Is There An Abstract Wave Function?

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Abstract: This paper gives a hypothesis about state and a formulation about quantum system. This formulation has no inside conflict, needn't any abstract boundary and can connect state with reality. Its calculation consists with orthodox theory (OQM). It is used to explain double slits experiment and Stern-Gerlach experiment. Paper also shows a case, in which, this formulation and OQM give different result.

I. Introduction

Year 1926, mathematical formulation of quantum mechanics was completed. This formulation was exalted by N. Bohr and W. Heisenberg. They considered this formulation as a complete theory of microscopic world [1]. Year 1932, J. V. Neumann complemented projection postulate to make orthodox quantum mechanics (OQM) [2].

OQM very consists with experiment. But it contains serious difficulty. From analysis of N. Bohr [3], W. Heisenberg [4], J. V. Neumann [2], A. Einstein [1, 5], Margenau [1] and many other physicists showed key difficulty of OQM:

Discrete values require wave function collapse. The collapse requires an wave function $\psi = \sum_n C_n . \phi_n$ as a combination of vector system which are orthogonal, normalization, completeness. But this wave function never appears in experiment so it is only an abstract wave function (AWF). It obeys Schrodinger's equation. But this equation contrasts with the wave function collapse. So, OQM contains inside conflict. On other hand, there are both spontaneous and measurement processes in OQM make some difficult in metaphysical aspect. First is the meaning of wave function. OQM only describes ability, not reality. So, OQM is only theory about epistemology, not about reality. The second, to use the projection postulate, we need an abstract boundary (AB) between quantum and classical, object and apparatus, matter and consciousness… Experiment and theory can't determine this boundary.

Disentanglement the difficulties of OQM is very important. Because, an exact solution reveals deeply properties of the world. To solve conflict between spontaneous process and measure process, it is one of three ways:

Remove one of two processes then use other in all case.

Modify one or both processes so that one become a consequence of other.

Modify one process and reject other.

Marengau wanted to reject measure process [1] but only spontaneous process is not fully to describe phenomena of atom. Many world theory [6], and consistent history theory reject the collapse but they are couldn't test.

Ensemble theory [8] is driven by second way, it modified the meaning of wave function. But this theory got not many successes. Objective collapse theories [9, 10, 11, 12, 13] modify the Schrodinger's equation to contain the collapse. Decoherence theory [14, 15] is also belong this way. On this way, we must mention hidden variable theory [16, 17], relational theory [18], transactional theory [19, 20], time-symmetric theory [21, 22]. However, every of them is also restricted. Some theories are until theory about epistemology, not about reality. Some are not valid with basic symmetry principles. All of them are couldn't test.

This paper gives a way to modify OQM. By analyzing OQM, we will see that in frame of OQM, there is no way to test the validity of the Schrodinger's equation. So this paper's approximation belongs third way. It rejects the abstract wave function (AWF), of course, it rejects spontaneous process. State of system is determined by eigenvectors Hamilton operator. However, the state is couldn't predict.

Formulation which is built in this paper consists with calculations by OQM. But it has no inside conflict. This formulation also connects the state vector to reality. Other result, in this formulation, we needn't any AB to get value of observable quantity.

Apply this formulation we can explain natural double slits experiment and Stern-Gerlach experiment. At the end of this paper, we give a supposed experiment. With this experiment, viewpoint of OQM and the formulation in this paper give difference results.

II. Hypothesis

Because OQM fits very good with experiment, so, we should start our analysis by OQM. We consider 6 cases:

First, we measure energy of system in stationary state. From OQM, after measurement, state of system is an eigenvector of Hamilton operator.

Second, we measure energy in time-dependent state. From principle about observable quantities of OQM, at moment t, we get an eigenvalue of Hamilton operator H(t). From projection postulate of OQM, after measurement, state of system is an eigenvector of operator H(t).

Third, measure quantity A which it commutes with Hamilton operator H. From projection postulate, after measure, state of system is an eigenvector of operator A. Because operator A commutes with operator H, state of system is also eigenvector of H.

