Failure mode and effects analysis based on D numbers and TOPSIS

Tian Bian[a], Haoyang Zheng[a], Likang Yin[a,b], Yong Deng[a,c,d,*], Sankaran Mahadevan[d]

[a] *School of Computer and Information Science, Southwest University, Chongqing 400715, China*
[b] *School of HanHong, Southwest University, Chongqing 400715,China*
[c] *Institute of Integrated Automation, School of Electronic and Information Engineering, Xi'an Jiaotong University, Xian, Shaanxi, 710049, China*
[d] *School of Engineering, Vanderbilt University, Nashville, TN, 37212, USA*

Abstract

Failure mode and effects analysis (FMEA) is a widely used technique for assessing the risk of potential failure modes in designs, products, process, system or services. One of the main problems of FMEA is to deal with a variety of assessments given by FMEA team members and sequence the failure modes according to the degree of risk factors. The traditional FMEA using risk priority number (RPN) which is the product of occurrence (O), severity (S) and detection (D) of a failure to determine the risk priority ranking order of failure modes. However, it will become impractical when multiple experts give different risk assessments to one failure mode, which may be imprecise or incomplete or the weights of risk fac-

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[*] Corresponding author Yong Deng: School of Computer and Information Science, Southwest University, Chongqing 400715, China. Tel. : +86 23 68254555; fax: +86 23 68254555.

Email address: `ydeng@swu.edu.cn, ydeng@xjtu.edu.cn, prof.deng@hotmail.com` (Yong Deng)

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tors is inconsistent. In this paper, a new risk priority model based on D numbers, and technique for order of preference by similarity to ideal solution (TOPSIS) is proposed to evaluate the risk in FMEA. In the proposed model, the assessments given by FMEA team members are represented by D numbers, a method can effectively handle uncertain information. TOPSIS method, a novel multi-criteria decision making (MCDM) method is presented to rank the preference of failure modes respect to risk factors. Finally, an application of the failure modes of rotor blades of an aircraft turbine is provided to illustrate the efficiency of the proposed method.

Keywords: Failure modes and effects analysis, risk priority numbers, Dempster-Shafer evidence theory, D numbers, MCDM, TOPSIS, rotor blades

1. Introduction

Failure mode and effects analysis (FMEA) is widely used to define, identify and eliminate known or potential failures, errors and so on from the system, design and process to prevent the unexpected failure happen[1, 2, 3, 4]. It can not only examine all possible ways in which a system failure can occur, potential effects of failures on system and seriousness of these effects, but also provide information for helping designers identify the critical potential failure modes and adjust the existing programs to decrease the probability of failure rates and avoid hazardous accidents which may enhance reliability and safety of a product or a system[5, 6, 7, 8, 9]. In the 1960s FMEA was first developed as a formal design methodology by the aerospace industry[10]. Due to its reliability, safety and simplicity, FMEA plays an important role in the design of industrial products such as structures operating in power, aeronautics and astronautics [11, 12, 13, 14, 15] and the solution of various reliability problems in many industries such as aerospace, nuclear, chemical and manufacturing[16, 17, 18, 19, 20].

The traditional risk assessment of FMEA is the risk priority number (RPN) which involves the failure occupancy (*O*), detection(*D*), and severity (*S*) to identify the risk degree of failure modes for a product or a system. Alougth it has been widely applied, FMEA still exits some important shortcomings and limitations when evaluated by RPN. For example, the weights of *O*, *S* and *D* haven't been considered and some assessment information provided by FMEA team members may be uncertain. To improve the traditional FMEA, many other risk assessment methods based on multi-criteria decision making (MCDM) methods[21, 22] have been proposed, such as analytic hierarchy process (AHP)[23], technique for ordering preference by similarity to ideal solution (TOPSIS)[24, 25, 26], decision making trial and evaluation laboratory (DEMATEL)[27, 28] and so on. Furthermore, to deal with the imprecise assessment information for risk factors, D-S evidence theory [29, 30] been adopted to quantify the imprecision and uncertainty[31, 32, 33, 34, 35], Yang et al.[4] used the modified D-S evidence theory to aggregate the different information which may be inconsistent, imprecise and uncertain. However, the basic belief assignments (BBAs) constructed by Yang et al.'s method become highly conflicting evidence which is inconsistent with the rules put forward by Dempster[29, 36]. Su et al.[36] solved the problem by modifying the original evidences and obtained a more accurate result. Even so, defects still exist by using the D-S evidence theory in practical application while there are many limits in constructing the frame of discernment and BPAs[37, 38, 39]. Liu et al.[40] proposed a new risk priority model for the risk assessment using a more appropriate representation of uncertain information called D numbers[37, 38, 41] and an modified grey relational analysis method[42, 43, 44], which converting the GRP method[45, 46, 47] to the double reference points (the positive ideal alternative and negative ideal alternative). However, taking the lowest and highest levels of the risk factors to be the positive and negative reference sequences is less accurate than TOPSIS[48, 49], which takes the highest and lowest value of the risk factors' assessments to be the positive ideal and negative ideal solutions. For example, in Liu's method the positive and the negative reference sequence are expressed as X_0^+=(1,1,...,1) and X_0^-=(10,10,...,10), it will have a greater error then taking the lowest and highest value of each column to be the positive and the negative reference sequence. Meanwhile, when determining the grey relation matrices in Liu's method the value of distinguishing coefficient ξ is variable, it also will produce the deviation of the result. And the calculation process of TOPSIS is less complicated, depicted in a simple mathematical form. In this paper, a new risk priority model is proposed for the risk evaluation in FMEA based on D numbers and TOPSIS owing to its more precisely quantify of the positive and negative reference sequences and straightforward calculation.

