# Using Waves to determine Primes, Composites, Number's Factors, and their Distributions.

William R. Blickos

**Sections:**



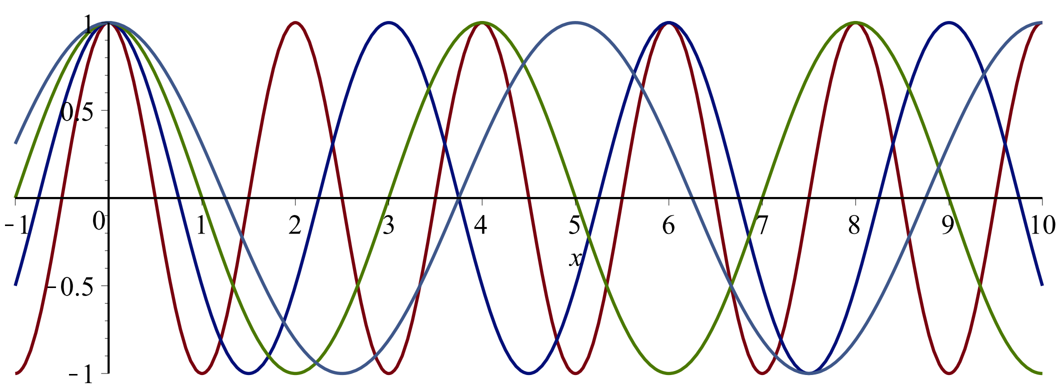
**1 - Introduction**

This paper provides a method using periodic functions to check for primality, count factors, list factors, calculate the prime distribution, and determine the Nth prime. It describes this method in a straightforward manner, from one equation to the next, using graphs between each key step to help quickly visualize the reasoning and result of each maneuver. It concludes with brief afterthoughts and ongoing questions about the technique.

**2 - Initial Functions and Strategy**

Begin with the set of basic Cos functions of the form $Cos\left(\frac{2\cdot\pi\cdot x}{k}\right)$, with wave number k, such that k is an integer $\geq 1$. Waves 2 through 5 are shown as a reference.

$$plot\left(\left[\cos\left(\frac{2\,\pi\,x}{k}\right)\$(k=2..5)\right], x=-1..10\right)$$

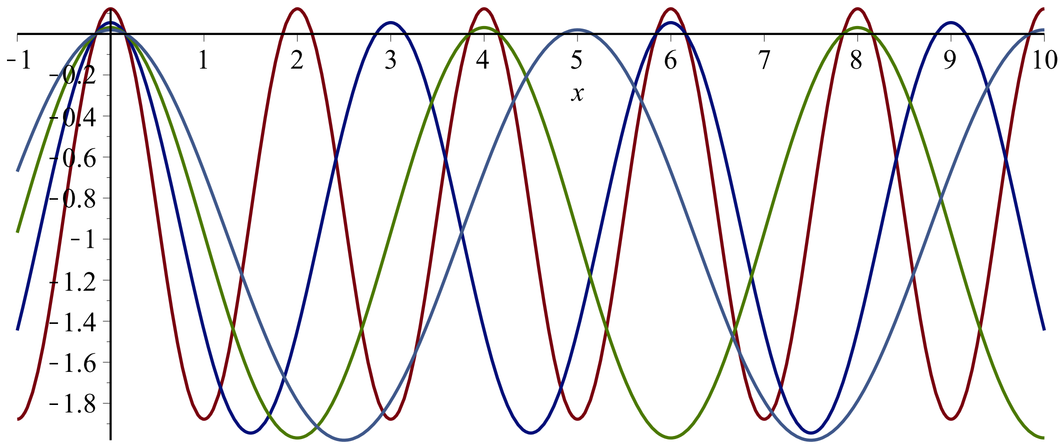



The first step, forthcoming, serves 2 purposes. It shifts the waves down such that their values remain positive above the multiples of each's individual wave number, but become negative below integers that are not multiples of each's wave number. Secondly, it sets a constant width for each crest such that wave crests that share an integer also share x-intercepts.

### 3 - Wave Peak Restricting

This is achieved by choosing the crest half-width, d, evaluating each wave at d, and sliding it down by that amount. Note, for the techniques that follow this step to work, d must meet the requirement that 0<d≤1/2. So, using $Cos\left(\frac{2\cdot\pi\cdot d}{k}\right)$, and the limiting requirement, convenient choices of d for simplification purposes are 1/2π, 1/π, and 1/2. For this paper, d=1/2π was used. Again, waves 2 through 5 are shown as a reference.

$$plot\left(\left[\cos\left(\frac{2\cdot\pi\cdot x}{k}\right)-\cos\left(\frac{1}{k}\right)\$(k=2..5)\right], x=-1..10\right)$$

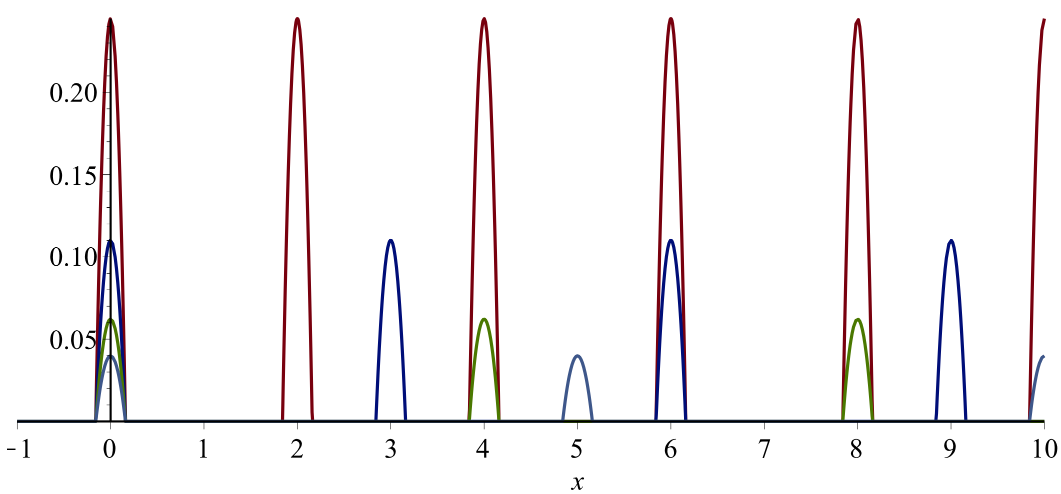



The reason for this restriction on d is due to the following steps, which amount to the facts that a choice of d=0 leaves no information above the axis to work with, and a choice of d>1/2 leaves information above the axis in places where there should be none and thus creates noise.