Fourth, we measure quantity B which it doesn't commute with Hamilton operator. Call V_{mear} is interaction potential operator between system and apparatus. Operator $H_{mear} = H + V_{mear}$ is Hamilton operator of system during measurement. From OQM, after measurement, state of system is an eigenvector of operator B. If we measure B second time, serial after first time, we get the same result as first time. Because we do serially two measurement, so after first measurement, Hamilton operator of system is H_{mear}. Because the second measurement give a sure result, so operator B commutes with operator H_{mear}. So, after first measurement, state of system is an eigenvector of operator Hamilton during measurement H_{mear}.

Fifth, system in stationary state and there is no any measurement. In this case, from OQM, state of system is found from Schrodinger's equation. Of course, state of system is an eigenvector of Hamilton operator.

Sixth, system is time-dependent and there is no any measurement. State of system is also found from Schrodinger equation. Call ϕ_n is an eigenvector of Hamilton operator H(t) at moment t. State of system can be present as a combination $\psi = \sum_n C_n.\phi_n$. This combination is not eigenvector of operator H(t). However, we note that: "*there is no any way to test directly the validity of Schrodinger's equation in time-dependent state*". Because there is no any observable operator which it takes combination $\psi = \sum_n C_n.\phi_n$ as an eigenvector.

So, in frame of OQM, there is no any measurement to get combination $\psi = \sum_n C_n.\phi_n$. It means that we can't test state of system in time-dependent state.

From these analysis, we suggest that: "*in time-dependent state*, *system doesn't obey the Schrodinger's equation*". But how is about state of system? From above analysis again, we suggest "*at a moment, state of system is an eigenvector of Hamilton operator*". This hypothesis rejects the existence of AWF and spontaneous process. We can present this hypothesis in a more elegant way. This way based on symmetry property. That: "*state of system is always invariant with action of Hamilton operator*".

We must note that this hypothesis is applied for all case, include degenerate states. At moment t, Hamilton operator H(t) with eigenvectors ϕ_{ns}. In which, $s = 1,2 \dots gn$ and gn is degenerate degree of energy level E_n. A combination of ϕ_{ns} is also an eigenvector of Hamilton operator. Of course, that combination can be a state of system. So, at a moment, state of system can be present as: $\psi = \sum_s C_{ns}.\phi_{ns}$.

To describe statistics property of quantum systems, we need admit that: "*every eigenvector of Hamilton operator has corresponding probability to be state of system*". Now we show the way to find this probability. To do this, again, we start from OQM. Follow OQM, state of system is ruled by Schrodinger's equation:

$$i\hbar\frac{\partial}{\partial t}\Psi = H.\Psi \qquad (1)$$

Solution of (1) can be presented in form:

$$\Psi = \sum_n\sum_s C_{ns}.|\phi_{ns}\rangle \qquad (2)$$

In which, vectors ϕ_{ns} are eigenvectors of Hamilton operator H. Of course, they are orthogonal, normalization and completeness:

$$H(t).|\phi_{ns}\rangle = E_n(t).|\phi_{ns}\rangle \qquad (3)$$

$$\langle\phi_{mp}|\phi_{ns}\rangle = \delta_{mn}.\delta_{sp} \qquad (4)$$

And:

$$\sum_{ns}|\phi_{ns}\rangle.\langle\phi_{ns}| = 1 \qquad (5)$$

Probability that $|\phi_{ns}\rangle$ to be state of system as:

$$w_{ns} = |C_{ns}|^2 \qquad (6)$$

Normalization condition:

$$\sum_n\sum_s w_{ns} = 1 \qquad (7)$$

Probability that combination $\psi = \sum_s C_{ns}.\phi_{ns}$ to be state of system as

$$w_\psi = \sum_s|C_{ns}|^2 \qquad (8)$$

Take (2) into (1) we have:

$$i\hbar\sum_n\sum_s C_{ns}.\frac{\partial}{\partial t}|\phi_{ns}\rangle + i\hbar\sum_n\sum_s|\phi_{ns}\rangle.\frac{\partial}{\partial t}C_{ns} = \sum_n\sum_s E_nC_{ns}.|\phi_{ns}\rangle \qquad (9)$$