The rest of this paper is organized as follows. In Section 2, a brief review about the traditional FMEA and its main shortcomings is given. Basic concepts of D numbers and TOPSIS are described. The risk priority model for FMEA based on D numbers and TOPSIS is developed in Section 3. In Section 4, an example is devoted to illustrate the proposed model and finally, some conclusions are provided in Section 5.

2. Preliminaries

2.1. FMEA

2.1.1. Traditional FMEA procedure

The procedures for carrying out an FMEA can be divided into several steps as shown in Fig. 1. These steps are briefly explained here[42, 50, 51]:

Step 1: Identify what the system is supposed to do when it is operating properly.

Step 2: Divide the system into sub-systems and/or assemblies to localise the search for components.

Step 3: Identify components and relations among components use schematics,blue prints and flow charts.

Step 4: List complete component for each assembly.

Step 5: Identify environmental and practical pressures that can affect the system. Consider how these pressures might affect the performance of individual components.

Step 6: Determine failure modes of each component and assess the effects of failure modes on assemblies, sub-systems, and the entire system.

Step 7: Define the hazard level of each failure mode.

Step 8: Evaluate the probability. This can also be done by employing qualitative evaluations in the absence of solid quantitative statistical information.

Step 9: Calculate the risk priority number (RPN), which is given as the multiplication of the index representing the probability, severity and detectability.

Step 10: Make a decision whether action needs to be taken according to the RPN.

Step 11: Propose recommendations to enhance the system performance, which may fall into two categories:

- Preventive actions: prevent failure from occurring.
- Compensatory actions: minimizing the cost in the event that a failure occurs.

Step 12: Summarise the analysis, which can be accomplished in a tabular form.

2.1.2. Shortcomings in traditional FMEA

Traditionally, the prioritization of failure modes is determined by calculating the risk priority number (RPN) [52, 53, 54], which is defined as follows:

$$RPN = S \times O \times D \tag{1}$$

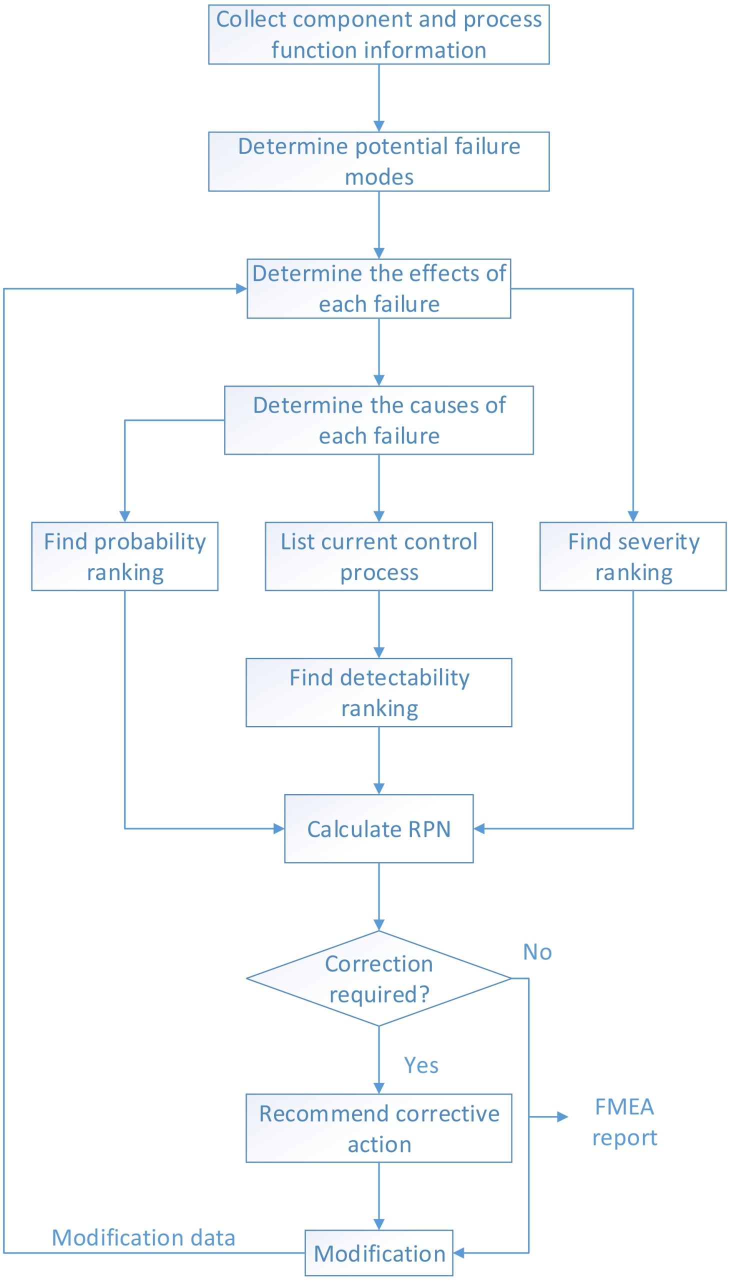

Figure 1: FMEA process[42].

where O is the probability of occurrence of a failure mode, S is the severity of a failure effect and D is the probability of a failure being detected[55, 56]. In general, each risk factor has 10 numerical ratings from 1 to 10[57]. Table. 1- 3[4, 58, 59] showed the probability scales of O, S and D. The failure mode with higher RPN is assumed to be more significant and should be given a higher priority than those having lower one. Although traditional FMEA has been acknowledged to be a useful tool in system, design, process and service, traditional RPN method has also been criticized for many shortcomings [40, 60].

- The relative importance among O, S and D is not taken into consideration in determining the priority of the failure modes. However, the weights of the risk factors may be different in practical applications[42, 61].

