# An improvement in monotonicity of the distance-based total uncertainty measure in belief function theory

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**Abstract**

Measuring the uncertainty of evidences is an open issue in belief function theory. Recently, a distance-based total uncertainty measure for the belief function theory, indicated by $TU^I$, is presented. Some experiments show the efficiency of the $TU^I$ to measure uncertainty degree. In this paper, numerical example and theoretical analysis are illustrated that the monotonicity in $TU^I$ is not satisfied. To address this issue, an improved uncertainty measure $TU_E^I$ is proposed. The monotonicity for $TU_E^I$ is theoretically proved. Finally, through experimental comparison we show that $TU_E^I$ also has the desired high sensitivity to the evidence changes, which further indicates that the proposed $TU_E^I$ is better than $TU^I$.

*Keywords:* Uncertainty measure, Belief function, Dempster-Shafer evidence theory, Entropy

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## 1. Introduction

Uncertainty is widely existing in the real world. A variety of theories have been developed to deal with various types of uncertainties. Among them, Dempster-Shafer theory of evidence [1, 2], also called belief function theory, is a popular mathematical tool to represent and handle uncertain information [3, 4, 5], since it has an advantage of directly expressing the "uncertainty" by assigning the probability to the subsets of the set composed of multiple objects, rather than to each of the individual objects.

In belief function theory, measuring the uncertainty contained in an evidence is an open issue [6, 7]. So far, researchers have proposed many measures to quantify the uncertainty of evidences, for example aggregated uncertainty (AU) [8, 9] and ambiguity measure (AM) [10]. But most of these uncertainty measures are criticized because of the low sensitivity, high computing complexity, concealment of conflict and non-specificity, or many other aspects. In addition, a basic difficulty for an uncertainty measure is that it is usually required to be consistent between the frameworks of belief function theory and probability theory. Recently, Yang and Han [11] proposed a new distance-based total uncertainty measure, indicated by $TU^I$, solely based on the framework of belief function theory but without considering the switch between the frameworks of belief function theory and probability theory. A number of experiments and simulations have shown that $TU^I$ has high sensitivity and could provide rational results in practical applications.

As stated in [11], the monotonicity is a key feature for an effective uncertainty measure. However, in this paper we find that the monotonicity of $TU^I$ is not satisfied. First, a counter example is given to show that the monotonicity is violated in $TU^I$. To address this issue, a new total uncertainty measure, denoted as $TU^I_E$, is proposed to improve the monotonicity of $TU^I$. The proposed $TU^I_E$ is mathematically proved to satisfy the desirable monotonicity. Finally, a number of experiments presented in [11] are illustrated to compare the proposed $TU^I_E$ and other uncertainty measures. The results show that $TU^I_E$ has a high sensitivity and good performance as same as $TU^I$. In addition, since $TU^I_E$ has the monotonicity which is violated in $TU^I$, $TU^I_E$ therefore is better than $TU^I$ and could provide much more rational results in practical applications.

## 2. Basics of belief function theory

Without loss of the generality, a few of basic concepts in belief function theory are introduced as follows.

A frame of discernment (FOD) is a set of mutually exclusive and collectively exhaustive events, which is indicated by $\Theta = \{\theta_1, \theta_2, \cdots, \theta_i, \cdots, \theta_n\}$. The power set of $\Theta$ is denoted as $2^\Theta$. Given a FOD, a mapping $m : 2^\Theta \to [0, 1]$ is a mass function, also called basic probability assignment (BPA), defined on $\Theta$, if it satisfies

$$m(\emptyset) = 0 \quad \text{and} \quad \sum_{A \subseteq \Theta} m(A) = 1. \tag{1}$$

If $m(A) > 0$, $A$ is called a focal element, and the union of all focal elements is called the core of the BPA. In belief function theory, the basic probability number $m(A)$ measures the belief exactly assigned to $A$ and represents how strongly the evidence supports $A$.

