

Giving a first estimate as to quantum ‘number’ N(quantum) of a Pre-Planckian state of space-time, prior to inflation; plus application to Entropy and Graviton physics, with Neutrinos thrown in.

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Abstract

Using the idea of a quantum ‘number’ N, which is due to $\int_{r_1}^{r_2} \sqrt{2m \cdot (E_N - U)} \cdot dr = \hbar N \pi$, with the walls mixing the space-time import of interior dynamics of a pre Planckian state, using energy $E_N \sim \Delta E \approx \frac{\hbar}{\delta g_n \delta t} \equiv \frac{\hbar}{a_{\min}^2 \phi_{\text{initial}} \cdot \delta t}$

And the U ~ potential. Energy expression given by Padmanabhan for an inflaton state ϕ_{initial} when the scale factor $a(t) \sim a_{\min} t^\gamma$, we obtain the term, N, for quantum exited state, prior to the exiting of the Pre Planck state, to inflationary physics. From there we bridge this creation of N, in Pre Planckian space-time, to Semi classical and brane theory versions of entropy, and also its linkage to both graviton and nonstandard beyond the standard model physics.

1. Introduction.

We are examining the guts of the ‘semi classical’ integral given by Kompaneyetes, [1], as, if the wave functions vanish at the walls of the ‘containment space’ for evaluation of this integral.

$$\int_{r_1}^{r_2} \sqrt{2m \cdot (E_N - U)} \cdot dr = \hbar N \pi \quad (1)$$

The idea here is to make use of a Pre Planckian energy level which we will render as [2]

$$E_N \sim \Delta E \approx \frac{\hbar}{\delta g_n \delta t} \equiv \frac{\hbar}{a_{\min}^2 \phi_{\text{initial}} \cdot \delta t} \quad (2)$$

The scale factor assumed, here is given by Padmabhan, for [3]

$$a(t) \sim a_{\min} t^\gamma \quad (3)$$

Also, from [3] we have

$$\begin{aligned} V(\phi) &\equiv V_0 \cdot \exp \left[-\sqrt{\frac{16\pi G}{\gamma}} \cdot \phi \right] \\ \phi &\approx \sqrt{\frac{\gamma}{4\pi G}} \cdot \ln \left(\sqrt{\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)}} \cdot t \right) \\ t &= r / c \end{aligned} \quad (4)$$

If we use a simple mid point approximation,[4] in the Pre Planckian space-time we will be looking at

$$\int_{r_1}^{r_2} \sqrt{2m \cdot (E_N - U)} \cdot dr = \hbar N \pi \sim (r_2 - r_1) \cdot \sqrt{2m \cdot (E_N - U)}(r^*); r_2 < r^* < r_1 \quad (5)$$

Using Eq. (5) and giving a precise delineation as to $\sqrt{2m \cdot (E_N - U)}(r^*) \approx \text{function}(\phi(r(t)))$, allows us to identify N, in Eq. (5).

2. Formal treatment of N, in Eq. (5) due to Eq. (4) and Eq. (3).

$$\begin{aligned} \int_{r_1}^{r_2} \sqrt{2m \cdot (E_N - U)} \cdot dr &= \hbar N \pi \\ &\approx \sqrt{2m} \cdot (r_2 - r_1) \cdot \left[\frac{\hbar}{a_{\min}^2 \cdot \delta t} \right] \cdot \left(\frac{1}{\ln \left(\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot (r/c) \right)} \right)^{\sqrt{\frac{\gamma}{4\pi G}}} \cdot \left(1 - \frac{V_0 \cdot (a_{\min}^2 \cdot \delta t)^2}{\hbar^2 \cdot \frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot \left(\frac{r}{c} \right)^2} \cdot \left(\ln \left(\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot (r/c) \right) \right)^{\sqrt{\frac{\gamma}{\pi G}}} \right)^{\sqrt{\frac{\gamma}{\pi G}}} \\ &\sqrt{\frac{m}{2}} \cdot (r_2 - r_1) \cdot \left[\frac{\hbar}{a_{\min}^2 \cdot \delta t} \right] \cdot \left(\frac{1}{\ln \left(\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot (r^*/c) \right)} \right)^{\sqrt{\frac{\gamma}{4\pi G}}} \cdot \left(1 - \frac{V_0 \cdot (a_{\min}^2 \cdot \delta t)^2}{\hbar^2 \cdot \frac{16\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot \left(\frac{r^*}{c} \right)^2} \cdot \left(\ln \left(\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot (r^*/c) \right) \right)^{\sqrt{\frac{\gamma}{\pi G}}} \right)^{\sqrt{\frac{\gamma}{\pi G}}} \quad (6) \end{aligned}$$