Multiple $\langle\phi_{mp}|$ with (9) and note to (3) (4) we get the equation of C_{ns}:

$$i\hbar\sum_n\sum_s C_{ns}.\left\langle\phi_{mp}\middle|\frac{\partial}{\partial t}\phi_{ns}\right\rangle + i\hbar.\frac{\partial}{\partial t}C_{mp} = E_m.C_{mp} \qquad (10)$$

Use equations (10) (6) and initial condition we can determine probability that eigenvector $|\phi_{ns}\rangle$ of Hamilton operator to be state of system. Now we consider two special case.

We consider system in stationary state. Note that, in stationary, energy of system doesn't depend on time. And, the vector $|\phi_{ns}\rangle$ depent on time in form: $exp(-\frac{i}{\hbar}.E_n.t)$. So product $\left\langle\phi_{mp}\middle|\frac{\partial}{\partial t}\phi_{ns}\right\rangle = \frac{i}{\hbar}.E_n.\left\langle\phi_{mp}\middle|\frac{\partial}{\partial t}\phi_{ns}\right\rangle$.

Equation (10) give result $C_{mp} = const$, dosen't depent on time. So, at moment t, vector $|\phi_{ns}\rangle$ is state of system, it is state of system forever.

The second case, we consider system in two serial amounts of time T1=[t0,t1] and T2=[t1,t2]. Hamilton Operator in T1 is H1, and in T2 is H2. These operators do not depend explicitly on time. From above analysis, C_{ns} dosen't vary in amount T1 and T2. Now, we consider the affect of continue properties on this case. We call solution of Schrodinger's equation in T1 and T2 are: $\psi^1 = \sum_n \sum_s C_{ns}.|\phi_{ns}\rangle$ and $\psi^2 = \sum_n \sum_s C_{ns}.|\varphi_{ns}\rangle$. The continue propertiy in time meaning that:

$$\sum_n \sum_s C_{ns}.|\phi_{ns}\rangle|_{t1} = \psi^1|_{t1} = \psi^2|_{t1} = \sum_n \sum_s C_{ns}.|\varphi_{ns}\rangle|_{t1} \quad (11)$$

Combine (11) with (10), (6) and initial condition, we can find C_{ns} and probability to be system state of eigenvector $|\phi_{ns}\rangle$ of Hamilton operator.

Not that (10) (6) and (11) are driven from OQM. It ensures that calculations from above hypothesis consist with OQM. Special, (11) is driven from continue property of AWF but it doesn't mean that eigenvectors of Hamilton operator to be continue. In other word, in frame of above hypothesis, state is not surely continuous.

III. Discussion

1. Difficulties of OQM

Hypothesis about state of system with equations (10) (6) (11) can be considered as a formulation. With this formulation, as show above, it consists with calculation from OQM. But it has no difficulties as OQM.

In this formulation, we have no AWF, have no collapse. Of Course, there is no conflict between two processes spontaneous and measurement.

In this formulation, state of every object can be found from eigenvectors of Hamilton operator. This hypothesis is used for all case. Of course, it can be used for very large number bodies or very small number bodies. It rejects AB between classical and quantum, between microscopic system and macroscopic system, between object and apparatus, between matter and consciousness…

In this formulation, it has no AWF, state of system is an eigenvector of Hamilton operator at moment. This vector can be measured. So, we can connect this vector to reality.

The reality from this formulation is very difference from classical physics. The first, it is discontinuous. The second, it is couldn't predicted. The third, it is couldn't divided. Totally, this formulation gives a discontinuous, indeterminate, nonlocal reality.

2. Double slits experiment

Now we use above hypothesis to consider the double slits experiment [23]. It is one of the most important experiment in physics. In this experiment, they radiated photons and then catch them by a screen. These photons are radiated in two cases. The first, there is only one slit which photon pass through before reaches to screen. The second, there are two slits on the way of photons.