- The calculation of multiplication of RPNs is questionable. Small variations may lead to vastly different effects on the RPN. For example, if O and S are both 10, then a 1-point difference in detection rating results in a 100-point difference in the RPN; if O and S are equal to 1, then the same 1-point difference results in only a 1-point difference in the RPN; Hence, the conclusion acquired is meaningless[61, 62].

- The RPN considers only three factors mainly in terms of safety, but it makes no sense why other important factors are not taken into account[24].

- Different operation of O, S and D may produce exactly the same value of RPN, but their hidden risk impacts may be totally ignored. For example, two different failures with the values of 2, 3, 4 and 2, 2, 6 correspond to O, S, D, respectively, having the same RPN value of 24. The hidden risk impact of the two failures, however, may be different and a high risk failure mode may be overlooked in some cases[42, 61].
- It is difficult or even impossible to give exact numerical evaluations associated with the risk factors. The FMEA team members often give inconsistent assessments to the same risk factors, some of which may be uncertain, ambiguous and incomplete because of different background and experience[2].
- The RPNs are not continuous. Many empty elements exist in the RPN scales because many numbers between 1 to 1000 can not be obtained by the product of O, S and D. It comes the problem in exploring the meaning of different RPNs[61, 63].

2.2. D numbers

To overcomes these existing deficiencies in Dempster-Shafer theory and appears to be more effective in representing various types of uncertainty, a new representation of uncertain information which is called D numbers[37, 38] is introduced below. D numbers is an extension of Dempster-Shafer theory. It is defines as follows.

Table 1: Traditional FMEA scale for occurrence(O).

Rating	Probability of failure	Possible failure rate
10	Extremely high: Failure almost inevitable	⩾1/2
9	Very high	1/3
8	Repeated failures	1/8
7	High	1/20
6	Moderately high	1/80
5	Moderate	1/400
4	Relatively low	1/2,000
3	Low	1/15,000
2	Remote	1/150,000
1	Nearly impossible	⩽1/1,500,000

Definition 1. *(D Numbers[37, 38, 41])* Let a finite nonempty set Ω denote the problem domain. D number function is a mapping formulated by

$$D: \quad \Omega \to [0,1] \tag{2}$$

with

$$D(\varnothing) = 0 \quad and \quad \sum_{B \subseteq \Omega} D(B) \leqslant 1 \tag{3}$$

where $\varnothing$ is an empty set and B is a subset of Ω. Compared with the definition of the mass function, the structure of the expression seems to be similar. However, in D numbers the elements in set Ω is different from the concept of frame of discernment in D-S theory, the elements do not require to be mutually exclusive. In addition, the completeness constraint is

Table 2: Traditional FMEA scale for severity(S).

Rating	Effect	Severity for effect
10	Hazardous without warning	Highest severity ranking of a failure mode, occurring without warning and consequence is hazardous
9	Hazardous with warning	Higher severity ranking of a failure mode, occurring with warning,consequence is hazardous
8	Extreme	Operation of system or product is broken down without compromising safe
7	Major	Operation of system or product may be continued but performance of system or product is affected
6	Significant	Operation of system or product is continued and performance of system or product is degraded
5	Moderate	Performance of system or product is affected seriously and the maintenance is needed
4	Low	Performance of system or product is small affected and the maintenance may not be needed
3	Minor	System performance and satisfaction with minor effect
2	Very minor	System performance and satisfaction with slight effect
1	None	No effect

released in D numbers. The information is acceptable to be incomplete if $\sum_{B\subseteq\Omega} D\left(B\right) < 1$.

Furthermore, for a discrete set $\Omega = \{b_1, b_2, \cdots, b_i, \cdots, b_n\}$, where $b_i \in R$ and when $i \neq j$, $b_i \neq b_j$. A special form of D numbers can be expressed

Table 3: Traditional FMEA scale for detection(D).

Rating	Detection	Likelihood of detection by design control
10	Absolute uncertainty	Potential occurring of failure mode cannot be detected and subsequent failure mode
9	Very remote	The possibility of detecting the potential occurring of failure mode is very remote/mechanism and subsequent failure mode
8	Remote	The possibility of detecting the potential occurring of failure mode is remote/mechanism and subsequent failure mode
7	Very low	The possibility of detecting the potential occurring of failure mode is low/mechanism and subsequent failure mode
6	Low	The possibility of detecting the potential occurring of failure mode is low/mechanism and subsequent failure mode
5	Moderate	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
4	Moderately high	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
3	High	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
2	Very high	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode
1	Almost certain	The possibility of detecting the potential occurring of failure mode is moderate/mechanism and subsequent failure mode

by

$$\begin{aligned} D(\{b_1\}) &= v_1 \\ D(\{b_2\}) &= v_2 \\ \cdots \quad & \cdots \\ D(\{b_i\}) &= v_i \\ \cdots \quad & \cdots \\ D(\{b_n\}) &= v_n \end{aligned} \tag{4}$$

or simply denoted as $D = \{(b_1, v_1), (b_2, v_2), \cdots, (b_i, v_i), \cdots, (b_n, v_n)\}$, where

$v_i > 0$ and $\sum\limits_{i=1}^{n} v_i \leqslant 1$.

Below is the combination rule, a kind of add operation to combine two D numbers.