### 4 - De-noising

During a following summation, the information below the axis generates unwanted noise. To accommodate this, it is removed ahead of time via addition of the absolute values of the functions to themselves.

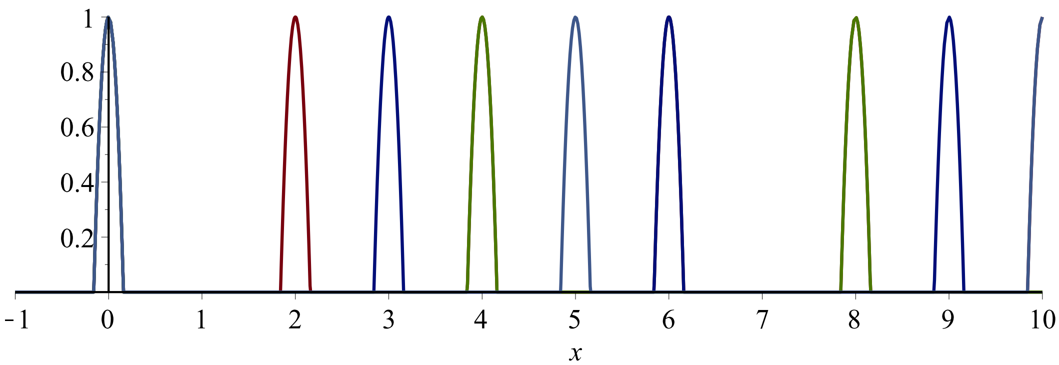
$$plot\left(\left[\cos\left(\frac{2\cdot\pi\cdot x}{k}\right)-\cos\left(\frac{1}{k}\right)+\left|\cos\left(\frac{2\cdot\pi\cdot x}{k}\right)-\cos\left(\frac{1}{k}\right)\right|\$(k=2..5)\right], x=-1..10\right)$$



**5 - Renormalizing Peaks to 1**

Due to the process so far, the peaks no longer have a value of 1. To normalize all the values to 1, divide by 2 to counteract the de-noising, and divide by $1 - \cos\left(\frac{1}{k}\right)$ to counteract the wave peak restricting. The result is:

$$plot\left(\left[\frac{\left(\cos\left(\frac{2\cdot\pi\cdot x}{k}\right) - \cos\left(\frac{1}{k}\right) + \left|\cos\left(\frac{2\cdot\pi\cdot x}{k}\right) - \cos\left(\frac{1}{k}\right)\right|\right)}{2\cdot\left(1 - \cos\left(\frac{1}{k}\right)\right)}\$(k=2..5)\right], x=-1..10\right)$$



**6 - Summation**

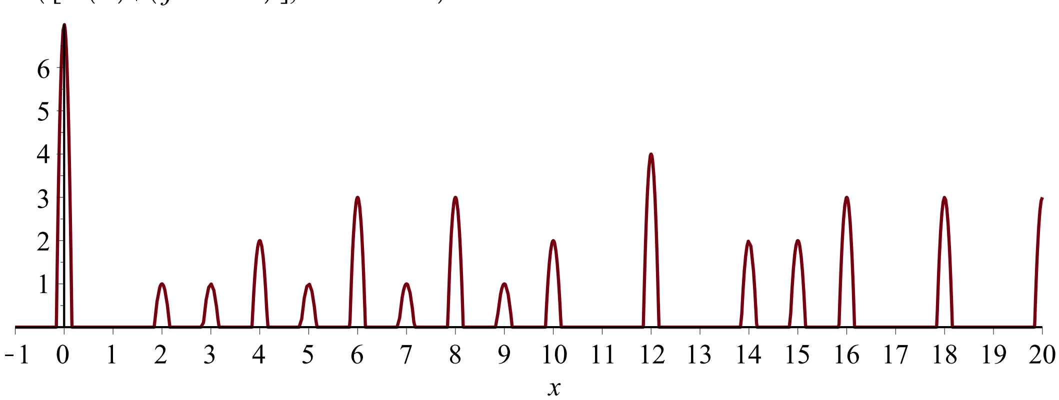
Now, every wave that is a factor of an integer x, or that x is a multiple of its wave number if it's preferred to think of it that way, contributes a value of 1 above that integer. So, total the number of ones at each integer and define the first main function F(x) to be:

$$F := x \to \sum_{k=2}^{j} \frac{\left(\cos\left(\frac{2\cdot\pi\cdot x}{k}\right) - \cos\left(\frac{1}{k}\right) + \left|\cos\left(\frac{2\cdot\pi\cdot x}{k}\right) - \cos\left(\frac{1}{k}\right)\right|\right)}{2\cdot\left(1 - \cos\left(\frac{1}{k}\right)\right)}$$

$$x \to \sum_{k=2}^{j} \frac{\cos\left(\frac{2\,\pi\,x}{k}\right) - \cos\left(\frac{1}{k}\right) + \left|\cos\left(\frac{2\,\pi\,x}{k}\right) - \cos\left(\frac{1}{k}\right)\right|}{2 - 2\cos\left(\frac{1}{k}\right)} \qquad \textbf{(1)}$$