Result of experiment, photons make traces on the screen. With the first case, traces make an shape which it is uniform of slit. With the second, traces make interference pattern. Note, when intensity of light beam so small that there is single photon in a pulse, traces still make interference pattern.

There was argument that every photon interferes itself or two photons interfere each other? This paper gives another way to understand. "*Not photons, but electromagnetic field makes interference pattern. Both photon and atom on screen, their state are always eigenvectors of their operator Hamilton".*

Before photon reaches to screen, state of electromagnetic field is a free wave. If this wave passes through only one slit, it doesn't make interference pattern. If this wave passes two slits, this wave makes interference pattern.

When photon reaches to atoms on screen, they interact each other. This moment, Hamilton operator of electromagnetic field and atom on screen change. From above hypothesis, state of both change. There is infinite ability. One of them, atom absorbs photon, electromagnetic field changes into ground state, photon disappears, trace on screen was made. Here, trace on screen is result of change of state of atom, not photon's wave function collapse as OQM.

In frame of this formulation, both photon and atom interact as ensembles. They are couldn't divided. So, every photon can be only absorbed by only one atom. So every absorbed photon makes only one trace on screen.

Base interaction quantum field theory [24], at the higher intensity position, the absorbed probability of photon is higher. So, At the higher intensity position, the higher trace density on screen. From this, with the first case, traces make shape as slit's shape. And with the second, traces make interference pattern.

This explanation can use for not only photon. This can be extended for every microscopic particle.

3. Stern-Gerlach experiment

Stern-Gerlach experiment [25] was performed in 1922. In this experiment, an apparatus makes a non-uniform magnetic field. First configuration magnetic Bz is oriented on z-axis. One Ag atom beam passes through apparatus. Then a screen records atoms. On this screen, there are two beam.

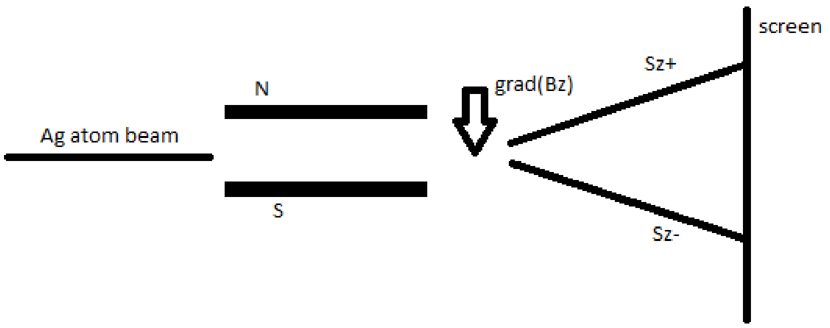

Figure 1: diagram of Stern-Gerlach experiment.

If we use second apparatus, it also makes non-uniform magnetic field. And it is also oriented on z-axis. We stop one beam. Other beam passes through the second apparatus. The screen records atoms. On this screen, we have only one beam.

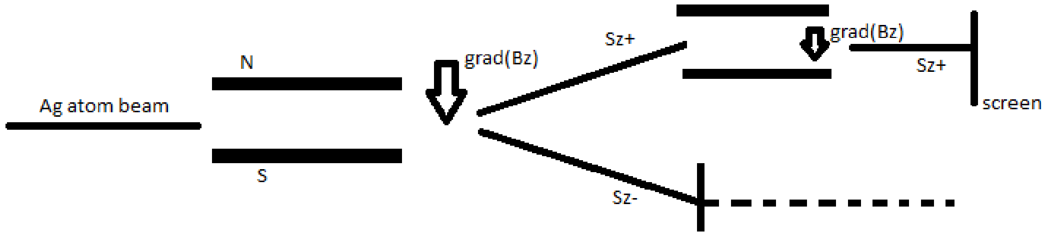

Figure 2: Stern-Gerlach experiment with two magnetic fields same orientation.

If the second magnetic field is oriented on x-axis y-axis, on the screen, we have two beams.

