THE METHOD OF CALCULATING THE VELOCITY FIELD AND SHEAR STRESSES IN INCOMRESSIBLE FLUID

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The velocity field culculation method is based on the use of two special cases of the Newtonian fluid motion equations, not including the Navier-Stokes equation. Two shear stress calculation ways are considered. The first way is the differentiation on the velocity field equation, and the second one requires the solution of the first-order differential equation. The second way provides the distribution of shear stresses for any continuous medium, including Newtonian fluid.
Culculation equations for a laminar flow in a round pipe are found. It is shown that parabolic velocity distribution along the radius is a special case of more general equation.
The factors affecting the shear stresses for the three flow models are found. Stresses are determined by the linear velocity gradients in the laminar flow. In the 3D vortex, they can be found by various equations, which include vorticity. Total stresses for the averaged turbulent flow are culculated by summing the previously found stresses.
The equations of the method are incomplete and may be used for the accurate solution of simple problems.
Keywods: incompressible fluid, Poiseuille, flow regimes, velocity gradient, vorticity.

1. Introduction

Among a large number of fluids medium, a Newtonian fluid in which shearing stresses are proportional to the velocity gradient $\tau = \mu \cdot grad\ u$ has the most value for practice [1-4]. Water, air and other ideal gases refer to such fluids. Ability to calculate shearing stresses and velocity fields for the different modes of flow including their geometry allows extending the area of the use of theoretical methods of calculation. Implementing this approach is presently made difficult in connection with absence of the closed system of equations for any flow mode. At the same time, presence of already known equations of motion allows to move up on the way of theoretical description of connections between gravities, pressure, inertia and friction. Shearing stresses that specify forces of viscid friction occupy an important place in the results of calculation of any mechanic process characterizing the power expended [1, 2, 4]. Their correct calculation allows in theory to expect the effect of dissipation of mechanical energy and implies a transition from the calculation of the idealized flow (without regard for influence of viscid friction) to the calculation of motion of the real flow.

The problem of calculation of the velocity field is related to the computation of shearing stresses and executed in theory only for some exactly problems solved. Absence of general methods of calculation resulted in the need to use experimental methods and also numerical simulation. The numerous computer programs (Flow vision, Phoenix and others) that give good results in the tested range of influencing factors are used for this purpose. The drawback of this method of calculation is instability of solutions especially for new tasks. It is believed that reason for problems are calculation equations used as a basis of numerical simulation (Navier - Stokes and Reynolds) [1, 2].

Therefore, the negligible quantity of exact solutions for one flow mode only and the issues of the use of numerical methods require development of new calculation methods.

2. Analysis of literary data and formulation of the problem

During the research, basic attention is paid to the determination of the velocity fields that characterize not only the fluid flow but also flow of great number of concomitant processes. For example, processes of heat exchange during air motion in an atmosphere that have impact on a local weather [5]; in the elements of technical constructions using a fluid medium as a working body [6]; processes in a magnetic hydrodynamic oscillator accompanied by influence of the thermal and magnetic field [7] and so on.

Some works are dedicated to the theoretical and numerical simulation of flow of the rotating streams existing in an unlimited medium (for example, natural vortices, interaction of curl source with a plane) [8]. For technical applications, research of flows in cyclones is crucial. Velocity field in such devices is determined by efficiency of falling of particulate matters at the flow friction against the wall. The work analysis of cyclones is performed by numerical and experimental methods [9, 10].

Other area of the use of the velocity fields are different devices available in environment. They include aerial vehicles with a special geometrical shape that provides the required velocity field and field of shearing stresses and also different technical constructions that can not have a good extent of streamlining. For example, they include high-rise buildings, radio engineering antenna and sunny collectors [11]. Researches alone this line are conducted in medicine aimed at determining the influence of speed profile upon the throughput capacity of blood vessels during the laminar flow mode [12].

A key factor that influences the flow of liquid is viscidity that determines not only the velocity field, but also field of tangential stress.

The calculation of tangential stress is carried out in theory only for a few exactly formulated problems during the laminar flow mode or numerically [1, 2, 4, 13].

In practice, no tangential stresses are estimated in corresponding planes, and their total effect is the hydraulic resistance that specifies the loss of mechanical energy of flow and does not take into account local effects.

