LOGICAL TREATMENT FOR THE OSCILLATORY SEQUENCE
1, 2, 3, 4, 3, 2, 1, 2,...
TO FIND ANY TERM AND A COMPUTER PROGRAM TO ASSIST THE OPERATION

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Abstract. This paper’s aim is to tackle any oscillatory sequence in a new way. I introduced a new method to find any term of the specific sequence given on a title. This method not only focus for that problem, but also can apply in such a sequence whose terms are repeated after some period.

1. Introduction

We have a sequence as 1, 2, 3, 4, 3, 2, 1,... and we are trying to find any term of that sequence.
First of all, we plot the graph of the above sequence as;
Along X-axis = \(N\) = Domain of the sequence
Along Y-axis = \(t_N\) = Range of the sequence

![Figure 1. scatter diagram](image)

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1.1. Steps.

- Draw four lines of equation $y = 1$, $y = 2$, $y = 3$ and $y = 4$.
- Then, write all the points lying on $y = 1$, $y = 2$, $y = 3$ and $y = 4$ separately.
- It can be tabulated as,

<table>
<thead>
<tr>
<th>$S_{tN}$</th>
<th>Lying points</th>
<th>Naming</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1, 7, 13, 19, .. to $i$ terms</td>
<td>$t_i$</td>
</tr>
<tr>
<td>2</td>
<td>2, 6, 8, 12, 14, 18, .. to $j$ terms</td>
<td>$t_j$</td>
</tr>
<tr>
<td>3</td>
<td>3, 5, 9, 11, 15, 17, .. to $k$ terms</td>
<td>$t_k$</td>
</tr>
<tr>
<td>4</td>
<td>4, 10, 16, .. to $l$ terms</td>
<td>$t_l$</td>
</tr>
</tbody>
</table>

where, $S_{tN} = $ Newly formed sequence for the range of the original sequence

**Note:** From the above table, we have seen that they are in new sequences.

2. Calculations

We are going to find the general terms for the above newly formed sequences.

- The general term for the sequence $S_1 = 1, 7, 13, 19, ...$ can be found by,
  \[ t_i = 1 + (i - 1)6 \]
  \[ = 6i - 5 \quad |i = 1, 2, 3, ... \]
  Since, the $S_1$ is in arithmetic progression.
- The general term for the sequence $S_4 = 4, 10, 16, ...$ can be found by,
  \[ t_l = 4 + (l - 1)6 \]
  \[ = 6l - 2 \quad |l = 1, 2, 3, ... \]
  Since, the $S_4$ is in arithmetic progression.

From the **Appendix**:

- The general term for the sequence $S_2 = 2, 6, 8, 12, 14, 18, ...$ is,
  \[ t_j = 3j \quad when \ j \ is \ even. \]
  \[ = 3j - 1 \quad when \ j \ is \ odd. \ |j = 1, 2, 3, ... \]
- The general term for the sequence $S_3 = 3, 5, 9, 11, 15, 17, ...$ is,
  \[ t_k = 3k \quad when \ k \ is \ odd. \]
  \[ = 3k - 1 \quad when \ k \ is \ even. \ |k = 1, 2, 3, ... \]

**Note:** Never confuse with the values of $i, j, k, l$ with the values of domain of the sequence. Because, they all are different.

3. Step Continue

Since, $t_i$, $t_j$, $t_k$ and $t_l$ are the no. of the terms ($N$) of the original sequence.

where values of $N$ can be 1, 2, 3, 4, 5, ...

After, we test the value of $N$ with a general formula of newly formed sequences.

Then, we can say which value of N is the last term.

- In the sequence $S_1$:
  \[ t_i = 6i - 5 = N \]
  \[ \therefore i = \frac{N + 5}{6} \]
**Condition:** If the value of $i$ is found to be a natural number (there is no remainder) then, the *last term should be 1.*
If not then, test the next operation.

- In the sequence $S_2$:

  $$t_j = 3j = N$$

  $$\therefore j = \frac{N}{3} \text{ where, } j \text{ must be even}$$

**Condition:** If the value of $j$ is found to be a natural number (there is no remainder) and $j$ is even then, the *last term should be 2.*
If not then, test the next operation.

  $$t_j = 3j - 1 = N$$

  $$\therefore j = \frac{N + 1}{3} \text{ where, } j \text{ must be odd}$$

**Condition:** If the value of $j$ is found to be a natural number (there is no remainder) and $j$ is odd then, the *last term should be 2.*
If not then, test the next operation.