Associated with each BPA, the belief function $Bel$ and plausibility function $Pl$ express the lower bound and upper bound of the support degree to each proposition in a BPA, respectively. They are defined as

$$Bel(A) = \sum_{B \subseteq A} m(B), \tag{2}$$

$$Pl(A) = 1 - Bel(\bar{A}) = \sum_{B \cap A \neq \emptyset} m(B), \tag{3}$$

where $\bar{A} = \Omega - A$. Obviously, $Pl(A) \geq Bel(A)$, for each $A \subseteq \Theta$, and $[Bel(A), Pl(A)]$ is called the belief interval of $A$.

In belief function theory, two independent BPAs can be combined by Dempster's rule of combination, denoted by $m = m_1 \oplus m_2$, which is shown as follows

$$m(A) = \begin{cases} \frac{1}{1-K} \sum\limits_{B \cap C = A} m_1(B) m_2(C)\ , & A \neq \emptyset; \\ 0\ , & A = \emptyset. \end{cases} \tag{4}$$

with

$$K = \sum_{B \cap C = \emptyset} m_1(B) m_2(C), \tag{5}$$

where $K$ is a normalization constant, called conflict coefficient of the two BPAs. Note that the Dempster's rule of combination is only applicable to such two BPAs which satisfy the condition $K < 1$.

## 3. The distance-based total uncertainty measure $TU^I$ and its deficiency in monotonicity

Recently, Yang and Han [11] have proposed a distance-based total uncertainty measure, indicated by $TU^I$, for BPAs directly and totally based on the framework of belief function theory. $TU^I$ is formally defined as follows.

**Definition 1** **($TU^I$).** *Let $m$ be a BPA defined on the FOD $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, the total uncertainty of $m$ is*

$$TU^I(m) = 1 - \frac{1}{n} \cdot \sqrt{3} \cdot \sum_{i=1}^{n} d^I\left([Bel(\{\theta_i\}), Pl(\{\theta_i\})], [0,1]\right) \tag{6}$$

*with*

$$d^I\left([a_1, b_1], [a_2, b_2]\right) = \sqrt{\left[\frac{a_1 + b_1}{2} - \frac{a_2 + b_2}{2}\right]^2 + \frac{1}{3}\left[\frac{b_1 - a_1}{2} - \frac{b_2 - a_2}{2}\right]^2} \tag{7}$$

*where $\sqrt{3}$ is the normalization factor which equals to $1/d^I\left([0,0],[0,1]\right)$.*

Theoretically, the measure $TU^I$ utilizes the information of belief intervals of all elements in FOD to measure the total uncertainty in a BPA. For more clear understanding, Eq.(6) can be rewritten as

$$\begin{aligned} TU^I(m) &= 1 - \tfrac{1}{n}\sum_{i=1}^{n} \tfrac{d^I([Bel(\{\theta_i\}),Pl(\{\theta_i\})],[0,1])}{d^I([0,0],[0,1])} \\ &= \sum_{i=1}^{n}\left[1 - \tfrac{d^I([Bel(\{\theta_i\}),Pl(\{\theta_i\})],[0,1])}{d^I([0,0],[0,1])}\right] \Big/ n \\ &= \sum_{i=1}^{n}\left[\tfrac{d^I([0,0],[0,1]) - d^I([Bel(\{\theta_i\}),Pl(\{\theta_i\})],[0,1])}{d^I([0,0],[0,1])}\right] \Big/ n \end{aligned} \tag{8}$$

Now it is easily found that there are two steps in $TU^I$: (i) calculating the uncertainty caused by every element in FOD, i.e. $u(\{\theta_i\}) = \frac{d^I([0,0],[0,1]) - d^I([Bel(\{\theta_i\}),Pl(\{\theta_i\})],[0,1])}{d^I([0,0],[0,1])}$; and (ii) normalizing the uncertainty, i.e. $\sum_{i=1}^{n}\left[u(\{\theta_i\})\right] \Big/ n$.

In [11], the authors have shown that $TU^I$ has a range of $[0, 1]$ and it is monotonous for the changes of BPAs. The monotonicity of uncertainty measures is a basic property in belief function theory. Given an uncertainty measure $UM$ and two BPAs $m_1$ and $m_2$ over FOD $\Theta$, if

$$\forall A \subseteq \Theta : \quad [Bel_{m_1}(A), Pl_{m_1}(A)] \subseteq [Bel_{m_2}(A), Pl_{m_2}(A)]$$

there exists $UM(m_1) \leq UM(m_2)$, then $UM$ satisfies the monotonicity. However, by some numerical examples, we find that $TU^I$ actually fails to satisfy the monotonicity. Let us consider the following example.