This type of calculations is made by means of reference books on hydraulic resistances that contain the equations and charts obtained as a result of experiments pursued [14]. One of examples of such approach is the use of Bernoulli equation to which a seventh sumand is added with regard for influence of viscidity [2, 4]. This summand substantially changes physical and geometrical interpretation of problem definition and is fully confirmed by practice for any flow mode.

Method modification is used in some computer programs where the computing of construction of flow part and its hydraulic resistance are superposed. Such approach increases velocity and quality of computation allowing quickly to compare the different variants of construction according to the size of resistance; however, principle of calculation of mechanical energy dissipation remains unchanged.

The use of entropy concept is another design way of dissipation that is widely spread in technical and statistical thermodynamics for the calculation of the real processes in heat-engines, heat exchange processes during the phase transitions and so on. However, such way of dissipation estimate (viscid friction) did not find application at the flow of incompressible fluids [15].

For the turbulent flow mode, the design method based on motion equation of an average turbulent flow (Reynolds equations) and different semi empiric theories of turbulence is used [16, 17]. This approach will be achieved by means of the computer programs, it will give acceptable exactness and the use of visual aids in the fields of stable solutions. Most practically important applications refer exactly to this flow mode, however, at present, there are no exactly found solutions. One of reasons for unsatisfactory situation is that there are no summands available in calculation equations that take into account the rotation of liquid particles [1, 2].

The marked problems resulted in the appearance of other design methods that are not associated with theoretical or numerical solutions. The use of similarity concept of the physical phenomena and its application to the solution of hydrodynamics and heat exchange can be referred to the number of the methods like these [1, 3, 18]. This method is based on three theorems of similarity and does not use the physical laws of maintenance in an explicit form. Criteria similarities are calculation instrument that is widely used in the solutions of convective heat exchange for the account of combined influence of the fields of velocity flow and thermal flux. Such approach is justified if the definition of local parameters of a process is not required and the calculation error is assumed about 20%.

In order to simplify calculations, numerous empiric equations that are obtained as a result of experimental data processing and have a narrow application domain are used. In these equations, the theory of dimension requirements is not met and they can not qualify for universal status [18].

Therefore, the known examples of exact solutions for the laminar mode show direct connection between tangential stresses level and the losses of mechanical energy of flow, however, there are no general calculation equations suitable for this purpose and other modes of flow [1, 3].

In order to formulate the marked problems, a method that allows carrying out the calculation of three fields - linear and angular speeds - is needed, and also it includes the field of tangential stresses. In order to perform this method, two general systems of equations and also their special cases are used in this paper. Equation of motion in stresses (Navier) is the first system that describes the field of tangential stresses for any continuous medium. The second equation is a special case of Navier equations re-arranged as it applies to a Newtonian fluid and contains the linear and angular velocity of movement of particles [19]. Therefore, this method differs from the known methods in other calculation equations and does not use Navier-Stokes equation.

Classic Navier equation can be re-arranged to select pressure in a separate summand. Then it will assume the following form [1, 4]:

$$
\begin{aligned}
&X-\frac{1}{\rho}\frac{\partial p_x}{\partial x}+\frac{1}{\rho}\left(\frac{\partial \tau_{yx}}{\partial y}+\frac{\partial \tau_{zx}}{\partial z}\right)=\frac{du_x}{dt}\\
&Y-\frac{1}{\rho}\frac{\partial p_y}{\partial y}+\frac{1}{\rho}\left(\frac{\partial \tau_{xy}}{\partial x}+\frac{\partial \tau_{zy}}{\partial z}\right)=\frac{du_y}{dt},\\
&Z-\frac{1}{\rho}\frac{\partial p_z}{\partial z}+\frac{1}{\rho}\left(\frac{\partial \tau_{xz}}{\partial x}+\frac{\partial \tau_{yz}}{\partial y}+\right)=\frac{du_z}{dt},
\end{aligned}
\qquad (1)
$$

where normal tension and pressure are bound by correlations: $p_{xx}=-p_x$, $p_{yy}=-p_y$, $p_{zz}=-p_z$ [1, 2, 4].