Then the quantum number, N is then, for the Pre Planckian space-time reducible to the following value. Our claim, in simple language is that the quantum number, N, as given in Eq. (6) and Eq. (7) will have in its own way, if not equal to zero, even if we have a black body treatment of pre Planckian space-time, a connection with a semi classical treatment of Entropy, and we will delineate the semi classical treatment of entropy, below, in order to make the point that a non zero value for the N (quantum number) as given above, and in Eq. (7) if not equal to zero, will have a connection with the entropy, as given in Eq. (9) and Eq. (10). Our claim is that in doing so we will be able to say something about the formation of early universe gravitons and Neutrinos. I.e. our assumption will be in examining the existence of a Pre Planck space 'cosmological constant' parameter, dependent upon temperature, T, which even in the Pre Planckian era is significantly larger than today's cosmological "constant" value.

The value of m, in Eq. (7) below will be conflated with the mass of 'heavy gravitons' and will be considered to be non zero.

N(quantum - number)

$$\begin{aligned} &\approx \sqrt{\frac{m}{2}} \cdot (r_2 - r_1) \cdot \left[\frac{1}{\pi a_{\min}^2 \cdot \delta t} \right] \cdot \left(\frac{1}{\ln \left(\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot (r^*/c) \right)} \right)^{\sqrt{\frac{\gamma}{4\pi G}}} \cdot \left(1 - \frac{V_0 \cdot (a_{\min}^2 \cdot \delta t)^2}{\hbar^2 \cdot \frac{16\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot \left(\frac{r^*}{c} \right)^2} \cdot \left(\ln \left(\frac{8\pi G V_0}{\gamma \cdot (3\gamma - 1)} \cdot (r^*/c) \right) \right)^{\sqrt{\frac{\gamma}{\pi G}}} \right)^{\sqrt{\frac{\gamma}{\pi G}}} \quad (7) \end{aligned}$$

Here, $r^* \approx \text{Planck-length} / 2$. We will next discuss the application of this nonstandard value of N, quantum number, to the two values of entropy, as given in [], earlier and discuss its relationship to both graviton physics and the matter of Neutrino production.

3. Considering the semi classical definition of entropy.

Kolb and Turner [5] have a temperature T related entropy density which leads to that we are able to state total entropy as the entropy density time's space time volume $V_4 \approx (r^*)^3 \cdot \delta t$. The quantity g_* as given by Kolb and Turner, may have an upper bound of 120, or conceivably could be higher. Note that Hector De La Vega, in [6] claims that g_* is undefinable in Pre Planckian space-time. Our view is nuanced upon the idea of a finite region of space, , the quantum bounce as an initial starting point, which is referenced by the author in [5]. Having said that, the following is used in a finite volume of Pre Planckian space-time which we write up as [5]

$$S_{total} \equiv S_{Density} \cdot V_4 = \frac{2\pi^2}{45} \cdot g_* \cdot T^3 \cdot V_4 \quad (8)$$

Here we set, even in Pre Planckian space-time $T \approx 10^{32} K$. We can compare this with Thanu Padmanadan's [7] treatment of entropy which is with regards to micro canonical ensemble as defined via

$$\begin{aligned} \exp(S_{total}) &= g(E) = \frac{A}{N!} \cdot \int d^{3N}x \cdot \left[E - \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j) \right]^{\frac{3N}{2}} \\ &\leq \frac{A}{N!} \cdot \int d^{3N}x \cdot [E]^{\frac{3N}{2}} \approx \left[\frac{A}{N!} \cdot \int d^{3N}x \right] \cdot \left[\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \right]^{\frac{3N}{2}} \end{aligned} \quad (9)$$