**Example 1** Assume there are three BPAs defined on FOD $\Theta = \{\theta_1, \theta_2\}$:

$$m_1(\{\theta_1\}) = 0.4, \quad m_1(\{\theta_2\}) = 0.3, \quad m_1(\{\theta_1, \theta_2\}) = 0.3;$$

$$m_2(\{\theta_1\}) = 0.4, \quad m_2(\{\theta_2\}) = 0.2, \quad m_2(\{\theta_1, \theta_2\}) = 0.4;$$

$$m_3(\{\theta_1\}) = 0.4, \quad m_3(\{\theta_2\}) = 0.0, \quad m_3(\{\theta_1, \theta_2\}) = 0.6.$$

Then, according to the belief function $Bel$ and plausibility function $Pl$, we can obtain the belief intervals of each proposition in $m_1$, $m_2$ and $m_3$, respectively. For $m_1$, we have

$$[Bel_{m_1}(\{\theta_1\}), Pl_{m_1}(\{\theta_1\})] = [0.4, 0.7],$$
$$[Bel_{m_1}(\{\theta_2\}), Pl_{m_1}(\{\theta_2\})] = [0.3, 0.6],$$
$$[Bel_{m_1}(\{\theta_1, \theta_2\}), Pl_{m_1}(\{\theta_1, \theta_2\})] = [1, 1].$$

For $m_2$, we have

$$[Bel_{m_2}(\{\theta_1\}), Pl_{m_2}(\{\theta_1\})] = [0.4, 0.8],$$
$$[Bel_{m_2}(\{\theta_2\}), Pl_{m_2}(\{\theta_2\})] = [0.2, 0.6],$$
$$[Bel_{m_2}(\{\theta_1, \theta_2\}), Pl_{m_2}(\{\theta_1, \theta_2\})] = [1, 1].$$

And for $m_3$, we have

$$[Bel_{m_3}(\{\theta_1\}), Pl_{m_3}(\{\theta_1\})] = [0.4, 1],$$
$$[Bel_{m_3}(\{\theta_2\}), Pl_{m_3}(\{\theta_2\})] = [0, 0.6],$$
$$[Bel_{m_3}(\{\theta_1, \theta_2\}), Pl_{m_3}(\{\theta_1, \theta_2\})] = [1, 1].$$

Therefore,

$$\forall A \subseteq \Theta : \ [Bel_{m_1}(A), Pl_{m_1}(A)] \subseteq [Bel_{m_2}(A), Pl_{m_2}(A)] \subseteq [Bel_{m_3}(A), Pl_{m_3}(A)].$$

Based on the monotonicity as aforementioned, there should exist TotalUncertainty$(m_1) \leq$ TotalUncertainty$(m_2) \leq$ TotalUncertainty$(m_3)$. However, in terms of the definition of $TU^I$, we have $TU^I(m_1) = 0.6394, TU^I(m_2) = 0.6536, TU^I(m_3) = 0.6000$. The result shows that the monotonicity between $m_1$ and $m_2$ is satisfied in $TU^I$, but the monotonicity between $m_3$ and $m_1$, $m_2$ is violated. Therefore, we can conclude that the monotonicity is not always satisfied in $TU^I$.

The above example clearly shows that $TU^I$ does not have the property of monotonicity, which contradicts the proof of monotonicity of $TU^I$ given in [11]. The fact drives us to recheck the details of that proof in [11]. Regrettably, there is one very subtle error made by the authors of that paper, and this invalidates the proof of monotonicity of $TU^I$. In their proof, it is based on a FALSE property of the distance function of interval numbers in Eq.(7):

if $\forall \theta_i \in \Theta : \ [Bel_{m_1}(\{\theta_i\}), Pl_{m_1}(\{\theta_i\})] \subseteq [Bel_{m_2}(\{\theta_i\}), Pl_{m_2}(\{\theta_i\})]$, then

$$d^I([Bel_{m_1}(\{\theta_i\}), Pl_{m_1}(\{\theta_i\})], [0,1]) \geq d^I([Bel_{m_2}(\{\theta_i\}), Pl_{m_2}(\{\theta_i\})], [0,1])$$

which implies that function $d^I([a,b],[0,1])$ is monotonically decreasing with respect to $(b-a)$ given a fixed $a$ or $b$, where $[a,b] \subseteq [0,1]$.

