Illusory Signaling under Local Realism with Forecasts

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Abstract G. Adenier and A.Y. Khrennikov (2016) show that a recent "loophole free" CHSH Bell experiment violates no-signaling equalities, contrary to the expected impossibility of signaling in that experiment. We show that a local realism setup, in which nature sets hidden variables based on forecasts, and which can violate a Bell Inequality, can also give the illusion of signaling where there is none. This suggests that the violation of the CHSH Bell inequality, and the puzzling no-signaling violation in the CHSH Bell experiment may be explained by hidden variables based on forecasts as well.

1 Introduction

G. Adenier and A.Y. Khrennikov (2016) [1] analyze data from a recent "loophole free" CHSH Bell experiment reported in Hensen et. al., 2015 [2]. They demonstrate violation of no-signaling equalities, contrary to the apparent closure of this loophole. In J.R. Dixon (2011) [3] we imagined a local realism setup in which nature sets hidden variables based on forecasts. And we provided a "proof of concept" that in this setup, a Bell Inequality can be violated without any need to resort to nonlocal effects. Here we extend that proof of concept to show that the illusion of signaling can arise in the same setup, without actual signaling. This suggests that the violation of the CHSH Bell inequality in [2], and the puzzling no-signaling violation noticed in [1], may be explained by hidden variables based on forecasts as well.

For a convincing argument that "hidden variables based on forecasts by nature" may be behind "quantum" phenomena, in particular the violation of a Bell Inequality, we refer the reader to our previous paper [3]. The present paper seeks to further show that the setup in that paper can also give the illusion of signaling. The notation in this paper is relatively cumbersome, even with extensive abbreviations, compared to our previous paper. So our previous paper [3] may be a better starting point for the reader unfamiliar with our hypothesis.

In [3] we suggest experiments, which are relatively simple modifications of experiments already conducted, to test our hypothesis directly. And we hope such experiments will be carried out. We point out a recent experiment which provides anecdotal support for our hypothesis. In a recent test of local realism in D. Alsina and J.I. Latorre (2016) [4], the authors conclude:
"Experimental verification of Mermin inequalities for 3, 4 and 5 qubits has been tested on a 5 superconducting qubit IBM quantum computer. Results do show violation of local realism in all cases, with a clear degradation in quality as the number of qubits (and needed gates) increases."
We suggest that the reason for the degradation in quality may be that nature's forecasting abilities are limited, and its ability to achieve higher "quantum correlations" decreases as the complexity (number of qubits and gates) of the setup increases. (Here and in what follows we will use phrases like "quantum correlations". By this we mean numerical quantities which have values particular to quantum theoretical calculations, due to the fact that the underlying physical quantities are "correlated" in a sense that they vary in an interrelated manner that is unlikely due to chance. But the numerical quantities themselves may not technically fit the mathematical definition of "correlation".)

2 The Illusion of Signaling

In our "Proof of Concept" in [3] that the emitter in a Bell's Theorem setup sets hidden variables based on forecasts, and can thus achieve correlations predicted by quantum mechanics, we used the following notation. We point the reader to that paper for a more detailed discussion of the setup. We use g_{+-} to denote the event that the emitter guesses that the setting of the left detector will be $+1$ and that the setting of the right will be -1. We use e_{+-} to denote the event that the emitter sends a particle to the left in state $+1$ and to the right in state -1. We use d_{+-} to denote the event that the setting of the left detector will be $+1$ and that the setting of the right will be -1. Other guesses, detector settings, and emissions have analogous notation. We assume that the observed value at the left and right detectors, which we will denote by O_L and O_R, are the product of the emitted states and setting values. For example, if e_{++} and d_{+-}, then $O_L = (+1) \times (+1) = 1$ and $O_R = (+1) \times (-1) = -1$. In addition, to simplify notation: for a number y, we define $\overline{y} = 1 - y$, and abbreviate "with probability" by "w.p.".