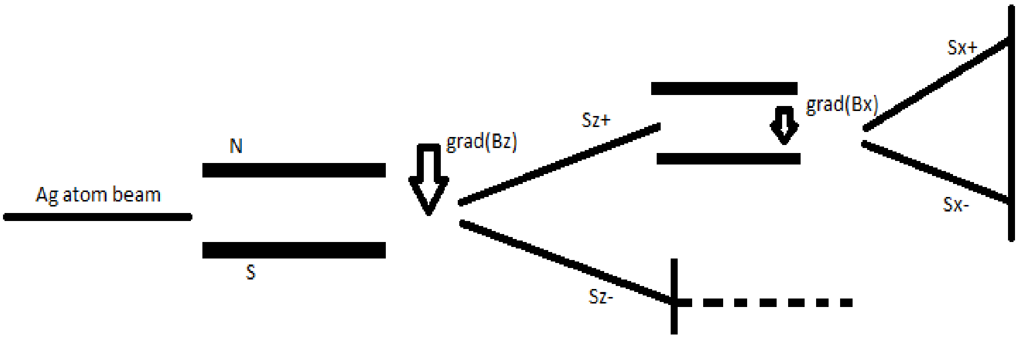

Figure 3: Stern-Gerlach experiment with two magnetic fields different orientation.

Now we use above hypothesis to explain these result. Before come into apparatus, atom is free, its operator Hamilton is:

$$H^1 = \frac{p^2}{2m} \qquad (12)$$

State of Ag atom can be presented as a wave in form:

$$\chi^1 = \sum_{p1,p2,p3}^{p1^2+p2^2+p3^2=const} C_{p1,p2,p3}.\exp[-\frac{i}{\hbar}(p1.x + p2.y + p3.z)] \ (13)$$

When atoms are in apparatus, its operator Hamilton is:

$$H^2 = \frac{p^2}{2m} + \mu z.Bz(z) \quad (14)$$

In which, μz is z-axis magnetic moment spin operator. Magnetics Bz varies on z.

$$\mu z = \mu_B.Sz \ (15)$$

Quantity μ_B is Magneton-Bohr constant. Sz is z-axis spin operator. Operator Sz commutes with Hamilton operator. So, Sz is conservation. From (16) we have state of atoms in apparatus:

$$\chi^2 = \begin{bmatrix}(\uparrow)\\0\end{bmatrix}.\eta_{nz+} \quad (16)$$

Or:

$$\chi^3 = [\begin{matrix}0\\(\downarrow)\end{matrix}].\eta_{nz-} \ (17)$$

Here, η_{nz+} are eigenvectors of operator $H^{z+} = \frac{p^2}{2m} + (Sz+).Bz(z)$. And η_{nz-} are eigenvector of operator $H^{z-} = \frac{p^2}{2m} + (Sz-).Bz(z)$. Constants $Sz+$ and $Sz-$ are two eigenvalues of operator Sz. Every vector χ^4 and χ^5 has corresponding probability to be state of Ag atom. As show above, these probability is constant.

When Ag atoms go out apparatus, it is free. Its operator Hamilton:

$$H^3 = H^1 \quad (18)$$

Eigenvectors χ^6 Hamilton operator of Ag atoms in this domain can be presented as:

$$\chi^6 = \{\begin{bmatrix}(\uparrow)\\0\end{bmatrix}.\sum_{p1,p2,p3}^{p1^2+p2^2+p3^2=const} C_{p1,p2,p3,z+}.\exp\left[-\frac{i}{\hbar}(p1.x + p2.y + p3.z)\right] + [\begin{matrix}0\\(\downarrow)\end{matrix}].\sum_{p1,p2,p3}^{p1^2+p2^2+p3^2=const} C_{p1,p2,p3,z-}.\exp[-\frac{i}{\hbar}(p1.x + p2.y + p3.z)]\}$$

(19)