Note that the N so mentioned in Eq.(9) is NOT the same as the N, in Eq. (7), I.e. frequently, we have $N = 1$, as a starting point, and the quantity $V_4 \approx (r^*)^3 \cdot \delta t$. This gives us the option of comparing what we get in entropy with Seth Lloyds[8]

$$I = S_{total} / k_B \ln 2 = [\#operations]^{3/4} = [\rho \cdot c^5 \cdot t^4 / \hbar]^{3/4} \quad (10)$$

Note that both Eq. (9) and Eq. (10) are not equal to zero, if we assume both are comparable, and if we assume

$$\begin{aligned} \left[\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \right]^{\frac{3N}{2}} \neq \epsilon^+ \approx 0 \\ \text{Iff} \\ \Lambda_{Max} = c_1 T (Max) \gg \Lambda_{Today} \end{aligned} \quad (11)$$

We can if we take the absolute value of (9) and (10) above get for small volume values good estimates as to the relative volume of the phase space in early universe cosmology where (9) and (3) give similar entropy values and we assume $\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \sim T^{00} V_4 \equiv \rho \cdot V_4 \gg \frac{1}{2} \cdot \sum_{i \neq j} U(x_i, x_j)$ [1], where U is a potential energy of self-interacting particles in an early universe cosmology.

4. Comments as to N(quantum number) and the value of Entropy, in Eq. (9) and Eq. (10)

The existence of a non zero quantum number, N(quantum) depends upon the existence of m (Pre Planckian) being not equal to zero. i.e. the presumed mass of a Pre Planckian mass for a ‘heavy’ graviton, will be in this case a precondition for the existence of a non zero quantum number, N, and also the existence of entropy, i.e. using a particle counting algorithm would exist along the lines of Ng [9]. This may exist if there exists a quantum bounce, along the lines of [10] and Loop quantum gravity [11]. As it is, the author also has asked if the initial radii goes to zero, that there still could be in that case, mostly due to entanglement the existence of entropy, at the start of the inflationary era, but [12] if not adhered to, i.e. the non existence of a non singular universe. would probably preclude the existence of massive gravitons, and if we do not have massive gravitons, it is difficult to see how to use the Ng infinite quantum statistics [9]. I.e. the Author has brought up the case where a singularity exists, but there is still non zero initial entropy, [13], but this choice probably does not permit massive gravitons initially.

We will be assuming an initial non zero, quantum bounce, and hence, massive gravitons.

Needless to say, the existence of a quantum bounce, infinite quantum statistics, and massive gravitons, may hinge in part upon

$$n(\text{graviton-count}) \sim \log \left[\frac{\Lambda_{Max} V_4}{8 \cdot \pi \cdot G} \right]^{\frac{3}{2}} \approx 10^{10} \quad (12)$$

$$\Leftrightarrow \Lambda_{Max} \propto \text{Number} \leq \infty, \text{Pre-Planckian}$$

The irony in all of this, may be that in a very small, pre Planckian regime of space-time that the quantum field theory value of the Cosmological constant, the value given by Peskins, page 790, as [14]

$$\langle 0 | H_{\text{vacuum-energy-hamiltonian-interaction}} | 0 \rangle \approx \Lambda^4 \leq \infty \quad (13)$$

I.e. the place for where the zero point vacuum energy interaction, giving an almost infinite value, may be correct would be in the quantum bounce regime of space-time.

A subsequent and open question we will wish to address, later is, is the following true, and if so, why? First of all, if Eq. (13) holds in the Pre Planckian state, with a finite volume, of less than one cubic length of Planckian length i.e. less than a Planck volume, then why would Eq. (13) hold? Or something similar. Secondly, what about Eq. (12)

Another bonus question will be the issues of if a graviton from the prior universe can survive transition to our present universe? This question I asked Dr. Paul Steinhardt, in 2009 in Paris in the meeting foundations of physics, , and neither he or I have an answer to this on.

