The relation of recurring decimal and primitive root

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1

We begin
\[ \frac{1}{7} = 0.14285714857 \ldots \]

10 = 3 + 7 is divided by 7 as 1 and the rest is 3. 30 is divided by 7 as 4 and the rest is 2. 20 is divided by 7 as 2 and the rest is 6. This is correspond to 3 \rightarrow 2 \rightarrow 6. So, that 3 is the primitive root of 7 equals to recurring decimal of \( \frac{1}{7} \). General case is not so easy.

\[ \frac{1}{13} = 0.076923076923 \ldots \]

case is just about \( \frac{1}{13} \) in 100 numeration. We cannot understand this case. 17 + 7 = 24 case, we calculate 24 numeration. 24 is divided by 17 as 1 and the rest is 7. 7 \times 24 = 168. 168 is divided by 7 as 9 and the rest is 15.

\[
7 \rightarrow 15 \rightarrow 3 \rightarrow 4 \rightarrow 11 \rightarrow 9 \rightarrow 12 \rightarrow 16 \rightarrow 10 \rightarrow 2 \rightarrow 14 \rightarrow 13 \rightarrow 6 \\
\rightarrow 8 \rightarrow 5 \rightarrow 1 \ldots
\]

In this case, 7 is primitive root of 17. So recurring decimal in 24 numeration is repeat 16.