Definition 2. *(Two D Numbers' Rule of Combination[37, 38])* Suppose D_1 and D_2 are two D numbers, indicated by

$$D_1 = \{(b_1^1, v_1^1), \cdots, (b_i^1, v_i^1), \cdots, (b_n^1, v_n^1)\}$$

$$D_2 = \{(b_1^2, v_1^2), \cdots, (b_j^2, v_j^2), \cdots, (b_m^2, v_m^2)\}$$

and the combination of D_1 and D_2, which is expressed as $D = D_1 \oplus D_2$, is defined as follows.

$$D(b) = v \tag{5}$$

with

$$b = \frac{b_i^1 + b_j^2}{2} \tag{6}$$

$$v = \frac{v_i^1 + v_j^2}{2} \Big/ C \tag{7}$$

$$C = \begin{cases} \sum\limits_{j=1}^{m}\sum\limits_{i=1}^{n}\left(\frac{v_i^1+v_j^2}{2}\right), & \sum\limits_{i=1}^{n} v_i^1 = 1 \quad and \quad \sum\limits_{j=1}^{m} v_j^2 = 1\,; \\ \sum\limits_{j=1}^{m}\sum\limits_{i=1}^{n}\left(\frac{v_i^1+v_j^2}{2}\right) + \sum\limits_{j=1}^{m}\left(\frac{v_c^1+v_j^2}{2}\right), & \sum\limits_{i=1}^{n} v_i^1 < 1 \quad and \quad \sum\limits_{j=1}^{m} v_j^2 = 1\,; \\ \sum\limits_{j=1}^{m}\sum\limits_{i=1}^{n}\left(\frac{v_i^1+v_j^2}{2}\right) + \sum\limits_{i=1}^{n}\left(\frac{v_i^1+v_c^2}{2}\right), & \sum\limits_{i=1}^{n} v_i^1 = 1 \quad and \quad \sum\limits_{j=1}^{m} v_j^2 < 1\,; \\ \sum\limits_{j=1}^{m}\sum\limits_{i=1}^{n}\left(\frac{v_i^1+v_j^2}{2}\right) + \sum\limits_{j=1}^{m}\left(\frac{v_c^1+v_j^2}{2}\right) & \\ \qquad + \sum\limits_{i=1}^{n}\left(\frac{v_i^1+v_c^2}{2}\right) + \frac{v_c^1+v_c^2}{2}\,, & \sum\limits_{i=1}^{n} v_i^1 < 1 \quad and \quad \sum\limits_{j=1}^{m} v_j^2 < 1\,. \end{cases} \tag{8}$$

where $v_c^1 = 1 - \sum\limits_{i=1}^{n} v_i^1$ and $v_c^2 = 1 - \sum\limits_{j=1}^{m} v_j^2$.

Definition 3. *(Multiple D Numbers' Rule of Combination[38])* Let D_1, D_2, $\cdots$, D_n be n D numbers, μ_j is an order variable for each D_j, indicated by tuple $< \mu_j, D_{\mu_j} >$, then the combination operation of multiple D numbers is a mapping f_D, such that

$$f_D(D_1, D_2, \cdots, D_n) = [\cdots [D_{\lambda_1} \oplus D_{\lambda_2}] \oplus \cdots \oplus D_{\lambda_n}] \tag{9}$$

where D_{λ_i} is the D_{μ_j} of the tuple $< \mu_j, D_{\mu_j} >$ having the i^{th} lowest μ_j.

In the meanwhile, an aggregate operation is proposed on this special D numbers, as such

Definition 4. *(D Numbers' Integration[37, 38, 41])* For $D = \{(b_1, v_1), (b_2, v_2), \cdots, (b_i, v_i), \cdots, (b_n, v_n)\}$, the integrating representation of D is defined as

$$I(D) = \sum_{i=1}^{n} b_i v_i \tag{10}$$

where $b_i \in R, v_i > 0$ and $\sum_{i=1}^{n} v_i \leqslant 1$

2.3. TOPSIS method

Technique for order preference by similarity to ideal solution (TOPSIS) which proposed by Hwang et al.[48], is one of the MCDM methods in conception and application. The standard TOPSIS method aims to select alternatives that have the shortest distance from the positive ideal solution and the negative ideal solution simultaneously[64]. The positive ideal solution maximizes the benefit criteria and minimizes the cost criteria, whereas the negative ideal solution minimizes the benefit criteria and maximizes the cost criteria[65]. The calculation procedure is described as follows.

Definition 5. Let us consider a decision matrix $D = (x_{ij})$, which consists of alternatives and criteria. Normalize the decision matrix[48]:

$$r_{mn} = \frac{x_{mn}}{\sqrt{\sum\limits_{n=1}^{j} x_{mn}^2}}, m = 1, ..., i; n = 1, ..., j. \tag{11}$$

Multiply the columns of the normalized decision matrix by the associated weights to obtain the weighted decision matrix[48] $A = v(ij)$:

$$v_{mn} = w_n \times r_{mn}, m = 1, ..., i; n = 1, ...j \tag{12}$$

where w_n is the weight for n criterion.

Determine the positive ideal and negative ideal solutions. The positive-ideal solution, assumed as A^+, and the negative ideal solution, assumed as A^-, are defined as follows[48]:

$$A^+ = \{v_1^+, v_2^+, ..., v_j^+\} = \{(\max_m v_{mn} | n \in K_b)(\min_m v_{mn} | n \in K_c)\} \tag{13}$$

$$A^- = \{v_1^-, v_2^-, ..., v_j^-\} = \{(\min_m v_{mn} | n \in K_b)(\max_m v_{mn} | n \in K_c)\} \tag{14}$$

where K_b is the set of benefit criteria and K_c is the set of cost criteria.

Obtain the separation measures of the existing alternatives from the positive ideal and negative ideal solutions. The separation measures based on Euclidean distance, S_m^+ and S_m^- , of each alternative from the positive ideal and negative ideal solutions, respectively, are derived from[48]:

$$S_m^+ = \sqrt{\sum_{n=1}^{j} (v_n^+ - v_{mn})^2}, m = 1, ..., i; n = 1, ..., j. \tag{15}$$

$$S_m^- = \sqrt{\sum_{n=1}^{j} (v_n^- - v_{mn})^2}, m = 1, ..., i; n = 1, ..., j. \tag{16}$$

Calculate the relative closeness to the ideal solution[48]:

$$C_m = \frac{S_m^-}{S_m^- + S_m^+}, m = 1, ..., i. \tag{17}$$

Rank the alternatives according to the relative closeness to the ideal solution: the alternatives with higher C_m are assumed to be more important and should be given higher priority.