The continuous condition (11) in this case:

$$\{\sum_n C_{nz+}.\begin{bmatrix}(\uparrow)\\0\end{bmatrix}.\eta_{nz+} + \sum_n C_{nz-}.[\begin{matrix}0\\(\downarrow)\end{matrix}].\eta_{nz-}\}\Big|_{t0} =$$

$$\{\begin{bmatrix}(\uparrow)\\0\end{bmatrix}.\sum_{p1,p2,p3}^{p1^2+p2^2+p3^2=const} C_{p1,p2,p3,z+}.\exp\left[-\frac{i}{\hbar}(p1.x+p2.y+p3.z)\right] +$$

$$[\begin{matrix}0\\(\downarrow)\end{matrix}].\sum_{p1,p2,p3}^{p1^2+p2^2+p3^2=const} C_{p1,p2,p3,z-}.\exp[-\frac{i}{\hbar}(p1.x+p2.y+p3.z)]\}\Big|_{t0} \quad (20)$$

As show above, with initial we can determine C_{z+} và C_{z-}. From them and use (21) we can find $C_{p1,p2,p3,z+}$ và $C_{p1,p2,p3,z-}$. Without any calculation, we see that, there are two large coefficient groups. The first are supported by η_{nz+}. They are fastened by vector $\begin{bmatrix}(\uparrow)\\0\end{bmatrix}$. The second are supported by η_{nz-}. They are fastened by vector $[\begin{matrix}0\\(\downarrow)\end{matrix}]$. So, we have two Ag atom beams on screen go out apparatus.

Now we consider experiment in configuration as figure 2. After the first apparatus, we stop one beam, other beam passes through second apparatus. Here, we have a boundary condition. It extinguishes one coefficient group. Suppose that is second group. So, state of system can be presented as:

$$\chi^{6a} = \begin{bmatrix}(\uparrow)\\0\end{bmatrix}.\sum_{p1,p2,p3}^{p1^2+p2^2+p3^2=const} C_{p1,p2,p3,z+}.\exp\left[-\frac{i}{\hbar}(p1.x+p2.y+p3.z)\right] \quad (21)$$

Hamilton operator of Ag atom in the second apparatus is:

$$H^4 = H^2 \qquad (22)$$

Now, eigenvectors χ^7 and χ^8 of Hamilton operator of Ag atom are similar χ^2 and χ^3. Call t1 is moment that Ag atoms go in the second apparatus. We use again the continuous condition:

$$\chi^{6a}|_{t1} = \{\sum_n C_{nz+}.\begin{bmatrix}(\uparrow)\\0\end{bmatrix}.\eta_{nz+} + \sum_n C_{nz-}.[\begin{matrix}0\\(\downarrow)\end{matrix}].\eta_{nz-}\}\Big|_{t1} \quad (23)$$

Not that state χ^{6a} correspond with eigenvector $\begin{bmatrix}(\uparrow)\\0\end{bmatrix}$ of operator Sz. So (23) gives $C_{nz-} = 0$. So, in the second apparatus, there is only one Ag atom beam.

Go out second apparatus, Ag atom free, its operator Hamilton:

$$H^5 = H^1 \qquad (24)$$

Eigenvectors of Hamilton operator of Ag atom in this domain are χ^9 similar χ^6. Use continuous condition and note that there is only one beam Ag atom in the second apparatus, then there is one Ag atom beam goes out apparatus. Of course, there is only one beam reaches to screen.

If the second magnetics is oriented x-axis, Hamilton operator of Ag atom in this apparatus is:

$$H^6 = \frac{p^2}{2m} + \mu x.Bx(x) \qquad (25)$$

In which, μx is x-axis magnetic moment operator.

$$\mu x = \mu_B.Sx \qquad (26)$$

With Sx is x-axis spin operator. And call $Sx+$ and $Sx-$ are eigenvalue of Sx. And $[\begin{matrix}\leftarrow\\0\end{matrix}]$, $[\begin{matrix}0\\\rightarrow\end{matrix}]$ are eigenvectors correspond eigenvalue $Sx+$ and $Sx-$ of Sx. State of Ag atom in the second apparatus are:

$$\chi^{11} = \begin{bmatrix}\leftarrow\\0\end{bmatrix}.\eta_{nx+} \qquad (27)$$

Or:

$$\chi^{12} = [\begin{matrix}0\\\rightarrow\end{matrix}].\eta_{nx-} \qquad (28)$$