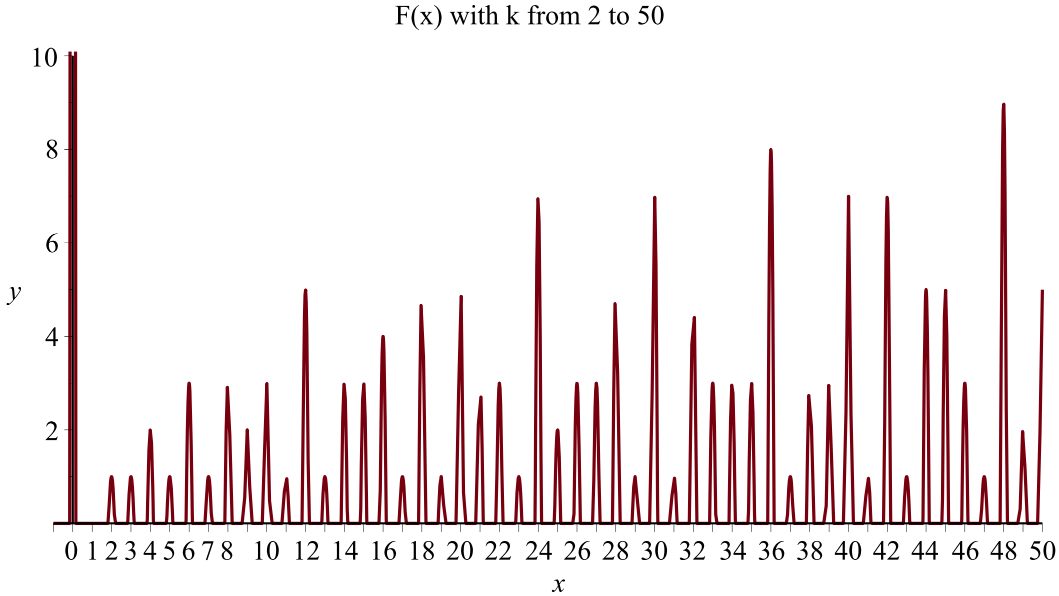
The result of adding the 2 through 8 waves for a reference is:

$plot([F(x)\$(j=8..8)], x=-1..20)$



In order to guarantee the function accuracy up to an integer x for all x, the summation must include all waves up to x.  Here is k from 2 to 50.