In the following, we will prove that the above property in the distance function $d^I$ is not satisfied unconditionally.

Assume there is an interval $[a, a+\Delta]$, where $a, \Delta \geq 0$ and $a+\Delta \leq 1$. According to Eq.(7), we have

$$\begin{aligned} d^I([a,a+\Delta],[0,1]) &= \sqrt{\left[\tfrac{a+a+\Delta}{2} - \tfrac{0+1}{2}\right]^2 + \tfrac{1}{3}\left[\tfrac{a+\Delta-a}{2} - \tfrac{1-0}{2}\right]^2} \\ &= \sqrt{\left[a+\tfrac{\Delta}{2} - \tfrac{1}{2}\right]^2 + \tfrac{1}{3}\left[\tfrac{\Delta}{2} - \tfrac{1}{2}\right]^2}. \end{aligned}$$

Then,

$$\frac{\partial d^I([a,a+\Delta],[0,1])}{\partial \Delta} = \left(a + \frac{2\Delta}{3} - \frac{2}{3}\right) \Bigg/ \left(2\sqrt{\left[a+\frac{\Delta}{2}-\frac{1}{2}\right]^2 + \frac{1}{3}\left[\frac{\Delta}{2}-\frac{1}{2}\right]^2}\right).$$

In order to let function $d^I([a,b],[0,1])$ monotonically decreasing with respect to $\Delta$, we must have

$$\frac{\partial d^I([a,a+\Delta],[0,1])}{\partial \Delta} < 0,$$

namely

$$a + \frac{2\Delta}{3} - \frac{2}{3} < 0,$$

hence,

$$\Delta < 1 - \frac{3a}{2}.$$

However, in the initial we just require $0 \leq \Delta \leq 1-a$. So, we have proved that the monotonicity of function $d^I([a,b],[0,1])$ mentioned above is not satisfied unconditionally. Further, because of the invalidation of the monotonicity of function $d^I([a,b],[0,1])$, the proof in [11] for the monotonicity of $TU^I$ is also invalid.

## 4. An improvement in monotonicity for $TU^I$

In the section, we propose a new total uncertainty measure which improves the $TU^I$ and overcomes the deficiency of $TU^I$ in monotonicity.

**Definition 2 ($TU_E^I$).** *Let $m$ be a BPA defined on the FOD $\Theta = \{\theta_1, \theta_2, \cdots, \theta_n\}$, the total uncertainty of $m$ is*

$$TU_E^I(m) = \sum_{i=1}^{n} \left[ \frac{d_E^I([0,0],[0,1]) - d_E^I([Bel(\{\theta_i\}), Pl(\{\theta_i\})],[0,1])}{d_E^I([0,0],[0,1])} \right] \Big/ n \tag{9}$$

*with*

$$d_E^I\left([a_1, b_1], [a_2, b_2]\right) = \sqrt{(a_1 - a_2)^2 + (b_1 - b_2)^2}. \tag{10}$$

*Since $d_E^I\left([0,0],[0,1]\right) = 1$, Eq.(9) can be simply rewritten as*

$$TU_E^I(m) = 1 - \frac{1}{n} \cdot \sum_{i=1}^{n} d_E^I\left([Bel(\{\theta_i\}), Pl(\{\theta_i\})], [0,1]\right). \tag{11}$$

Similar to $TU^I$, the proposed $TU_E^I$ also has a range of $[0,1]$.

**Property 1 (Range).** $0 \leq TU_E^I(m) \leq 1$ *for any BPA $m$ over FOD $\Theta$.*

Proof. Because $d_E^I\left([Bel(\{\theta_i\}), Pl(\{\theta_i\})], [0,1]\right) = 0$ if and only if $[Bel(\{\theta_i\}), Pl(\{\theta_i\})] = [0,1]$, $TU_E^I(m)$ attains its unique global maximum 1 when $[Bel(\{\theta_i\}), Pl(\{\theta_i\})] = [0,1]$, $\forall \theta_i \in \Theta$. Hence, only the vacuous BPA $m(\Theta) = 1$ has the maximum uncertainty of 1.