The no-signaling equalities of [1] (their equations (13) and (14)) are as follows in our notation:

$$\begin{aligned} 0 &= |P(O_L = +1|d_{--}) - P(O_L = +1|d_{-+})| + |P(O_L = +1|d_{+-}) - P(O_L = +1|d_{++})|. \\ 0 &= |P(O_R = +1|d_{--}) - P(O_R = +1|d_{+-})| + |P(O_R = +1|d_{-+}) - P(O_R = +1|d_{++})|. \end{aligned}$$

Under our assumptions about the emissions, detector settings, and resulting observed values, this is equivalent to:

$$0 = |P(e_{--} \text{ or } e_{-+}|d_{--}) - P(e_{--} \text{ or } e_{-+}|d_{-+})| + |P(e_{++} \text{ or } e_{+-}|d_{+-}) - P(e_{++} \text{ or } e_{+-}|d_{++})|. \quad (1)$$
$$0 = |P(e_{--} \text{ or } e_{+-}|d_{--}) - P(e_{--} \text{ or } e_{+-}|d_{+-})| + |P(e_{++} \text{ or } e_{-+}|d_{-+}) - P(e_{++} \text{ or } e_{-+}|d_{++})|. \quad (2)$$

Recall that to achieve the exact correlation predicted by quantum mechanics, we assumed that the emitter might generate an independent random variable X (a pseudorandom variable is adequate), with $P(X = 1) = r$ and $P(X = 0) = \overline{r}$. We assume the emitter follows the following strategy, proven to be able to achieve the exact quantum correlation, denoted by q, in [3].

When $X = 0$:

$$\begin{array}{llllllll} \text{If} & g_{+-} & \text{then} & e_{+-} & \text{w.p.} & j_1 & \text{and} & e_{-+} \;\; \text{w.p.} \;\; \overline{j_1}. \\ \text{If} & g_{-+} & \text{then} & e_{--} & \text{w.p.} & j_2 & \text{and} & e_{++} \;\; \text{w.p.} \;\; \overline{j_2}. \\ \text{If} & g_{--} & \text{then} & e_{+-} & \text{w.p.} & j_3 & \text{and} & e_{-+} \;\; \text{w.p.} \;\; \overline{j_3}. \\ \text{If} & g_{++} & \text{then} & e_{+-} & \text{w.p.} & j_4 & \text{and} & e_{-+} \;\; \text{w.p.} \;\; \overline{j_4}. \end{array}$$

When $X = 1$:

$$\begin{array}{llllllll} \text{If} & g_{+-} & \text{then} & e_{--} & \text{w.p.} & k_1 & \text{and} & e_{++} \;\; \text{w.p.} \;\; \overline{k_1}. \\ \text{If} & g_{-+} & \text{then} & e_{+-} & \text{w.p.} & k_2 & \text{and} & e_{-+} \;\; \text{w.p.} \;\; \overline{k_2}. \\ \text{If} & g_{--} & \text{then} & e_{--} & \text{w.p.} & k_3 & \text{and} & e_{++} \;\; \text{w.p.} \;\; \overline{k_3}. \\ \text{If} & g_{++} & \text{then} & e_{--} & \text{w.p.} & k_4 & \text{and} & e_{++} \;\; \text{w.p.} \;\; \overline{k_4}. \end{array}$$

Compare this to the specification of the strategy in [3], and one can see we have simply introduced notation j_1, j_2, j_3, j_4 and k_1, k_2, k_3, k_4 to represent the probability the emitter chooses (via random or pseudorandom toggling, incidental or intentional) one of the two possible emission configurations allowed under its guess. The reader may find it intuitively obvious that there is enough leeway under the strategy described in [3] for the illusion of signaling to arise. What follows is an admittedly cumbersome mathematical demonstration of that possibility.

Recall that in our setup we assumed that for $(cd) = (ab)$: $P(g_{cd}|d_{ab}) = p$, and for $(cd) \neq (ab)$: $P(g_{cd}|d_{ab}) = \frac{1}{3}\overline{p}$. To further simplify the notation, we will define $\dot{p} \equiv \frac{1}{3}\overline{p}$.