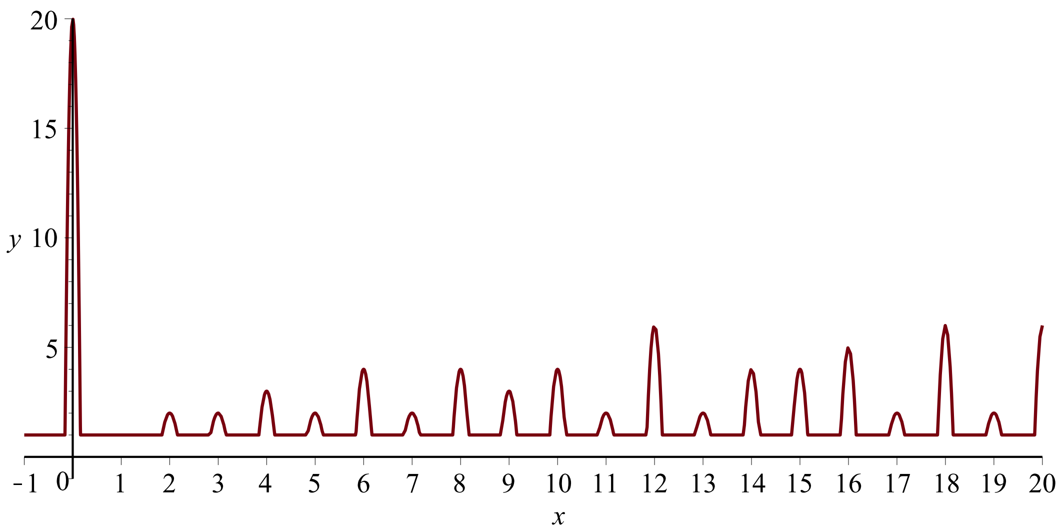
$plot(F(x), x=-1..50, y=0..10)$



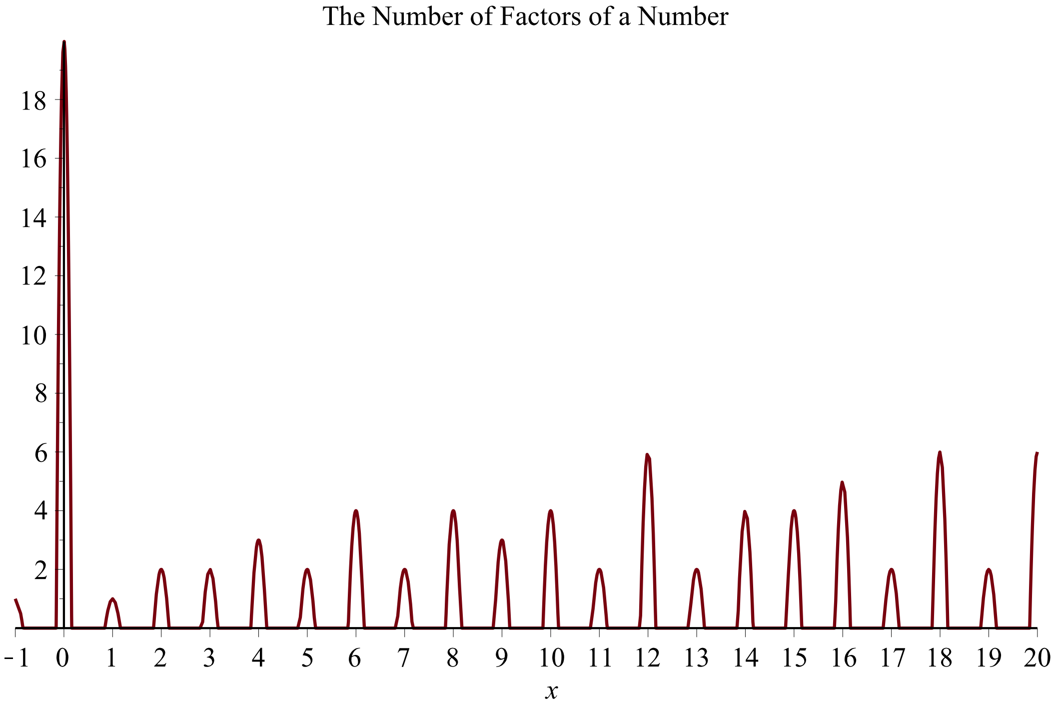

## 7 - A Waveform Prime Sieve and Factor Counting

At this point, the value of the function at an integer is equal to the number of factors of the integer, including the integer, but not including 1.  It acts as a Prime Sieve such that F(x) = 1 for all prime values of x such that x≤j, and that F(x) > 1 for all composites.  There are 2 easy ways to make the value of the function equal to "the number of factors of a number including 1 and the number".  The first, is to simply add a baseline of 1 to the function.  The second is to include the k = 1 wave.  Both are shown below.

$plot([F(x) + 1\$(j=20..20)], x=-1..20, y=-1..20)$



$$plot\left(\left[\sum_{k=1}^{j}\frac{\cos\left(\frac{2\cdot\pi\cdot x}{k}\right)-\cos\left(\frac{1}{k}\right)+\left|\cos\left(\frac{2\cdot\pi\cdot x}{k}\right)-\cos\left(\frac{1}{k}\right)\right|}{2\cdot\left(1-\cos\left(\frac{1}{k}\right)\right)}\$(j=20..20)\right],x=-1..20\right)$$




**8 - Factor Tagging**

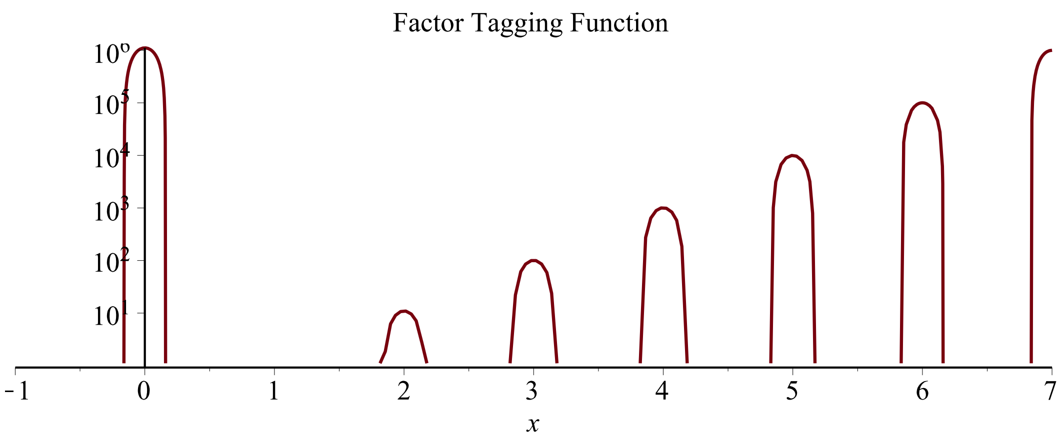
In order to find the specific factors of a number, take note that each factor is contributing a value of 1 to the value of F(x) at that integer. Next, include the fact that each number has a unique set of factors with no duplicates in that set. If each factor contributes a unique value instead of 1, and if the sums of the elements in those sets are also unique, then the resulting output at any integer uniquely corresponds to the specific set. Setting the unique value for each factor becomes that factor's "Tag". For each wave number k, consider a tag of $10^{(k-1)}$. That is, 2's tag is 10, 3's tag is 100, 4's tag is 1000, and so on. Now, adding each factor's tag value, instead of all factors contributing 1, gives that exclusive output. Note, there are certainly other tags that meet the mentioned requirements, but that this is one of the simpler ones, and as such, it is used in the following equation. Define a Factor Tagging Function, T(x), such that:

$$T := x \rightarrow \sum_{k=1}^{j}\frac{\left(10^{(k-1)}\right)\cdot\left(\cos\left(\frac{2\cdot\pi\cdot x}{k}\right)-\cos\left(\frac{1}{k}\right)+\left|\cos\left(\frac{2\cdot\pi\cdot x}{k}\right)-\cos\left(\frac{1}{k}\right)\right|\right)}{2\cdot\left(1-\cos\left(\frac{1}{k}\right)\right)}$$

$$x \rightarrow \sum_{k=1}^{j}\frac{10^{k-1}\left(\cos\left(\frac{2\,\pi\,x}{k}\right)-\cos\left(\frac{1}{k}\right)+\left|\cos\left(\frac{2\,\pi\,x}{k}\right)-\cos\left(\frac{1}{k}\right)\right|\right)}{2-2\cos\left(\frac{1}{k}\right)} \qquad \textbf{(2)}$$

A log plot of T(x) up to 7 for reference is as follows. *Note, the x = 1 value, is existent, but not visible at this graph's resolution.

$plot([T(x)\$(j=7..7)], x=-1..7)$




For example, the specific values of T(x) for 1 to 12 are as follows.