Similarly, since $d_E^I\left([Bel(\{\theta_i\}), Pl(\{\theta_i\})], [0,1]\right) = 1$ if $[Bel(\{\theta_i\}), Pl(\{\theta_i\})] = [1,1]$ or $[Bel(\{\theta_i\}), Pl(\{\theta_i\})] = [0,0]$, it is easily known that $TU_E^I$ gets the minimum value 0 when $m(\{\theta_i\}) = 1$ and $m(A) = 0$ $\forall A \neq \theta_i, A \subseteq \Theta$.

More important, the proposed $TU_E^I$ strictly satisfies the monotonicity.

**Property 2 (Monotonicity).** *Let $m_1$ and $m_2$ be two BPAs defined on $\Theta$, if $\forall A \subseteq \Theta : [Bel_{m_1}(A), Pl_{m_1}(A)] \subseteq [Bel_{m_2}(A), Pl_{m_2}(A)]$, then $TU_E^I(m_1) \leq TU_E^I(m_2)$.*

PROOF. Since $\forall A \subseteq \Theta : [Bel_{m_1}(A), Pl_{m_1}(A)] \subseteq [Bel_{m_2}(A), Pl_{m_2}(A)]$, there exists $\forall \theta_i \in \Theta : [Bel_{m_1}(\{\theta_i\}), Pl_{m_1}(\{\theta_i\})] \subseteq [Bel_{m_2}(\{\theta_i\}), Pl_{m_2}(\{\theta_i\})]$. Because $0 \le Bel_{m_2}(\{\theta_i\}) \le Bel_{m_1}(\{\theta_i\}) \le Pl_{m_1}(\{\theta_i\}) \le Pl_{m_2}(\{\theta_i\}) \le 1$, so $\forall \theta_i \in \Theta$ we have

$$(Bel_{m_1}(\{\theta_i\}))^2 \ge (Bel_{m_2}(\{\theta_i\}))^2$$
$$(1 - Pl_{m_1}(\{\theta_i\}))^2 \ge (1 - Pl_{m_2}(\{\theta_i\}))^2$$

then,

$$(Bel_{m_1}(\{\theta_i\}))^2 + (1 - Pl_{m_1}(\{\theta_i\}))^2 \ge (Bel_{m_2}(\{\theta_i\}))^2 + (1 - Pl_{m_2}(\{\theta_i\}))^2$$

so,

$$\sqrt{(Bel_{m_1}(\{\theta_i\}))^2 + (1 - Pl_{m_1}(\{\theta_i\}))^2} \ge \sqrt{(Bel_{m_2}(\{\theta_i\}))^2 + (1 - Pl_{m_2}(\{\theta_i\}))^2}$$

namely,

$$d_E^I\left([Bel_{m_1}(\theta_i), Pl_{m_1}(\theta_i)], [0,1]\right) \ge d_E^I\left([Bel_{m_2}(\theta_i), Pl_{m_2}(\theta_i)], [0,1]\right), \forall \theta_i \in \Theta$$

Therefore

$$\sum_{i=1}^{n} d_E^I([Bel_{m_1}(\{\theta_i\}), Pl_{m_1}(\{\theta_i\})], [0,1]) \ge \sum_{i=1}^{n} d_E^I([Bel_{m_2}(\{\theta_i\}), Pl_{m_2}(\{\theta_i\})], [0,1])$$

then,

$$1 - \frac{1}{n}\sum_{i=1}^{n} d_E^I([Bel_{m_1}(\{\theta_i\}), Pl_{m_1}(\{\theta_i\})], [0,1]) \le 1 - \frac{1}{n}\sum_{i=1}^{n} d_E^I([Bel_{m_2}(\{\theta_i\}), Pl_{m_2}(\{\theta_i\})], [0,1])$$

namely,

$$TU_E^I(m_1) \le TU_E^I(m_2)$$

The monotonicity of $TU_E^I$ has been proved.