Final question does the following make sense, in the Pre Planckian regime of space-time? . From already accepted for publication, [15] we have that

As is well known, a good statement about the number of gravitons per unit volume with frequencies between ω and $\omega + d\omega$ may be given by (assuming here, that $\bar{k} = 1.38 \times 10^{-16}$ erg/K, and K is denoting Kelvin temperatures, while we keep in mind that Gravitons have two independent polarization states) [15], [16]

$$n(\omega)d\omega = \frac{\omega^2 d\omega}{\pi^2} \cdot \left[\exp\left(\frac{2 \cdot \pi \cdot \hbar \cdot \omega}{kT}\right) - 1 \right]^{-1} \quad (14)$$

This formula predicts what was suggested earlier. A surge of gravitons commences due to a change of temperature.

Numerous authors including Carroll and Chen [17,18] have suggested in the Pre Planckian space-time that there is an increase in temperatures up to Planck temperature. Needless to say, we would then get, from [15] The first expression is a power of graviton based upon a rotating rod approximation, with the graviton given has having a mass of 10^{-60} - 10^{-62} grams, If so, then from Fontana [19] we have

$$P(\text{power}) = 2 \cdot \frac{m_{\text{graviton}}^2 \cdot \tilde{L}^4 \cdot \omega_{\text{net}}^6}{45 \cdot (c^5 \cdot G)} \quad (15)$$

First of all, we will integrate Eq.(14), and also give a Planck length value to the rotating rod, and then we get Eq.(15)

One can see the results of integrating Eq.(15), Note this expression for numerical production of gravitons is extremely sensitive to temperature, T, and se obtain , as cited in [15] the following

$$\langle n(\omega) \rangle = \frac{1}{\omega(\text{net value})} \int_{\omega_1}^{\omega_2} \frac{\omega^2 d\omega}{\pi^2} \cdot \left[\exp\left(\frac{2 \cdot \pi \cdot \hbar \cdot \omega}{\bar{k}T}\right) - 1 \right]^{-1} \quad (16)$$

Then, if $E_{\text{eff}} \equiv \langle n(\omega) \rangle \cdot \omega \equiv \omega_{\text{eff}}$; with $\hbar\omega \xrightarrow{\hbar=1} \omega \equiv |E_{\text{critical}}|$ being given in Eqn. (51) above we get a graviton burst as represented in Table 1, below. I.e. this was done assuming with T^* being an initial thermal background temperature of the pre inflationary universe condition of about 1/30 that of Planck temperature.

1st table, How to outline the existence of a relic graviton burst [15]

N1=1.794 E-6 for $Temp = T^*$	Power = 0
N2=1.133 E-4 for $Temp = 2T^*$	Power = 0
N3= 7.872 E+21 for $Temp = 3T^*$	Power = 1.058 E+16
N4= 3.612E+16 for $Temp = 4T^*$	Power \cong very small value
N5= 4.205E-3 for $Temp = 5T^*$	Power= 0

Do we see the same thing in terms of our present model ? Is such a burst feasible from the Pre Planckian regime?

5. Questions as to a link to Branes, and gravitons. And entropy generation. This is an open question, itself. Do branes apply in pre Planckian conditions? Brane world picture of early universe entropy formation

This is adapted from a lecture at ICGC-07 by Mathur [4]. We propose that branes and anti branes form the working component of an instanton. I.e. we obtain for D space time dimensions, and E the general energy $S \sim E^{(D-1/D)}$. Note that Mathur's classical and quantum gravity article[5] has a

string winding interpretation of energy along the lines of putting energy E into string windings which leads to an entropy defined in terms of an energy value of, if mass $m_i = T_p \prod L_j$ (with T_p being the tension of the i th brane, and L_j being spatial dimensions of a complex torus structure) This leads to entropy [20,21]

$$S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \quad (17)$$

Our claim is that this very specific value of entropy for (17) above will at about the onset of inflation lead to [8,22]

$$\left[\left[S_{Total} = A \cdot \prod_i^N \sqrt{n_i} \right] / k_b \ln 2 \right] \approx [\#operations]^{3/4} \approx 10^{10} \quad (18)$$