$[[T(x)\$(j=12..12)]\$(x=1..12)]$

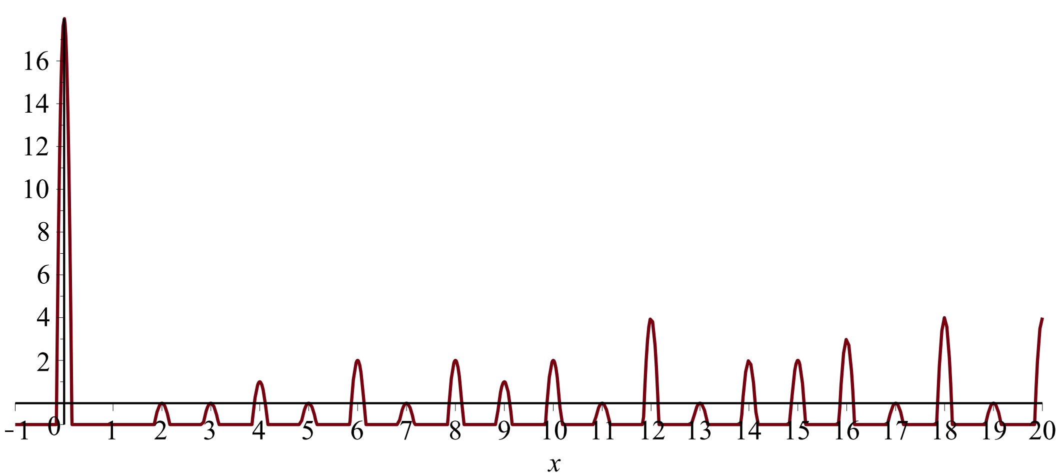
$[[1], [11], [101], [1011], [10001], [100111], [1000001], [10001011], [100000101], [1000010011], [10000000001], [100000101111]]$ **(3)**

This function generates an output in binary such that the 1s correspond to the factors from right to left. For example T(6) = 100111 shows the factors of 6 to be 1, 2, 3, and 6. While not discussed here, further associations can now be made between the decimal value of each binary string and its associated set. That is, [1] = 1 = [1], [11] = 3 = [1,2], [101] = 5 = [1,3], [1011] = 11 = [1,2,4], [10001] = 17 = [1,5], and so on. It is interesting to note, that it seems all the decimal values are primes, and that they span a subset of the primes. Questions on this are included in the afterthoughts section.

## 9 - Flip-Flopping to Filter the Composites from the Naturals

Next, the function F(x) can be manipulated to separate the composites from the naturals. This is done similarly to before through a process of further wave peak restricting, de-noising, and renormalizing. Shift F(x) down by 1, thus leaving information above the axis only above the composites.

$plot([F(x)-1\$(j=20..20)], x=-1..20)$

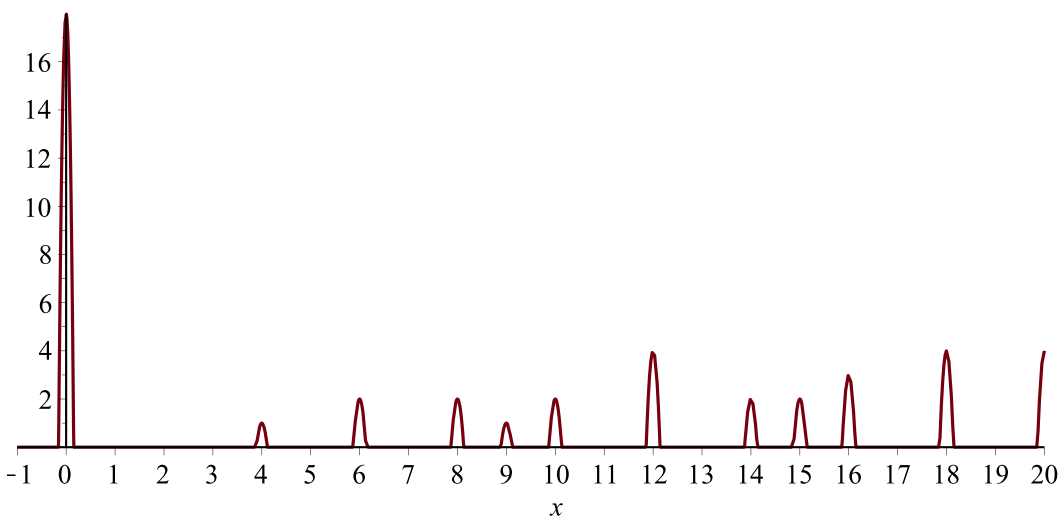


Then, remove the data below the axis via absolute value, and divide by 2 to counter that manipulation. Define that new function to be G(x).

$$G := x \rightarrow \left( \frac{(F(x) - 1 + |F(x) - 1|)}{2} \right)$$

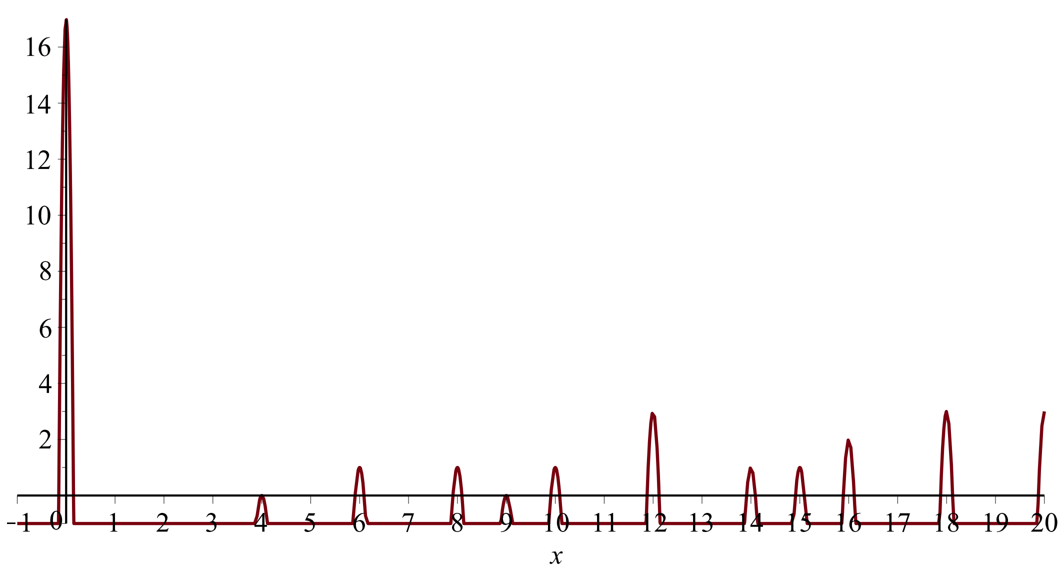
$$x \rightarrow \frac{1}{2} F(x) - \frac{1}{2} + \frac{1}{2} |F(x) - 1| \tag{4}$$