Now let us numerically illustrate the monotonicity of $TU_E^I$ through Example 1. For these three BPAs

$$m_1(\{\theta_1\}) = 0.4, \quad m_1(\{\theta_2\}) = 0.3, \quad m_1(\{\theta_1, \theta_2\}) = 0.3;$$

$$m_2(\{\theta_1\}) = 0.4, \quad m_2(\{\theta_2\}) = 0.2, \quad m_2(\{\theta_1, \theta_2\}) = 0.4;$$

$$m_3(\{\theta_1\}) = 0.4, \quad m_3(\{\theta_2\}) = 0.0, \quad m_3(\{\theta_1, \theta_2\}) = 0.6.$$

According to the definition of $TU_E^I$, we have $TU_E^I(m_1) = 0.5$, $TU_E^I(m_2) = 0.5528$, $TU_E^I(m_3) = 0.6$. Therefore, the monotonicity of uncertainty among $m_1$, $m_2$, $m_3$ is satisfied.

Besides, in [11] the authors have given many other experiments and simulations to illustrate that $TU^I$ is sensitive to the changes in evidences. In this paper, we have verified these examples 4 - 9 presented in [11]. The results are shown in Figures 1 - 6. From these figures, we can find that the proposed $TU_E^I$ has the good performance as same as $TU^I$, and $TU_E^I$ also has high sensitivity. However, since $TU_E^I$ has the monotonicity which is actually not satisfied in $TU^I$, $TU_E^I$ therefore is better than $TU^I$.

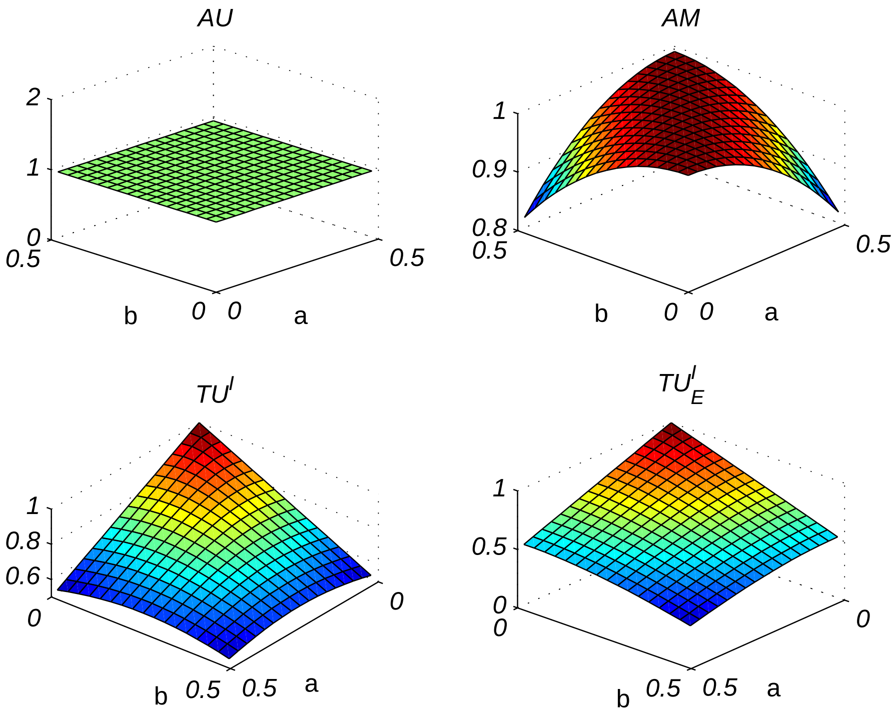


Figure 1: Comparison of different uncertainty measures through Example 4 from [11]