3. The proposed model for FMEA

Supposing k FMEA team membersTM_k(k=1,2,...,l) give assessments to i failure modes, FM_m(m=1,2,...,i), with n risk factors(RF_n)(n=1,2,...,j). Each FMEA team member evaluates the failure modes and identifies the proportion information of the n risk factors, satisfying the sum of n risk factors proportion equals to 1. The proposed model is composed of the following steps(shown in Figure 2):

Step 1: List all failure modes(FMs), relevant risk factors(RFs) and define appropriate numeric scales

In this paper, 10-point scales shown in Table 1-3 are employed to assess the risk factors of each failure modes. 7-point scale is adopted for evaluating the relative importance of the risk factors is shown in Table 4.

Step 2: Construct an assessment matrix by assessing failure modes and the risk factors' weights using D numbers

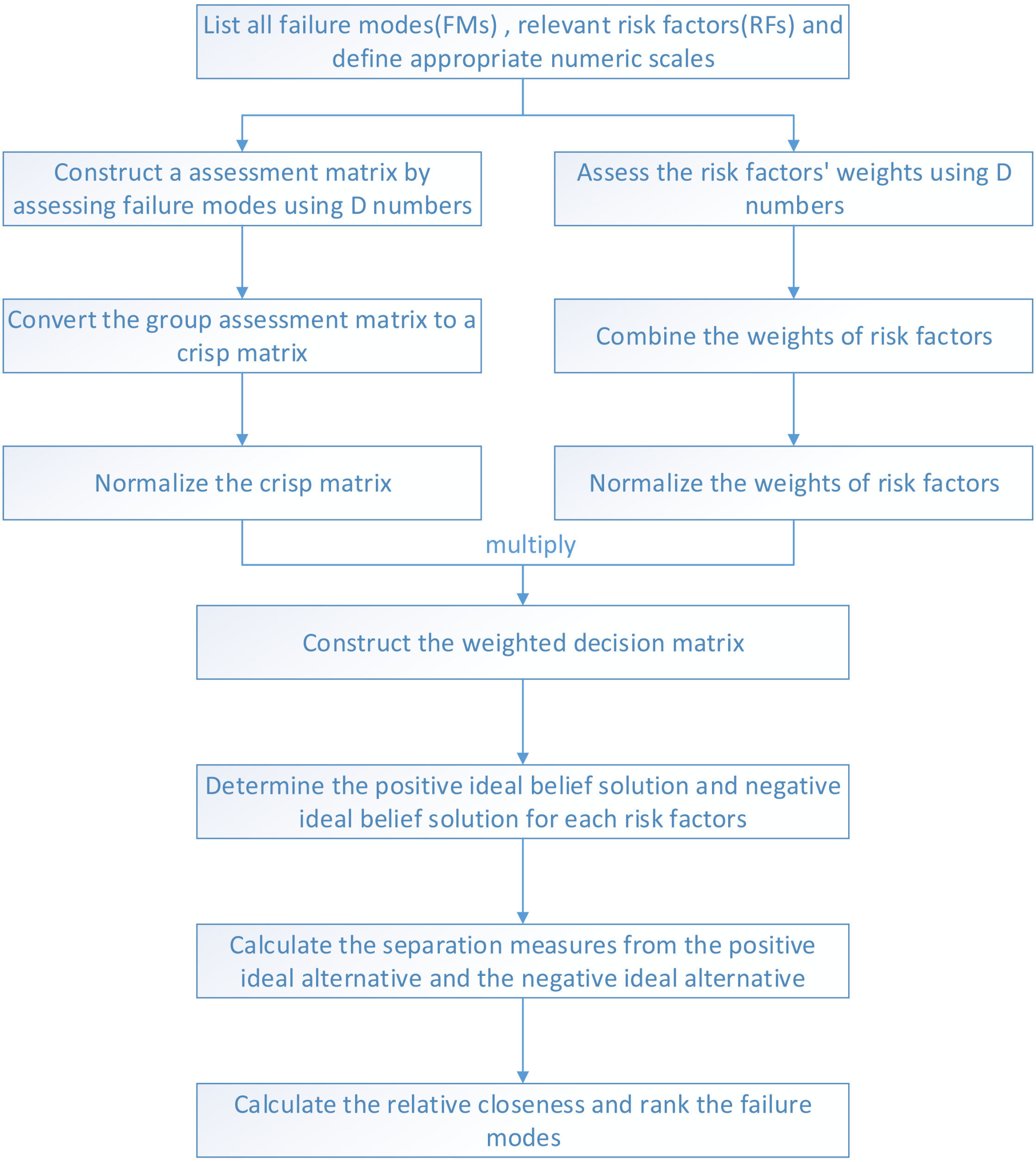

Figure 2: Flowchart of the proposed FMEA model.

Table 4: Linguistic scale for importance of risk factors.

