a radiation shift into the shortwave range of the spectrum.

Thus, contrary to the widespread opinion, a relativistic observer can see anything he likes: either the slowdown of the pace of a clock in motion relative to him (within his reference frame) or its acceleration. Each of the observers will account for the different observation results by ascribing to themselves a different speed within the limits of the speed at which they move relative to each other.

References