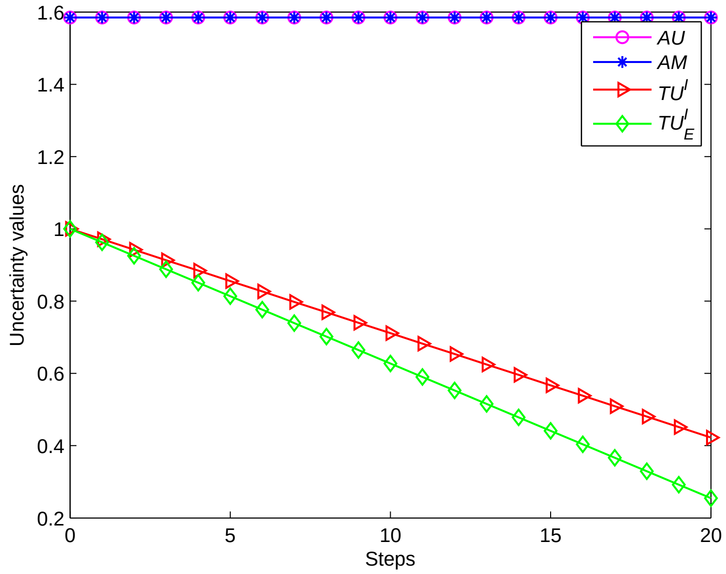


Figure 2: Comparison of different uncertainty measures through Example 5 from [11]

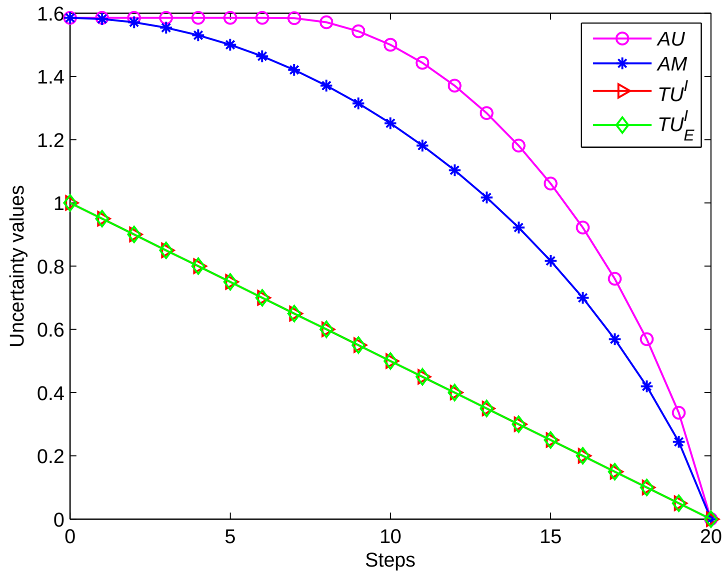


Figure 3: Comparison of different uncertainty measures through Example 6 from [11]

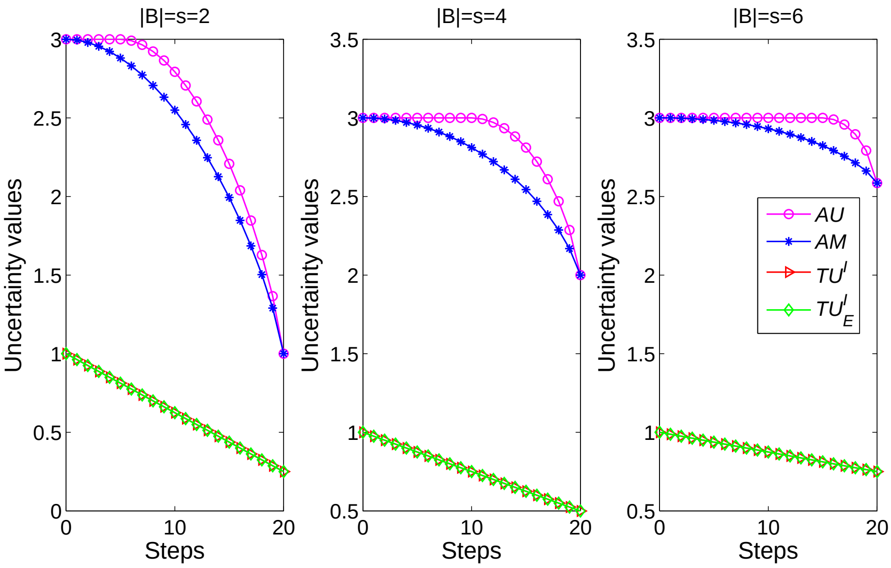


Figure 4: Comparison of different uncertainty measures through Example 7 from [11]