- In the sequence $S_3$:

  $$t_k = 3k = N$$

  $$\therefore k = \frac{N}{3} \text{ where, } k \text{ must be odd}$$

**Condition:** If the value of $k$ is found to be a natural number (there is no remainder) and $k$ is odd then, the *last term should be 3.*
If not then, test the next operation.

  $$t_k = 3k - 1 = N$$

  $$\therefore k = \frac{N + 1}{3} \text{ where, } k \text{ must be even}$$

**Condition:** If the value of $k$ is found to be a natural number (there is no remainder) and $k$ is even then, the *last term should be 3.*
If not then, test the next operation.

- In the sequence $S_4$:

  $$t_l = 6l - 2 = N$$

  $$\therefore l = \frac{N + 2}{6}$$

**Condition:** If the value of $l$ is found to be a natural number (there is no remainder) then, the *last term should be 4.*

4. **Axioms**

- If all $(n - 1)$ tests are fail then, the last test $n$ must be *true.*
  where, $n = \text{total number of test}.$
- If one test is passed then, all others remaining tests should be (or, must be) fail.
• The points lying on the line of the upper and lower bound of the oscillatory sequence are in arithmetic progression.

5. Program

With the help of above axioms, here is the program that can run the test and the program is written in Python 2.x.

```python
N = input("Enter the value of N as you like!: ")
if (N+5)%6==0 :
    print ("The last term is 1")
elif N%3==0 :
    if (N/3)%2==0:
        print("The last term is 2")
    else:
        print("the last term is 3")
elif (N+1)%3==0 :
    if (((N+1)/3)%2==0 :
        print("The last term is 3")
    else:
        print("The last term is 2")
else:
    print("The last term is 4")
```

6. Appendix

• For the sequence, $S_2 = 2, 6, 8, 12, 14, 18, \ldots$

Here,

\[
\begin{align*}
\text{Series of } S_2 & = 2 + 6 + 8 + 12 + 14 + 18 + \ldots + t_{j-1} + t_j \\
\text{Series of } S_2 & = \downarrow + 2 + 6 + 8 + 12 + 14 + 18 + \ldots + t_{j-1} + t_j \\
\hline
0 & = 2 + 4 + 2 + 4 + 2 + 4 + \ldots + (t_j - t_{j-1}) - t_j \\
\text{or, } t_j & = 2 + 4 + 2 + 4 + 2 + 4 + \ldots \text{to } j \text{ terms}
\end{align*}
\]

When $j$ is even,

\[
t_j = (2 + 2 + 2 + \ldots \text{to } (j/2) \text{ terms}) + (4 + 4 + 4 + \ldots \text{to } (j/2) \text{ terms})
\]

\[
= 2 * (j/2) + 4 * (j/2)
\]

∴ $t_j = 3j$

When $j$ is odd,

\[
t_j = 2 * ((j + 1)/2) + 4 * ((j - 1)/2)
\]

\[
= (j + 1) + 2 * (j - 1)
\]

∴ $t_j = 3j - 1$

• For the sequence, $S_3 = 3, 5, 9, 11, 15, 17, \ldots$

Here,
LOGICAL TREATMENT FOR THE OSCILLATORY SEQUENCE 1, 2, 3, 4, 3, 2, 1, 2, . . .

Series of $S_3 = 3 + 5 + 9 + 11 + 15 + \ldots + t_{k-1} + t_k$

Series of $S_3 = 3 + 5 + 9 + 11 + 15 + \ldots + t_{k-1} + t_k$

\[ 0 = 3 + 2 + 4 + 2 + 4 + 2 + \ldots + (t_k - t_{k-1}) - t_k \]

or, $t_k = 3 + (2 + 4 + 2 + 4 + 2 + \ldots \text{to } k \text{ terms})$

When $k$ is odd,

\[
t_k = 3 + (2 + 2 + 2 + \ldots \text{to } (k - 1)/2 \text{ terms}) + (4 + 4 + 4 + \ldots \text{to } (k - 1)/2 \text{ terms})
\]

\[
= 3 + 2 \times (\frac{(k - 1)}{2}) + 4 \times (\frac{(k - 1)}{2})
\]

\[
= 3 + 3k - 3
\]

\[\therefore t_k = 3k\]

When $k$ is even,

\[
t_k = 3 + 2 \times (\frac{(k - 1)}{2}) + 4 \times (\frac{(k - 1)}{2})
\]

\[
= 3 + k + 2 \times (k - 2)
\]

\[\therefore t_k = 3k - 1\]

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