Conversion of this equation is performed by the selection of rotors functions and gradients of linear velocities specifying rotatory and forward motion of particles. As a result, the system of equations is in the form of [19]:

$$
\begin{aligned}
&X-\frac{1}{\rho}\frac{\partial p_x}{\partial x}+\nu\left[\frac{\partial}{\partial y}\left(\left(\mathrm{rot}\,u\right)_z+2\frac{\partial u_x}{\partial y}\right)+\frac{\partial}{\partial z}\left(\left(\mathrm{rot}\,u\right)_y+2\frac{\partial u_z}{\partial x}\right)\right]=\frac{du_x}{dt},\\
&Y-\frac{1}{\rho}\frac{\partial p_y}{\partial y}+\nu\left[\frac{\partial}{\partial x}\left(\left(\mathrm{rot}\,u\right)_z+2\frac{\partial u_x}{\partial y}\right)+\frac{\partial}{\partial z}\left(\left(\mathrm{rot}\,u\right)_x+2\frac{\partial u_y}{\partial z}\right)\right]=\frac{du_y}{dt},\\
&Z-\frac{1}{\rho}\frac{\partial p_z}{\partial z}+\nu\left[\frac{\partial}{\partial x}\left(\left(\mathrm{rot}\,u\right)_y+2\frac{\partial u_z}{\partial x}\right)+\frac{\partial}{\partial y}\left(\left(\mathrm{rot}\,u\right)_x+2\frac{\partial u_y}{\partial z}\right)\right]=\frac{du_z}{dt},
\end{aligned}
\qquad (2)
$$

Where *u* is time averaged flow velocity, X, Y, Z is specific mass force.

Summands in square brackets (2) specify forces of viscid friction during joint influence of two types of motion: rotations of fluid /function $(rot\ u)_i$/ and its forward flow. The system (2) meets the turbulent flow definition if we consider flow velocity as time average. By elimination from (2) summands with regard for the rotation of particles, it is possible to obtain the equation for a laminar flow, and by elimination of forward velocity, it is possible to obtain equation for a clean rotation (3d vortex).

Basic assumptions for the system (2) are mass constancy of continuous medium and justice of the Newton's law for a viscid friction.

In the current paper, a method that is based on the idea of two systems of equations : equation of motion in tensions(1) and its special case for a Newtonian liquid (2) obtained without the use of additional assumptions is being examined.

3. Aim and research tasks

The research is designed to receive theoretical equations for the calculation of tangential stress and velocity field in the different modes with regard for different character of flow (without the rotation of particles, rotation of particles without forward velocity only and difficult flow consisting of rotatory and forward flow).

To achieve this goal, it is necessary to do the following tasks:

– to describe the calculation method of tangential stress and velocity fields of incompressible liquid on the basis of equations analysis (1) and (2);

–to compare the calculation equations obtained by means of this method with known variable for the special case of flow in a round pipe;

–to execute the analysis of factors influencing the velocity field during other flow modes.

4. Description of calculation method and its use for an individual task

Velocity field is described by derivatives from the velocity on coordinates and local time derivative. To find an equation of the velocity field, it is necessary to solve the equation of the second order simplifying it in accordance with an individual task.

Let us consider using this method by the example of the set flow in a horizontal round pipe.

In accordance with determination of laminar flow, there is no fine-scale rotation of particles of liquid and the function of rotor of speed must be equal to the zero. Then from (2) we will obtain:

$$
\begin{aligned}
&X - \frac{1}{\rho}\frac{\partial p_x}{\partial x} + 2\nu\left(\frac{\partial^2 u_x}{\partial y^2} + \frac{\partial^2 u_z}{\partial x \partial z}\right) = \frac{du_x}{dt}, \\
&Y - \frac{1}{\rho}\frac{\partial p_y}{\partial y} + 2\nu\left(\frac{\partial^2 u_y}{\partial z^2} + \frac{\partial^2 u_x}{\partial x \partial y}\right) = \frac{du_y}{dt}, \\
&Z - \frac{1}{\rho}\frac{\partial p_z}{\partial z} + 2\nu\left(\frac{\partial^2 u_z}{\partial x^2} + \frac{\partial^2 u_y}{\partial y \partial z}\right) = \frac{du_z}{dt}.
\end{aligned}
\tag{3}
$$