In which, η_{nx+} are eigenvectors of operator $H^{x+} = \frac{p^2}{2m} + (Sx+).Bx(x)$. And η_{nx-} are eigenvectors of operator $H^{x-} = \frac{p^2}{2m} + (Sx-).Bx(x)$. Call t2 is moment that Ag atoms go in the second apparatus. Use continuous condition, we have:

$$\chi^{6a}|_{t2} = \{\textstyle\sum_n C_{nx+}.\begin{bmatrix}\leftarrow\\0\end{bmatrix}.\eta_{nx+} + \sum_n C_{nx-}.[\begin{smallmatrix}0\\ \rightarrow\end{smallmatrix}].\eta_{nx-}\}\Big|_{t2} \quad (29)$$

From (29) we see C_{nx+} and C_{nx-} difference zero. So, in the second apparatus, there are two atom beams. They tend to two sides. So, after this apparatus and on the screen, we have two atom beams.

4. Supposed experiment

This section discuss about a supposed experiment. From this experiment, the difference from above hypothesis and OQM reveals.

We consider Stern-Gerlach experiment again with configuration as figure 3. But Ag atoms are prepared very special. When they pass the first apparatus, Ag's core loses energy. Then Ag's core change into state $[\begin{smallmatrix}0\\ \rightarrow\end{smallmatrix}]$. This makes an inhomogeneous magnetics B_{x2} which it oriented on axis x. Then Ag atoms pass the second apparatus.

After the first apparatus, both OQM and formulation in this paper give a same result. After the first apparatus, there are two Ag atom beams. We stop one beam. And we consider other beam. This beam is acted by magnetics B_{x2}. From two viewpoints, they drive two results.

From viewpoint of OQM, only apparatus makes the wave function collapse, the magnetics B_{x2} doesn't make the collapse. So, the state of Ag beam in magnetics B_{x2} is $\chi^{6b} = c_1.\begin{bmatrix}\leftarrow\\0\end{bmatrix} + c_2.[\begin{smallmatrix}0\\ \rightarrow\end{smallmatrix}]$. They are in only one beam. They are not separate into two beams. This beam passes the second apparatus. This apparatus separates them into two beams.

From viewpoint of formulation in this paper, every interaction changes Hamilton operator so it changes state of system. So, with magnetics B_{x2} Hamilton operator of Ag atom is:

$$H^{3a} = \frac{p^2}{2m} + \mu x.Bx2 \quad (30)$$

And, Ag atoms are separated into two beam. One beam is in state $c_3.\begin{bmatrix}\leftarrow\\0\end{bmatrix}$. Other is in state $c_4.[\begin{smallmatrix}0\\ \rightarrow\end{smallmatrix}]$. These beams pass the second apparatus. This apparatus separates them. The beam $c_3.\begin{bmatrix}\leftarrow\\0\end{bmatrix}$ is separated into two beams. The beam $c_4.[\begin{smallmatrix}0\\ \rightarrow\end{smallmatrix}]$ is too. So, after the second apparatus, there are 4 beams. Two beams are in state $c_5.\begin{bmatrix}(\uparrow)\\0\end{bmatrix}$. And two beams are in state $c_6.[\begin{smallmatrix}0\\ (\downarrow)\end{smallmatrix}]$. This result is not the same with the first viewpoint.

IV. Conclusion

Paper give a hypothesis about state of system. Then it built a mathematical formulation. It consists with OQM. But there is no inside conflict in this formulation. It can connect state of system with a reality. It can reject abstract boundary of OQM. Apply this formulation, we can explain double slits experiment, Stern-Gerlach experiment. Paper also give a supposed experiment that there is difference between OQM with this formulation.

Acknowledgements

I dedicate respectfully this work to my parents, my wife and my unborn daughter.

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