Rating	Importance	Description
7	Very high	The importance of risk factor is very high
6	High	The importance of risk factor is high
5	Medium high	The importance of risk factor is medium high
4	Medium	The importance of risk factor is medium
3	Medium low	The importance of risk factor is medium low
2	Low	The importance of risk factor is low
1	Very low	The importance of risk factor is very low

Since the different FMEA team members may deliver different view for the same risk factors based on their a variety of experiences and backgrounds, the assessments for risk factors and their relative weights may be uncertain and incompleteness inevitably. According to section 2, D numbers can be used to correspondingly supplement the assessments for risk factors and their relative weights. Presuming the assessment of FM_m with respect to RF_n can be converted as D_{mn} and the weights of n^{th} risk factor can be w_n. The assessment matrix given by the k^{th} FMEA team member can be constructed as follows:

$$D^k = \begin{bmatrix} \mathrm{FM}_1^{\mathrm{k}} \\ \mathrm{FM}_2^{\mathrm{k}} \\ \vdots \\ \mathrm{FM}_{\mathrm{i}}^{\mathrm{k}} \end{bmatrix} = \left\{ \begin{matrix} D_{11}^k & D_{12}^k & \cdots & D_{1j}^k \\ D_{21}^k & D_{22}^k & \cdots & D_{2j}^k \\ \vdots & \cdots & \cdots & \vdots \\ D_{i1}^k & D_{i2}^k & \cdots & D_{ij}^k \end{matrix} \right\}$$

Step 3: Convert the group assessment matrix to a crisp matrix

The group assessment matrix can be converted to a crisp matrix by using the combination and integration representation of D numbers, considering there are k TMs, combination process could be executed as:

$D_{ij} = D_{ij}^{\lambda_1} \oplus D_{ij}^{\lambda_2} \oplus \ldots \oplus D_{ij}^{\lambda_s} \oplus \ldots \oplus D_{ij}^{\lambda_k}$

Where the order variables λ_s(s=1,2,...,k) is determined by the weights of each FMEA team members[38].

Consequently, a crisp matrix X=(x_{ij}) is derived:

$$X = I(D) = \begin{bmatrix} \mathrm{X}_1 \\ \mathrm{X}_2 \\ \vdots \\ \mathrm{X}_\mathrm{i} \end{bmatrix} = \begin{Bmatrix} I(D_{11}) & I(D_{12}) & \ldots & I(D_{1j}) \\ I(D_{21}) & I(D_{22}) & \ldots & I(D_{2j}) \\ \vdots & \ldots & \ldots & \vdots \\ I(D_{i1}) & I(D_{i2}) & \ldots & I(D_{ij}) \end{Bmatrix}$$

Similarly, the weights of risk factors are combined in the same way,which is indicated in:

$$W = (I(w_1), I(w_2), ..., I(w_j)) \tag{18}$$

Step 4: Normalize the crisp matrix and the weights of risk factors

Normalize the decision matrix by Eq.(11). The normalized decision matrix R = (r_{mn}) is obtained:

$$R = \begin{Bmatrix} r_{11} & r_{12} & \ldots & r_{1j} \\ r_{21} & r_{22} & \ldots & r_{2j} \\ \vdots & \ldots & \ldots & \vdots \\ r_{i1} & r_{i2} & \ldots & r_{ij} \end{Bmatrix}$$

Besides, the weights of risk factors can be normalized as:

$$W_n = \frac{I(w_n)}{\sum\limits_{n=1}^{j} I(w_n)} \tag{19}$$

Step 5: Construct the weighted decision matrix

The weighted decision matrix can be obtained by assigning the normalized weights of risk factors to corresponding columns of the normalized decision matrix as follows:

$$A = \left\{ \begin{matrix} W_1 r_{11} & W_2 r_{12} & \dots & W_j r_{1j} \\ W_1 r_{21} & W_2 r_{22} & \dots & W_j r_{2j} \\ \vdots & \dots & \dots & \vdots \\ W_1 r_{i1} & W_2 r_{i2} & \dots & W_j r_{ij} \end{matrix} \right\} \tag{20}$$

Step 6: Determine the positive ideal belief solution and negative ideal belief solution for each risk factors

According to Eq.(13), (14), the positive ideal belief solution and negative ideal belief solution are determined:

$$A^+ = \{A_1^+, A_2^+, ..., A_j^+\} \tag{21}$$

$$A^- = \{A_1^-, A_2^-, ..., A_j^-\} \tag{22}$$

where A_j^+ is the maximum value with respect to risk factor RF_j and A_j^- is the minimum value with respect to risk factor RF_j.

Step 7: Calculate the separation measures from the positive ideal alternative and the negative ideal alternative

Based on the Euclidean distance we can respectively obtain the separation measures from the positive ideal alternative S_m^+ and the negative ideal alternative S_m^- by Eq.(15), (16),

$$S_m^+ = \sqrt{\sum_{n=1}^{j} (A_n^+ - A_{mn})^2, m = 1, 2, ..., i; n = 1, 2, ..., j.} \tag{23}$$

$$S_m^- = \sqrt{\sum_{n=1}^{j} (A_n^- - A_{mn})^2, m = 1, 2, ..., i; n = 1, 2, ..., j.} \tag{24}$$

Step 8: Calculate the relative closeness and rank the failure modes

The relative closeness of each failure mode is determined by Eq.(17) as follows:

$$C_m = \frac{S_m^-}{S_m^- + S_m^+}, m = 1, ..., i. \tag{25}$$

In the proposed FMEA model, the value of relative closeness can reflect the impact of failure mode and the relationship between the FMs. The higher the value of relative closeness, the smaller the effect of the failure mode. Hence, the influential of all the failure modes in the FMEA can be ranked by the ascending order of their relative closeness coefficients.

4. An application in the rotor blades of an aircraft turbine

In this section, a case of rotor blades for an aircraft turbine is employed to illustrate the validity and practicability of the proposed method[4]. Rotor blades are the crucial rotating components of an aircraft turbine, which plays an important role in the task of energy conversion. Since they are the thin-form, components moving in high-speed rotation, under the severe load conditions in complex work environments, rotor blades are one of the components which are most likely to be failed in aircraft turbines[40]. Imultaneously, with the development of the aviation industry, the Thrust-Weight Ratio (TWR) of aircraft turbines has grown constantly and the stress level of rotor blades has been a dramatic increase as well. Furthermore, their stabilization plays an essential role in the aircraft turbine security. For the sake of improving their safety and reliability, failure mode and effects analysis (FMEA) is prerequisite in their design[3, 4, 12].