$plot(G(x), x = -1..20)$



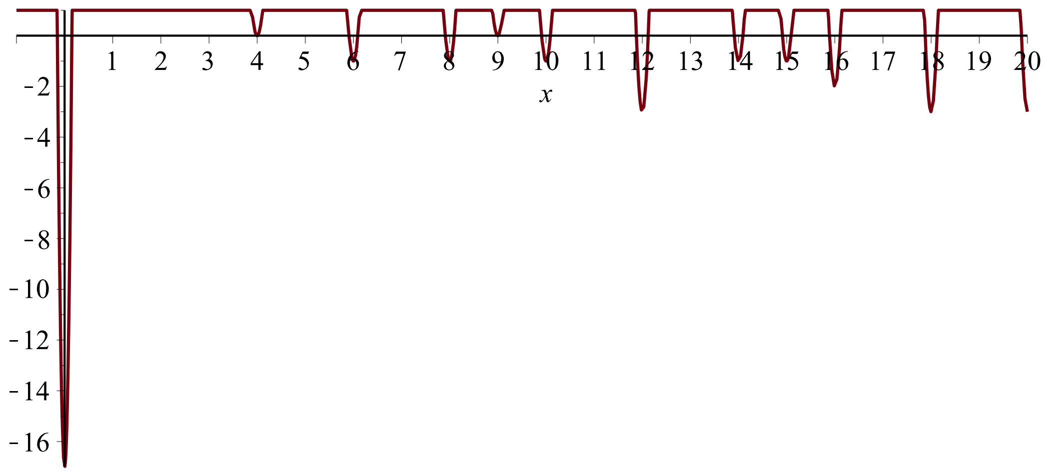
Now, the goal is to get all the composite peaks to have the same value, namely 1. This is accomplished by first shifting the function down by 1.

$plot(G(x) - 1, x = -1..20)$



Second, flip the function over the x axis.

$plot(-1 \cdot (G(x) - 1), x = -1..20)$

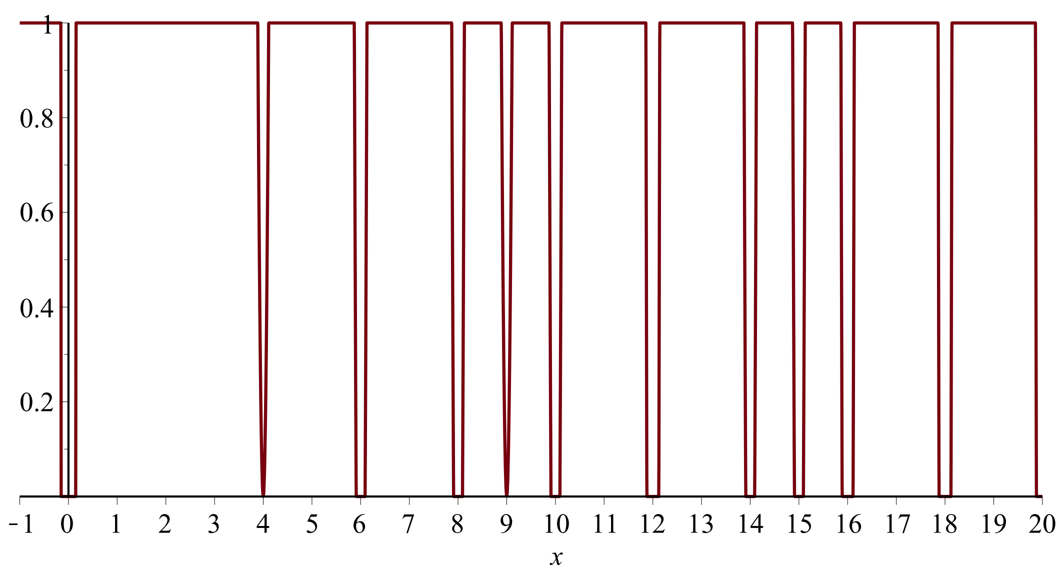



Again, add the absolute value and divide by 2 to counter the magnitude change. This chops off all the peaks which are now under the axis. This function was labeled as L(x) as a placeholder name simply to keep things neat in the paper.

$$L := x \rightarrow \frac{(1 - G(x) + |1 - G(x)|)}{2}$$

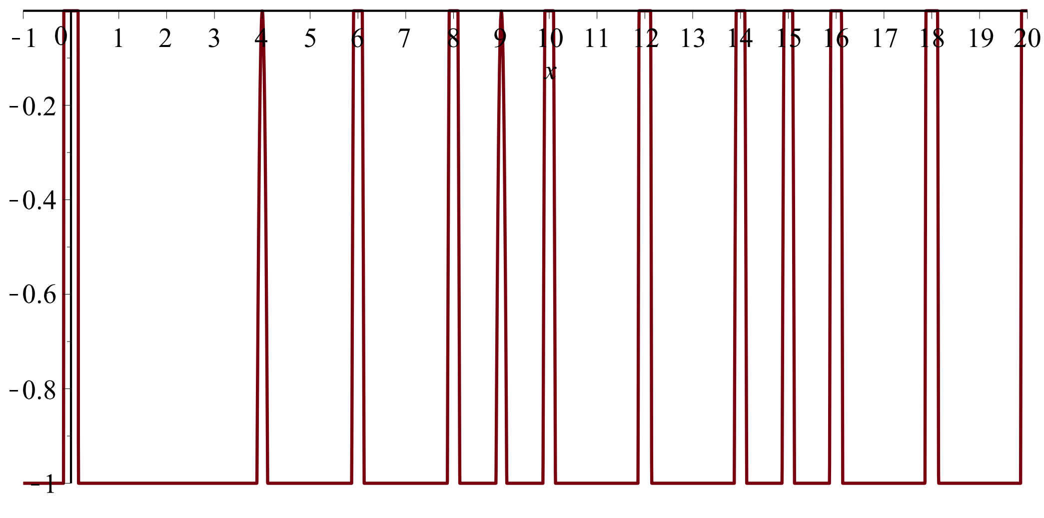
$$x \rightarrow \frac{1}{2} - \frac{1}{2} G(x) + \frac{1}{2} |1 - G(x)| \qquad \textbf{(5)}$$

$plot(\mathrm{L}(\mathrm{x}), x = -1..20)$



Flip the function back over.