Note that:

$$\begin{aligned} P(e_{--} \text{ or } e_{-+}|d_{--}) &= \overline{r}\left(\dot{p}\overline{j_1} + \dot{p}j_2 + p\overline{j_3} + \dot{p}\overline{j_4}\right) \\ &+ r\left(\dot{p}k_1 + \dot{p}\overline{k_2} + pk_3 + \dot{p}k_4\right). \end{aligned}$$

And:

$$\begin{aligned} P(e_{--} \text{ or } e_{-+}|d_{-+}) &= \overline{r}\left(\dot{p}\overline{j_1} + pj_2 + \dot{p}\overline{j_3} + \dot{p}\overline{j_4}\right) \\ &+ r\left(\dot{p}k_1 + p\overline{k_2} + \dot{p}k_3 + \dot{p}k_4\right). \end{aligned}$$

And so the first term of (1) can be simplified by:

$$P(e_{--} \text{ or } e_{-+}|d_{--}) - P(e_{--} \text{ or } e_{-+}|d_{-+}) = \overline{r}\left(j_2(\dot{p}-p) + \overline{j_3}(p-\dot{p})\right) + r\left(\overline{k_2}(\dot{p}-p) + k_3(p-\dot{p})\right). \tag{3}$$

Also note that:

$$\begin{aligned} P(e_{++} \text{ or } e_{+-}|d_{+-}) &= \overline{r}\left(pj_1 + \dot{p}\overline{j_2} + \dot{p}j_3 + \dot{p}j_4\right) \\ &+ r\left(p\overline{k_1} + \dot{p}k_2 + \dot{p}\overline{k_3} + \dot{p}\overline{k_4}\right). \end{aligned}$$

And:

$$\begin{aligned} P(e_{++} \text{ or } e_{+-}|d_{++}) &= \overline{r}\left(\dot{p}j_1 + \dot{p}\overline{j_2} + \dot{p}j_3 + pj_4\right) \\ &+ r\left(\dot{p}\overline{k_1} + \dot{p}k_2 + \dot{p}\overline{k_3} + p\overline{k_4}\right). \end{aligned}$$

And so the second term of (1) can be simplified by:

$$P(e_{++} \text{ or } e_{+-}|d_{+-}) - P(e_{++} \text{ or } e_{+-}|d_{++}) = \overline{r}\left(j_1(p-\dot{p}) + j_4(\dot{p}-p)\right) + r\left(\overline{k_1}(p-\dot{p}) + \overline{k_4}(\dot{p}-p)\right). \tag{4}$$

Further:

$$\begin{aligned} P(e_{--} \text{ or } e_{+-}|d_{--}) &= \overline{r}\left(\dot{p}j_1 + \dot{p}j_2 + pj_3 + \dot{p}j_4\right) \\ &+ r\left(\dot{p}k_1 + \dot{p}k_2 + pk_3 + \dot{p}k_4\right). \end{aligned}$$

And:

$$\begin{aligned} P(e_{--} \text{ or } e_{+-}|d_{+-}) &= \overline{r}\left(pj_1 + \dot{p}j_2 + \dot{p}j_3 + \dot{p}j_4\right) \\ &+ r\left(pk_1 + \dot{p}k_2 + \dot{p}k_3 + \dot{p}k_4\right). \end{aligned}$$

And so the first term of (2) can be simplified by:

$$P(e_{--} \text{ or } e_{+-}|d_{--}) - P(e_{--} \text{ or } e_{+-}|d_{+-}) = \overline{r}\left(j_1(\dot{p}-p) + j_3(p-\dot{p})\right) + r\left(k_1(\dot{p}-p) + k_3(p-\dot{p})\right). \tag{5}$$

Finally:

$$\begin{aligned} P(e_{++} \text{ or } e_{-+}|d_{-+}) &= \overline{r}\left(\dot{p}\overline{j_1} + p\overline{j_2} + \dot{p}\overline{j_3} + \dot{p}\overline{j_4}\right) \\ &+ r\left(\dot{p}\overline{k_1} + p\overline{k_2} + \dot{p}\overline{k_3} + \dot{p}\overline{k_4}\right). \end{aligned}$$

And:

$$\begin{aligned} P(e_{++} \text{ or } e_{-+}|d_{++}) &= \overline{r}\left(\dot{p}\overline{j_1} + \dot{p}\overline{j_2} + \dot{p}\overline{j_3} + p\overline{j_4}\right) \\ &+ r\left(\dot{p}\overline{k_1} + \dot{p}\overline{k_2} + \dot{p}\overline{k_3} + p\overline{k_4}\right). \end{aligned}$$

And so the second term of (2) can be simplified by:

$$P(e_{++} \text{ or } e_{-+}|d_{-+}) - P(e_{++} \text{ or } e_{-+}|d_{++}) = \overline{r}\left(\overline{j_2}(p-\dot{p}) + \overline{j_4}(\dot{p}-p)\right) + r\left(\overline{k_2}(p-\dot{p}) + \overline{k_4}(\dot{p}-p)\right). \tag{6}$$