Commented [WH1]:

Furthermore we claim that the interaction of the branes and anti branes will form an instanton structure, which is implicit in the treatment outlined in (18), and that the numerical counting given in (17) merely reflects that branes and anti branes, even if charge conjugates of each other have the same ‘wrapping number’ n_i . How to tie in the entropy with the growth of the scale function? Racetrack models of inflation, provide a power spectrum given by [22]

$$P \sim \frac{1}{150\pi^2} \cdot \frac{V(\phi)}{\epsilon} \quad (19)$$

This is assuming a slow roll parameter treatment with $\epsilon \ll 1$, and $t > t_p$. An increase in scalar power, is then proportional to an increase in entropy via [22]

$$\left| \frac{\Delta E}{l_p^3} \right| \sim \left| \frac{\Delta P \in 150\pi^2}{l_p^3} \right| \approx |\Delta S| \quad (20)$$

Now, Eq. (19), and Eq. (20) are in themselves suspect. Does one really believe that the parameterization, $\epsilon \ll 1$ Still hold ? in Pre Planckian space-time ? The term, Delta E, may be akin to the approximation given in Eq.(2) above, as well as the potential in Eq. (4) above. Our summary conclusion is that for the BRANE theory approximation, i.e. racetrack to hold, in Pre Planckian space-time, that Eq. (20) must give , through Eq. (2) and Eq. (4) a similar initial entropy value as Eq. (8) which we estimate will give a value of initial entropy $\sim 10^{10}$, which is what we would expect, if Eq.(18) holds. If this is not the case, then we have some major analytical problems. We also are concerned about if the Eq. (7) value for quantum numbers has any fidelity with a value of Eq. (7), being numerically similar to Eq. (20) above.

6. Finally the question of Non standard neutrinos, revisited

The major take away from this, is if there is a coupling between gravitons and photons, could there be a similar linkage as to early universe neutrinos, as well ? Originating, in part in processes initiated in the start of the Pre Planckian space-time regime ?

K. Meissner and H. Nicolai [23] recently postulated an extension of the SM (standard model) involving a classically conformal Langrangian. The outstanding feature of their model is that if we extend the standard model the way they intend to with the usual Higgs doublet Φ and one extra weak scalar field $\tilde{\phi}$ we write as[23]

$$\tilde{\phi}(x) = \phi(x) \cdot \exp\left[\frac{ia(x)}{\sqrt{2}\mu}\right] \quad (21)$$

Where the field $a(x)$ then gives rise to a (pseudo-) Goldstone particle associated “with the spontaneous breaking of a new global $U(1)_L$ (modified Lepton number) symmetry [23]. This boson, the so called ‘Majoran’ shares many properties with the axion” . And furthermore we use conformal symmetry to eliminate in their conformal Lagrangian contributions from all but $\tilde{\phi}^4$ terms in their Lagrangian so eventually we have masses for particles like ‘neutrinos’ which are heavier than the SUSY neutrino candidate, but with the same ‘branching ratio “ for a particle signature which is like the Higgs but with a lower cross section”.

Quoting what was said in the abstract. This ties in with possible new species of detectable neutrinos in ways which lead to an extension of the standard model, since the derived ‘axion’[23] is coupled to photons to the tune of $f_a = 9(10^{15} GeV)$

We direct the interested reader to [24]. Where Crowell gives analogous arguments for quantum gravity detection, in the beginning chapters of his books.