From the last equation of the system (3), written in cylindrical coordinates (*r*, *z*), it is possible to get a correlation by the gradient of pressure along an axis *z* and distribution of speed along a radius for a horizontal pipe in a form of $\frac{\partial p_z}{\partial z} = 2\mu \frac{\partial^2 u_z}{\partial r^2}$. After double integration, we will obtain:

$$u_z(r) = \frac{1}{4\mu}\frac{\partial p_z}{\partial z}\cdot r^2 + c_1 r + c_2 \cdot \tag{4}$$

Use of standard boundary conditions /for $r = 0$, $\frac{\partial u_z}{\partial r} = 0$, and at $r = r_0, u_z = 0$/ results in $c_1 = 0$ and equation(4) acquires a form of $u_z(r) = \frac{1}{4\mu}\frac{\partial p_z}{\partial z}\cdot\left(r^2{}_0 - r^2\right)$, that has been obtained before and driven to [1, 4]. The difference of an equation(4) from the known one consists in a presence of an additional constant that specifies more difficult distribution speeds along the radius of pipe. In general case (or in other boundary conditions) a constant c_1 can differ from a zero, and equation of velocity diagram will differ from an ideal parabola. It is possible to suppose that influence of this constant must show up nearby a wall, specifying a flow in a viscid sublayer [1, 2].

It is necessary to mark that standard boundary conditions are not fully correct as they suppose that the parabolic law of velocity distribution are expanding to all radius of pipe. Experimental researches of flow nearby a wall show that on small distances the law of velocity change changes and work for linear [2, 4].

Defining tangential stresses is possible by means of the Newton's law for a viscid friction. Using equation (4), we will obtain:

$$\tau = \mu \frac{du_z(r)}{dr} = \partial p_z \big/ \partial z \cdot \frac{r}{2} + c' \cdot \tag{5}$$

Equation (5) within a constant coincides with the known solution obtained as a result of application of equation of equilibrium to the individual task [1, 4].

The calculation of tangent tensions can be also made by means of the system of equations of motion in tensions. As it follows from (1), tangential stresses are under a sign a derivative and in order to solve an individual task, it is necessary to distinguish and solve differential equation and also to formulate and apply boundary conditions. We will take advantage of cylindrical coordinates (*r, θ, z)* in which the system *(1) for* a coordinate *is in* the form of [1]:

$$Z - \frac{1}{\rho}\frac{\partial p_z}{\partial z} + \frac{1}{\rho}\left(\frac{\partial \tau_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \tau_{\theta r}}{\partial \theta} + \frac{\tau_{zr}}{r}\right) = \frac{du_z}{dt} \cdot \tag{6}$$

We will maximally simplify this equation supposing that mass forces are unavailable, a flow is set and axisymmetrical.

Then we will obtain:

$$\frac{\partial \tau_{zr}}{\partial r} + \frac{\tau_{zr}}{r} = \frac{\partial p_z}{\partial z} \cdot \tag{7}$$

Because at the permanent diameter of pipe $\frac{\partial p}{\partial z} = const$, the solution of equation (7) is in the form of :

$$\tau_{zr} = \frac{c_1}{r} + \frac{dp/dz \cdot r}{2}. \tag{8}$$

Then at $c_1 = 0$ we will obtain end-point $\tau_{zr} = dp/dz \cdot \frac{r}{2}$ that coincides with the known formula.

A general integral (8) shows that the examined method of calculation brings to more general result as compared to the known solution for Poiseuille flow [2, 4]. A difference is in an availability of an additional constant that cannot be equal to the zero in general case. Reason for difference is caused by more general character of the system (1) that refers to any continuous medium rather than just to the Newtonian liquid. Physical sense of the first summand in equation (8) is not clear and requires the conduct of additional research.

Community of equations of example can be extended to receive other solutions. For this purpose, it is necessary to decrease the number of limitations and take into account the influence of inclination of pipe (variable Z), acceleration of stream (variable du_z/dt) , etc.