This rotor blades includes two subsystems: the compressor rotor blades and the turbo rotor blades. In this paper we only analysis the compressor rotor blades to compare with Liu et al.[40] and Yang et al.[4] method. Supposing there are three FMEA team members, TM_1, TM_2 and TM_3. As a result, there are eight failure modes(FM_m,m=1,2,...,8) needed to be assessed by the FMEA team members. For applying the proposed model to the FMEA, the FMEA team members should assess the risk factors(O, S and D) and their relative weights by D numbers based on the numeric scales defined in Tables 1-4. The assessment results given by the three FMEA team members are presented in Table 5. Firstly, combining the individual assessments of the FMEA team members into a group assessment by using Eq. (9), as shown in Table 6.

The crisp matrix are then aggregated using Eq.(10) and the results are tabulated in Table 7. Then normalize the crisp matrix and the weights of risk factors by Eq.(11), for instance:

Table 5: Evaulation information on failure modes provided by the FMEA team members.

Team members	TM_1			TM_2			TM_3		
Risk factors	O	S	D	O	S	D	O	S	D
FM_1	{(3, 0.4), (4, 0.6)}	{(7, 0.8)}	{(2, 1.0)}	{(3, 0.9), (4, 0.1)}	{(7, 1.0)}	{(2, 1.0)}	{(3, 0.8), (4, 0.2)}	{(7, 1.0)}	{(2, 1.0)}
FM_2	{(2, 1.0)}	{(8, 1.0)}	{(4, 1.0)}	{(2, 1.0)}	{(8, 0.7), (9, 0.3)}	{(4, 1.0)}	{(2, 1.0)}	{(8, 1.0)}	{(4, 1.0)}
FM_3	{(1, 0.9)}	{(10, 1.0)}	{(3, 1.0)}	{(1, 1.0)}	{(10, 1.0)}	{(3, 1.0)}	{(1, 1.0)}	{(10, 1.0)}	{(3, 0.9)}
FM_4	{(1, 1.0)}	{(6, 0.8),{(3, 1.0)}	{(1, 1.0)} (7,0.1)}	{(6, 1.0)}	{(2, 0.3), (3, 0.7)}	{(1, 1.0)}	{(6, 1.0)}	{(3, 1.0)}	
FM_5	{(1, 1.0)}	{(3, 1.0)}	{(1, 0.5), (2, 0.5)}	{(1, 1.0)}	{(3, 1.0)}	{(1, 0.7), (2, 0.3)}	{(1, 1.0)}	{(2, 0.3), (3, 0.6)}	{(1, 1.0)}
FM_6	{(2, 1.0)}	{(6, 1.0)}	{(5, 1.0)}	{(2, 1.0)}	{(6, 1.0)}	{(5, 1.0)}	{(2, 1.0)}	{(6, 1.0)}	{(5, 1.0)}
FM_7	{(1, 0.7)}	{(7, 1.0)}	{(3, 1.0)}	{(1, 1.0)}	{(7, 0.8), (8, 0.1)}	{(3, 1.0)}	{(1, 1.0)}	{(7, 1.0)}	{(3, 1.0)}
FM_8	{(3, 1.0)}	{(5, 0.6),	{(1, 1.0)} (6,0.4)}	{(3, 1.0)}	{(5, 0.75), (6, 0.2)}	{(1, 1.0)}	{(3, 1.0)}	{(5, 0.8), (7, 0.2)}	{(1, 1.0)}
Weights	{(6, 1.0)}	{(7, 1.0)}	{(5, 1.0)}	{(7, 1.0)}	{(7, 1.0)}	{(5, 1.0)}	{(6, 1.0)}	{(7, 1.0)}	{(5, 1.0)}

Table 6: Group assessments of the FMEA team members and group weights of risk factors.

Failure modes	O	S	D
FM_1	{(3, 0.3), (3.5, 0.5), (4, 0.2)}	{(7, 0.533)}	{(2, 1.0)}
FM_2	{(2, 1.0)}	{(8, 0.567), (8.5, 0.433)}	{(4, 1.0)}
FM_3	{(1, 0.544)}	{(10, 1.0)}	{(3, 0.544)}
FM_4	{(1, 1.0)}	{(6, 0.4), (6.25, 0.335)}	{(2.5, 0.433), (3, 0.567)}
FM_5	{(1, 1.0)}	{(2.75, 0.331), (3, 0.35)}	{(1, 0.3), (1.25, 0.3), (1.5, 0.2), (1.75, 0.2)}
FM_6	{(2, 1.0)}	{6, 1.0)}	{(5, 1.0)}
FM_7	{(1, 0.522)}	{(7, 0.6), (7.5, 0.367)}	{(3, 1.0)}
FM_8	{(3, 1.0)}	{(5, 0.183), (5.25, 0.175), (5.5, 0.25), (5.75, 0.233), (6, 0.067), (6.25, 0.058)}	{(1, 1.0)}
Weights	{(6.75, 1.0)}	{(7, 1.0)}	{(5, 1.0)}

$$r_{11}=\frac{x_{11}}{\sqrt{\sum_{m=1}^{8} x_{m1}^2}}=\frac{3.450}{\sqrt{3.450^2+2^2+0.544^2+1^2+1^2+2^2+0.522^2+3^2}}=0.6150$$

Table 7: Comparative sequences for the failure modes.