The linkage to Gravitons is by analogy. Here is a summary as to what may be happening in the transition from Pre Planckian to Planckian physics. As stated by [25, 26] (Crowell, 2010), the way to delineate the evolution of the VeV is to consider an initially huge VeV, due to inflationary geometry. Note by [27] (Poplawski, 2011):

$$\rho_\Lambda = H\lambda_{QCD} \quad (22)$$

Where λ_{QCD} is 200MeV and similar to the QCD scale parameter of the SU(3) gauge coupling constant, where H a Hubble parameter. Here if there is a relationship between Eq.(22) (35) above and $\rho_{vacuum} = [\Lambda/8\pi \cdot G]$ then the formation of inputs into our vacuum expectation values $V \sim 3\langle H \rangle^4 / 16\pi^2$, and equating $V \sim 3\langle H \rangle^4 / 16\pi^2$ with $V(\phi) \sim \phi^2$ would be consistent with an inflaton treatment of inflation which has similarities to [28] (Kuchiev and Yu, 2008). Then equate vacuum potential with vacuum expectation values as:

$$\rho_{vacuum} = [\Lambda/8\pi \cdot G] \approx \rho_\Lambda \approx H\lambda_{QCD} \Leftrightarrow V \sim 3\langle H \rangle^4 / 16\pi^2 \sim V_{inf} \approx \phi^2 \quad (23)$$

Different models for the Hubble parameter, H exist, and are linked to how one forms the inflaton. We are expecting that the ‘axion’ like Majoran, as stated in Eq. (21) may indeed be one of the constituent evolutionary fields, would be added to the potential as given in Eq. (23), and this is a detail which would have to be worked out via analysis.

7. Conclusion. What is an issue here, and what may be gained by completing the analysis

What we are doing is investigating the following interrelations

- Initially, a quantum number as defined in Eq. (1) is established, via Eq. (2) and Eq.(4) for the purpose of measuring quantum 'numbers' in Eq. (7) as an indirect measure of quantum geometry.
- We next Utilize the structure of Ng based infinite quantum statistics, [9] with $S \sim n$, with n a counting number, and state that Eq.(12) would have as an upper bound a numerical quality . This is for semi classical procedures, and it also has counterpart as mentioned as for Branes. Which are mentioned next.
- As for the Branes, the open question is, are Branes even pertinent in the Pre Planckian regime of space-time ? The document indicates a way this may be the case.
- This is done with the expectation of numerical scaling with respect to graviton counting. i.e. S (initial graviton count) $\sim 10^{\wedge} 10$ initially. I.e. the open question being if this is pertinent to both semi classical and branes, even if we look at what was postulated by Carroll and Chen [17, 18].
- The Majoran, and the variant of the Neutrino [23] , as a close cousin of the inflaton have to be worked into the context of Eq. (23). The open question being is the scalar field, as realized by Eq. (21) so identifiable, with the structures as brought up later in that paragraph?
- I.e. the nature of the quantum number, N (quantum) as delineated in Eq. (7) has to be compared to if it is connected to entropy counting

All this requires hard work. The author judges that the rewards for doing it, are immense and that it is time to seriously initiate investigations along these lines> The issues of the interplay between $SO(4)$ and gravity in [28] also need to be considered, since it, up to a point may influence the peak frequencies as seen next.

Above all we need to consider as given by Buonanno [29,30,31]

$$f_{Peak} \cong 10^{-8} \cdot [\beta/H_*] \cdot [T_*/16GeV] \cdot [g_*/100]^{1/6} Hz \quad (1.59)(55) \quad (24)$$

By conventional cosmological theory, limits of g_* are at the upper limit of 100-120, at most, according to Kolb and Turner (1991). [5] $T_* \sim 10^2 GeV$ is specified for nucleation of a bubble, as a generator of GW. Early universe models with $g_* \sim 1000$ or so are not in the realm of observational science, yet, according to Hector De La Vega (2009) in personal communications with the author, at the Colmo, Italy astroparticle physics school , ISAPP, [6] a signal for GW and/or gravitons may be to consider how to obtain a numerical count of gravitons and/or neutrinos for

$$h_0^2 \Omega_{gw}(f) \cong \frac{3.6}{2} \cdot \left[\frac{n_f [graviton] + n_f [neutrino]}{10^{37}} \right] \cdot \left(\frac{\langle f \rangle}{1kHz} \right)^4 \quad (25)$$

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