Let us consider the example of the use of method for finding distribution of velocity in a sloping pipe. In this case, gravity has influence on a flow that can be estimated by means of the system (3) in(r, z) coordinates. The special case of equation of motion of signs is in the form of:

$$\frac{\partial^2 u_z}{\partial r^2} = \frac{1}{2\mu}\left(\frac{\partial p_z}{\partial z} - \rho Z\right)$$

After a double integration, we will obtain:

$$u_z(r) = \frac{1}{4\mu}\left(\frac{\partial p_z}{\partial z} - \rho Z\right) r^2 + c_1 r + c_2. \tag{9}$$

The equation (9) shows that distribution of velocity along a radius changes due to the change of parabola coefficient, and size of constant of r^2 depends on direction of vectors of velocity and gravity.

We will carry out the analysis of factors influencing on the velocity field and tangential stresses. Then system (2) with forward speed unavailable will be in the form:

$$\begin{gathered}
X - \frac{1}{\rho}\frac{\partial p_x}{\partial x} + 2\nu\left[\frac{\partial \omega_z}{\partial y} + \frac{\partial \omega_y}{\partial z}\right] = \frac{du_x}{dt}, \\
Y - \frac{1}{\rho}\frac{\partial p_y}{\partial y} + 2\nu\left[\frac{\partial \omega_z}{\partial x} + \frac{\partial \omega_x}{\partial z}\right] = \frac{du_y}{dt}, \\
Z - \frac{1}{\rho}\frac{\partial p_z}{\partial z} + 2\nu\left[\frac{\partial \omega_y}{\partial x} + \frac{\partial \omega_x}{\partial y}\right] = \frac{du_z}{dt}.
\end{gathered} \tag{10}$$

Expressions in square brackets are tangential stresses from a rotatory flow along a corresponding coordinate and can be re-arranged to select function of rotor of ω. For a coordinate x conversion of this summand into equation(10) looks like :

$$\left[\frac{\partial\omega_z}{\partial y}+\frac{\partial\omega_y}{\partial z}\right]-\frac{\partial\omega_y}{\partial z}+\frac{\partial\omega_y}{\partial z}=\left[(rot\ \omega)_x+2\frac{\partial\omega_y}{\partial z}\right].$$

As a result of (10) it will be in the form of:

$$\begin{aligned}
&X-\frac{1}{\rho}\frac{\partial p_x}{\partial x}+2\nu\left[(rot\ \omega)_x+2\frac{\partial\omega_y}{\partial z}\right]=\frac{du_x}{dt},\\
&Y-\frac{1}{\rho}\frac{\partial p_y}{\partial y}+2\nu\left[(rot\ \omega)_y+2\frac{\partial\omega_z}{\partial x}\right]=\frac{du_y}{dt},\\
&Z-\frac{1}{\rho}\frac{\partial p_z}{\partial z}+2\nu\left[(rot\ \omega)_z+2\frac{\partial\omega_x}{\partial y}\right]=\frac{du_z}{dt}.
\end{aligned}\tag{11}$$

The system(11) shows that viscid friction from rotatory motion is characterized by the sum of two summands first of which is the function of double rotor, and second – is the function of single rotor from corresponding linear velocity of u_i.

Let us add up corresponding summands in square brackets from equations (3) and (11). Then we will obtain:

$$\begin{aligned}
&X-\frac{1}{\rho}\frac{\partial p_x}{\partial x}+2\nu\left[\frac{\partial^2u_x}{\partial y^2}+\frac{\partial^2u_z}{\partial x\partial z}+(rot\ \omega)_x+2\frac{\partial\omega_y}{\partial z}\right]=\frac{du_x}{dt},\\
&Y-\frac{1}{\rho}\frac{\partial p_y}{\partial y}+2\nu\left[\frac{\partial^2u_y}{\partial z^2}+\frac{\partial^2u_x}{\partial x\partial y}+(rot\ \omega)_y+2\frac{\partial\omega_z}{\partial x}\right]=\frac{du_y}{dt},\\
&Z-\frac{1}{\rho}\frac{\partial p_z}{\partial z}+2\nu\left[\frac{\partial^2u_z}{\partial x^2}+\frac{\partial^2u_y}{\partial y\partial z}+(rot\ \omega)_z+2\frac{\partial\omega_x}{\partial y}\right]=\frac{du_z}{dt}.
\end{aligned}\tag{12}$$

The systems (2) and (12) are equivalent but have a different form of record, while the first two summands in brackets specify a viscid friction at a forward (laminar) flow and second two summands specify a viscid friction at rotatory motion of particles. All four summands in brackets specify a viscid friction at an averaged turbulent flow. It allows to use the different forms of equation of motion depending on the known basic data.