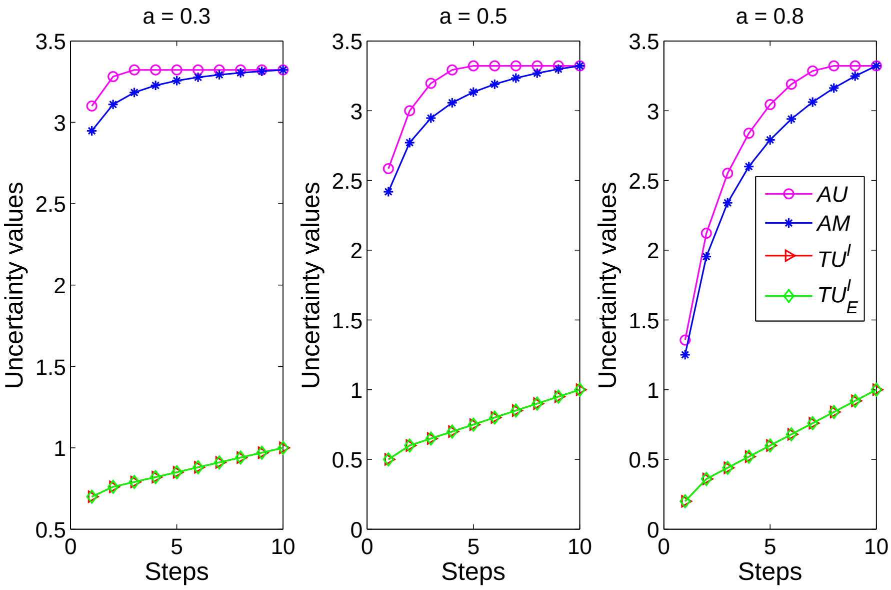


Figure 5: Comparison of different uncertainty measures through Example 8 from [11]

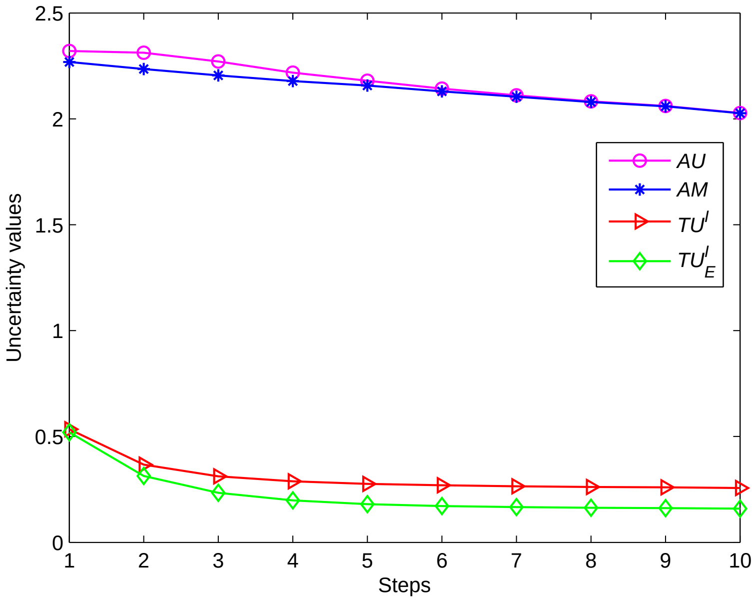


Figure 6: Comparison of different uncertainty measures through Example 9 from [11]

## 5. Conclusion

In this paper, we investigated the issue of uncertainty measuring in belief function theory. Though the uncertainty measure $TU^I$ proposed in [11] has many merits, we have found a counter example to show that the monotonicity for uncertainty measures is violated in $TU^I$ and then theoretically pointed out that $TU^I$ actually does not satisfy the monotonicity. Then, a new uncertainty measure $TU_E^I$ is proposed to improve $TU^I$ in monotonicity. The monotonicity of $TU_E^I$ is mathematically proved. A number of numerical examples have further verified that $TU_E^I$ is also highly sensitive to the changes in evidences. Therefore, the proposed $TU_E^I$ is better than $TU^I$ and could provide much more reasonable results in practical applications.

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