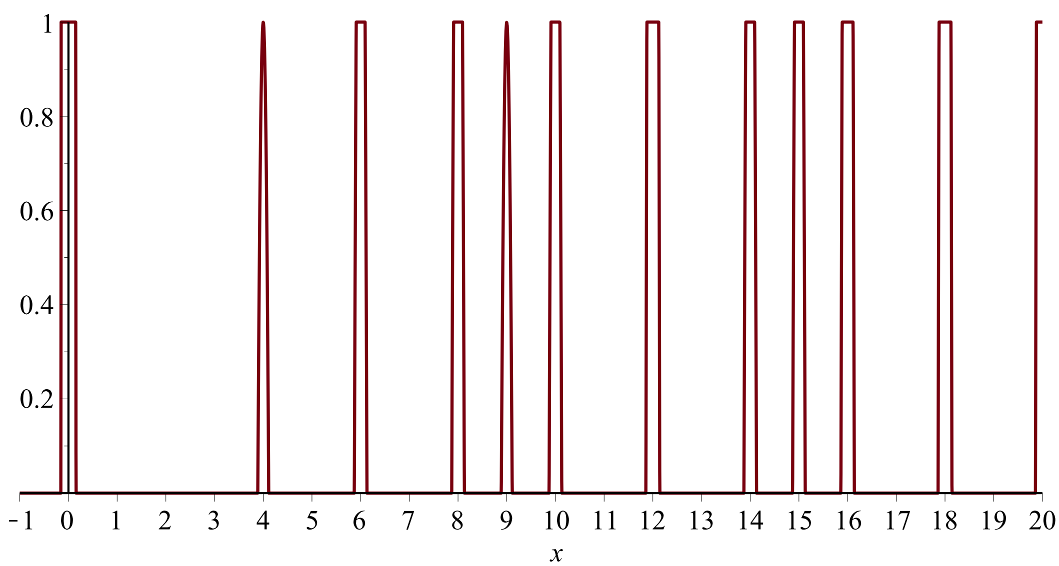
$plot(-1 \cdot \mathrm{L}(\mathrm{x}), x=-1..20)$



Finally, shift it back up by 1.

$plot(((-1 \cdot \mathrm{L}(\mathrm{x}))+1), x=-1..20)$

H(x), The Composite Function



This function now has a peak value of 1 for all composites and only the composites, and is labeled H(x).
$H := x \rightarrow (-1 \cdot \mathrm{L}(\mathrm{x}))+1$

$$x \rightarrow -L(x)+1 \tag{6}$$

Sums taken over this function can be used to give the number of composites ≤ a number, and as such it can be used to determine the prime distribution.

**10 - The Prime Distribution**

Noting that the number of primes ≤ a number is equal to that number, minus the number of composites ≤ the number, minus 1, the formula for the Prime Distribution, P(x), is:

$$P := x \to x - 1 - \sum_{n=1}^{x} H(n)$$

$$x \to x - 1 - \left( \sum_{n=1}^{x} H(n) \right) \tag{7}$$

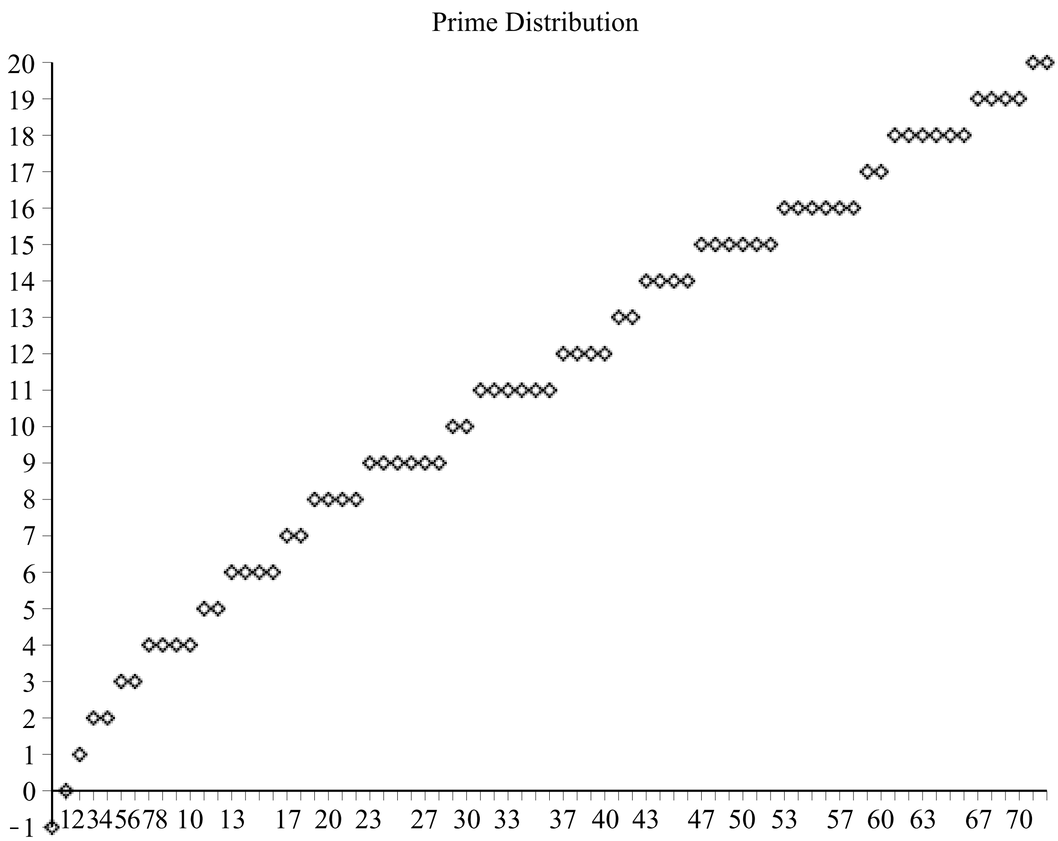
As an example, P(72) outputs 20, which coincides with 71 being the 20th prime.

$$P(72)$$

$$20 \tag{8}$$

Point plotting P(x) shows the familiar Prime Distribution. P(x) gives the exact distribution for all x as long as the initial restriction on j in F(x) is abided throughout the calculation.