Failure modes	O	S	D
FM_1	3.45	3.731	2
FM_2	2	8.217	4
FM_3	0.544	10	1.632
FM_4	1	4.494	2.784
FM_5	1	1.963	1.325
FM_6	2	6	5
FM_7	0.522	6.955	3
FM_8	3	5.313	1
Weights	0.630	0.373	0.267

By this analogy, the normalized decision matrix is obtained as follows:

$$R = \left\{ \begin{array}{ccc} 0.6150 & 0.2092 & 0.2440 \\ 0.3565 & 0.4607 & 0.4881 \\ 0.0970 & 0.5607 & 0.1991 \\ 0.1783 & 0.2520 & 0.3396 \\ 0.1783 & 0.1099 & 0.1617 \\ 0.3565 & 0.3364 & 0.6101 \\ 0.0930 & 0.3899 & 0.3661 \\ 0.5348 & 0.2979 & 0.1220 \end{array} \right\}$$

According to Eq. (19), the weights of risk factors can be expressed as below:

W=(W_1, W_2, W_3)=$(0.3600, 0.3733, 0.2667)$

In this example, the final decision matrix with weighted is obtained by Eq. (20) as shown in Table 8.

Table 8: weighted normalized sequences for the failure modes.

Failure modes	O	S	D
FM_1	0.2214	0.0781	0.0651
FM_2	0.1283	0.1720	0.1302
FM_3	0.0349	0.2093	0.0531
FM_4	0.0642	0.0941	0.0906
FM_5	0.0642	0.0410	0.0431
FM_6	0.1283	0.1256	0.1627
FM_7	0.0335	0.1455	0.0976
FM_8	0.1925	0.1112	0.0325

According to Eqs. (21), (22) and Table 8, the positive ideal solution and the negative ideal solution are generated as follows.

$A^{+}= \{0.2214,0.2093,0.1627\}$

$A^{-} =\{0.0335,0.0410,0.0325\}$

Then, the separation measures from the positive ideal alternative, S_m^+ and the negative ideal alternative S_m^- are calculated by Eqs. (23), (24) for all the failure modes identified in the FMEA. Finally, the relative closeness of each failure mode C_m, can be calculated using Eq. (25). The results to determine risk priority ranking of the eight failure modes are shown in Table 9. As can be seen from Table 9, FM_2 has the highest relative closeness value in the failure modes of compressor rotor blades and thus should be given a top risk priority, followed by FM_6, FM_1, FM_8, FM_3, FM_7, FM_4 and FM_5. Therefore, the priority ranking of the eight failure modes is $FM_2 > FM_6 > FM_1 > FM_8 > FM_3 > FM_7 > FM_4 > FM_5$.

Table 9: Results of the proposed method and risk priority ranking

Failure modes	S^+	S^-	C	The proposed method	Yang's method	Liu's method
FM_1	0.1636	0.1943	0.5429	3	3	3
FM_2	0.1054	0.1889	0.6418	1	1	2
FM_3	0.2163	0.1696	0.4394	5	4	1
FM_4	0.2079	0.0844	0.2888	7	6	7
FM_5	0.2595	0.0324	0.1111	8	8	8
FM_6	0.1252	0.1819	0.5924	2	2	4
FM_7	0.2088	0.1231	0.3709	6	5	5
FM_8	0.1655	0.1738	0.5122	4	7	6

In the previous literature, a method based on D numbers and grey relational projection was proposed[40] and the risk priority ranking gained by this method is $FM_3 > FM_2 > FM_1 > FM_6 > FM_7 > FM_8 > FM_4 > FM_5$. This is obvious that only three rankings of the eight failure modes(FM_1, FM_4 and FM_5) is the same, which has the same conclusion with the comparison between the method of Liu et al.[40] and the approach of Yang et al.[4]. However, as shown in Table 9, a risk evaluation method proposed by Yang et al.[4] is taken to compare with the proposed model. Comparing the results obtained from Table 9, it can be found that except for FM_8, the ranking orders of the other seven failure modes are the same. In other words, the rising of the FM_8's ranking has led to a fall in other FM's rankings by order. After conducting criticality assessment using method of Yang et al., FM_8 ranked only at the seventh place led to FM_3, FM_7 and

FM_4 ranked at fourth, fifth and sixth. However, a close look at the values of the risk factors for FM_4 and FM_8 reveals that FM_8 has the highest value of O and S. At the same time, looking at the values of O for FM_7 and FM_3 are a lot smaller than FM_8, which are fairly safe. The RPN of them also can be seen that FM_8 has a higher value than FM_7 and FM_3, following the similar logic, we can easily rank the FM_8 a higher place than FM_3, FM_4 and FM_7. Therefore, the proposed method is more logical and a more accurate ranking can be admissive. What's more, the proposed FMEA model based on D numbers can effectively cope with the uncertain information and the TOPSIS method has the double reference points as same as the modified GRP method but is more accurate and understandable. The computation processes are more straightforward.

5. conclusion

FMEA has been used in industrial settings as an operative tool for helping identify, rank and alleviate potential failures in both the products and the processes. Although the traditional FMEA has developed a lot, there still exist several shortcomings. To deal with the risk evaluation information of multiple experts, which may be inconsistent, fuzzy and uncertain. In this paper, a new FMEA model based on D numbers and TOPSIS is presented to address such issues. The proposed model overcomes the shortcomings of the conventional RPN method for assessing the risk of failure modes in FMEA. By using D numbers, it is more effective to address various types of uncertainties, such as imprecision, fuzziness, ignorance and so on, in the failure analysis process. What's more, it incorporates the generic advantages of the MCDM methods, which are able to avoid the unreasonable risk priory ranking methods of traditional FMEA.

The effectiveness of the proposed model has been illustrated by an application of risk priority ranking of failure modes in FMEA of aircraft turbine rotor blades. The results are consistent with the practical engineering background demonstrated that the combination of D numbers and TOPSIS for the risk evaluation in FMEA is more accurate than other risk ranking methods.

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