5. Short analysis of equations for the calculation of the velocity field and tangential (shearing) stresses

The method of calculation of the velocity field is based on the use of the special case of equation of motion in tensions for a Newtonian liquid (2), in which two types of summands are distinguished. One of the summands takes into account an influence of forward flow, and the second includes rotation of particles of liquid. Such approach allows dividing a difficult (turbulent) flow into two more simple flows [19].

Equations of the velocity field can be obtained by means of simplification (2) in accordance with an individual task. The example of such simplification showed for the Poiseuille flow that equation of the velocity field was more difficult as compared to the known one. Coincidence of two equations found in number of different ways can be obtained for some boundary conditions only.

A method allowed taking into account the influence of additional factor that is unavailable in the known solutions [1, 2]. This factor is a projection of gravity that results in an increase or reduction of constant of parabola depending on the angle of pipe slope in relation to horizon.

Determination of tangential stresses can be conducted by two ways. The first way is based on the Newton's law for a viscid friction and is taken to differentiation of equation of the velocity field.

The second way is based on the use of equation of motion in tensions. The analysis of the system (1) shows that at any mode of flow, the law of pair of tangential stresses apply, that is $\tau_{xy} = \tau_{yx}$, $\tau_{zy} = \tau_{yz}$, $\tau_{xz} = \tau_{zx}$ [1, 2]. Therefore, identical tangential stresses are included in (1) twice and are under a sign of a derivative that results in the necessity of equating and solving differential equations to find solution to them.

The use of the second way for the known individual task (Poiseuille flow) allowed formulating a differential equation that has an exact solution. Comparison of equations for the tangential stresses obtained by the first and second way shows the availability of some differences whose reasons consists in more general character of the system (1) [1-3].

From (3) it follows that the velocity field in the set laminar stream is characterized by the two types of individual variables of the second order. In a stream with a rotatory flow only, characteristic functions change and the velocity rate depends on a single rotor and gradient of angular speed of particles (or double and single rotor from linear speed). The velocity field of averaged turbulent flow is considered as a sum of previous summands. This method allows writing down the different forms of design equations, for example (2) and (12).

In equations for 3d vortex (10), and also for a turbulent flow, there is the function of velocity rotor available whose influence on tangential stresses is not clear enough and requires an additional analysis.

The approach examined in this work allows improving understanding of complex correlations between the different types of motions in a Newtonian liquid; However, the use of this method for any problem is complicated due to the absence of complete amount of the equations required.

6. Conclusions

1. The computing method of the velocity field is based on the use of the special cases of the equations of motion for a Newtonian liquid obtained by means of classic equation of motion of continuous medium in tensions (Navier). It shows that the computing of tangential stresses can be performed by two ways: by means of Newtonian equation for a viscid friction and by means of general equation of motion of continuous medium in tensions. Implementation of the first way reduces to differentiation of equation of the velocity field. To implement the second way, the classic method of solution of unknown quantity is used. For this purpose, it is required to distinguish corresponding summands from the general system of equations to equate and solve differential equation, and also to impose boundary conditions.

2. A method is used to solve equations of the velocity field and tangential stresses for the known task of laminar flow computing in a round pipe. Comparison of the solved equations with unknown has been made. It shows that the method brings to more general equations for both fields. Differences consist in presence of additional constants that arise up at integration of equations. It indicates that the method can be used for the account of new influences.

3. The analysis of equations has been performed to simplify the use of method as it applies to an averaged turbulent stream examined as a sum of two more simple flows: laminar and 3d vortex. Each of these flows has the regularities of change for the velocity field and tangential stresses. In a laminar flow, the velocity field is characterized by flexons from linear speeds of flow. The velocity field in a 3d vortex is

defined by gradients from an angular velocity (ω) or double rotor from linear speed (u). Tangential stresses in a turbulent stream are estimated as a sum of tensions for the previous two flows.

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