$with(plots):$
$pointplot(\{seq([x, P(x)], x = 0..72)\})$



**11 - Recursive Sequence for the Nth Prime**

Using the formula for the exact distribution of the primes, a recursive sequence, $Q_s(n)$, can be fashioned to determine the nth prime. Given that $Q_0 = 0$, then:

$Q_s(n) = n + Q_{s-1} - P(Q_{s-1})$

$$x \rightarrow x + Q_{s-1} - P(Q_{s-1}) \tag{9}$$

Note, the variable x was used in the math program that generated this paper due to the convenience of how the formula was previously input, hence the change in variable from input to output. This sequence always equals the nth prime for some term s where s < n. Every subsequent term will also be that prime. It seems the last terms of the sequence prior to it repeating will always be the numbers from the previous prime through the current prime. As an example, the terms $Q_2$ through $Q_{16}$ for $Q_{16}(20)$ are shown below. The reason the sequence is calculated as shown, and the reason why the $Q_1$ term is not in the list, is covered in the next section.

$q \| 0 := 0$

$\boldsymbol{q \| 1 := x}$

**for** $s$ **from** $2$ **to** $16$ **do** $q \| s := x + q \| (s-1) - P(q \| (s-1))$ **end do**;

32

41

48

53

57

61

63

65

67

68

69

70

71

71

71 **(10)**

**12 - A Reminder as to the case of P(0)**

The $Q_1$ term of the sequence is equal to x + $Q_0$ - P($Q_0$). Given that $Q_0$ is defined as 0 leads to P(0). Logically, the function P(x) represents the number of primes ≤ a number, and so we know that P(0) should be 0. The $Q_1$ term then simplifies to always being equal to x. However, the P(x) developed is accurate for all x ≥ 1, with P(0) = -1. This can be addressed in at least 2 ways. One, is the method used in the previous section, where the sequence is started at $Q_2$ and ran through $Q_s$ with $Q_1$ given as equal to x. The other method is to actually adjust P (x) so it keeps all its values for x ≥ 1, but gains P(0) = 0. One way to do that, for example, is back at the G(x) stage of the process. Multiplying the original G(x) by x, and then continuing from there through the entire process, gives the desired results. Thus, as a starting point, the new function $G_b(x)$ would be:

$$G_b := x \rightarrow \left( \frac{x \cdot ((F(x) - 1 + |F(x) - 1|))}{2} \right)$$

## 13 - Afterthoughts and Ongoing Questions

This concludes a general method for using waves and summation to create a prime sieve, count the number of factors of a number, give specific factors, sort the composites from the naturals, give the prime distribution, and determine the nth prime. As an afterthought, are some general comments and questions about the method.

First and foremost, is the question of how this method can be improved and streamlined. What are the pros and cons of starting with different periodic functions? Is there a better choice for d? Can the same results be achieved with less flip flops? Is it better to think of the absolute value portion of the functions as the positive roots of the quantities therein squared?

Secondly, what is the efficiency and time complexity of the method? What is the formula for the number of terms, s, of the sequence P(x) that are needed to converge to the prime?

Also of consideration were the decimal values gotten from considering the output of the tag function as a binary number and then converting that number to base 10. Are all those values indeed prime? What other tag multipliers would be good to use? As a note, I tried one, basically the reciprocal of the one used, that output all the ones to the right side of the decimal point, but found the whole number version more intuitive for simplicity and explanation.

Finally, how can existing problems in number theory be thought of or solved in terms of this method? As two examples, consider the Twin Prime Conjecture and Mersenne Primes. Showing that there are an infinite number of integer solutions to the system F(x) = F(x+2) = 1, would prove the Twin Prime Conjecture. Similarly, showing there are an infinite number of solutions to $F(2^m - 1) = 1$ would solve the Mersenne Prime Conjecture.

If anyone can shed light on these considerations, or wishes to discuss the method further, let me know.

P.S. Using trig functions as the periodics eliminates logic functions like ceiling and floor. However, one such function that was developed while considering these topics, that works fairly efficiently is:

$$f(x) = \left(-1^{\left\lceil \frac{2\cdot x}{k} \right\rceil}\right)\cdot\left(\frac{\left((4\cdot x) - \left(2\cdot\left\lceil \frac{2\cdot x}{k}\right\rceil\cdot k\right) + k\right)}{k}\right)$$ which would lead to:

$$F(x) = \sum_{k=1}^{j}\left(\frac{1}{2}\left(\left(\left(\left((-1)^{\left\lceil \frac{2\cdot x}{k}\right\rceil}\right)\cdot\left((4\cdot x) - \left(2\cdot\left\lceil\frac{2\cdot x}{k}\right\rceil\cdot k\right) + k\right)\right) + (1-k)\right) + \left|\left(\left(\left((-1)^{\left\lceil \frac{2\cdot x}{k}\right\rceil}\right)\cdot\left((4\cdot x) - \left(2\cdot\left\lceil\frac{2\cdot x}{k}\right\rceil\cdot k\right) + k\right)\right) + (1-k)\right)\right|\right)\right)$$

and so on.

End