As noted in [3], the solution for r as a function of p and the target quantum correlation q is:

$$r = \frac{q - 2p + 1}{-4p}.$$

In the special case where $q = \sqrt{2} - 1$ (the example considered in [3]), we have:

$$r = \frac{2p - \sqrt{2}}{4p} \quad \text{and} \quad \overline{r} = \frac{2p + \sqrt{2}}{4p}.$$

For p between $\frac{\sqrt{2}}{2}$ and 1, $0 < r < 1$ and $0 < \overline{r} < 1$. This is the range for p described in [3], and we restrict to the same range here. Define $\tilde{p} = p - \dot{p}$, and note that it is greater than 0 in the range for p we are considering.

Then:

$$\begin{aligned}
\text{equation (3)} &= \tilde{p}\left(\overline{r}(\overline{j_3} - j_2) + r(k_3 - \overline{k_2})\right), \\
\text{equation (4)} &= \tilde{p}\left(\overline{r}(j_1 - j_4) + r(\overline{k_1} - \overline{k_4})\right), \\
\text{equation (5)} &= \tilde{p}\left(\overline{r}(j_3 - j_1) + r(k_3 - k_1)\right), \\
\text{equation (6)} &= \tilde{p}\left(\overline{r}(\overline{j_2} - \overline{j_4}) + r(\overline{k_2} - \overline{k_4})\right).
\end{aligned}$$

And thus:

$$\begin{aligned}
\text{equation (3)} \neq 0 &\iff (2p + \sqrt{2})(\overline{j_3} - j_2) + (2p - \sqrt{2})(k_3 - \overline{k_2}) \neq 0, \\
\text{equation (4)} \neq 0 &\iff (2p + \sqrt{2})(j_1 - j_4) + (2p - \sqrt{2})(\overline{k_1} - \overline{k_4}) \neq 0, \\
\text{equation (5)} \neq 0 &\iff (2p + \sqrt{2})(j_3 - j_1) + (2p - \sqrt{2})(k_3 - k_1) \neq 0, \\
\text{equation (6)} \neq 0 &\iff (2p + \sqrt{2})(\overline{j_2} - \overline{j_4}) + (2p - \sqrt{2})(\overline{k_2} - \overline{k_4}) \neq 0.
\end{aligned}$$

If $k_1 = k_2 = k_3 = k_4 = \frac{1}{2}$, then this becomes:

$$\begin{aligned}
\text{equation (3)} \neq 0 &\iff (\overline{j_3} - j_2) \neq 0, \\
\text{equation (4)} \neq 0 &\iff (j_1 - j_4) \neq 0, \\
\text{equation (5)} \neq 0 &\iff (j_3 - j_1) \neq 0, \\
\text{equation (6)} \neq 0 &\iff (\overline{j_2} - \overline{j_4}) \neq 0.
\end{aligned}$$

Which is simply:

$$\begin{aligned}
\text{equation (3)} \neq 0 &\iff \overline{j_3} \neq j_2, \\
\text{equation (4)} \neq 0 &\iff j_1 \neq j_4, \\
\text{equation (5)} \neq 0 &\iff j_3 \neq j_1, \\
\text{equation (6)} \neq 0 &\iff \overline{j_2} \neq \overline{j_4}.
\end{aligned}$$

Which is satisfied by almost all choices of probabilities j_1, j_2, j_3, j_4. And the above differences can be different enough from 0 to allow statistical significance under typical sample sizes for a wide range of values of j_1, j_2, j_3, j_4. Further, the no-signaling equalities are violated if at least one of equations (3) and (4), and at least one of equations (5) and (6) are not 0.

3 Conclusion

We have shown that a local realism setup in which nature sets hidden variables based on forecasts, and which can violate a Bell Inequality, can also violate no-signaling equalities. But in our scenario, there is no signaling between the detectors. There is just an illusion of signaling. Thus we suggest the same hypothesis might explain the no-signaling violation G. Adenier and A.Y. Khrennikov (2016) [1] found in the "loophole free" CHSH Bell experiment reported in Hensen et. al. (2015) [2], despite the expected closure of that loophole. We again suggest experiments like those described in J.R. Dixon (2011) [3] to test our hypothesis directly.

4 References

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