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Degree of Dependence and Independence of the (Sub)Components of Fuzzy Set and Neutrosophic Set

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Abstract. We have introduced for the first time the *degree of dependence* (and consequently the *degree of independence*) between the components of the fuzzy set, and also between the components of the neutrosophic set in our 2006 book's fifth edition [1]. Now we extend it for the first time to the refined neutrosophic set considering the *degree of dependence or independence of subcomponets*.

Keywords: neutrosophy, neutrosophic set, fuzzy set, degree of dependence of (sub)components, degree of independence of (sub)components.

1 Refined Neutrosophic Set.

We start with the most general definition, that of a *n-valued refined neutrosophic set A*. An element x from A belongs to the set in the following way:

$$x\left(T_1, T_2, \dots, T_p;\ I_1, I_2, \dots, I_r;\ F_1, F_2, \dots, F_s\right) \in A, \quad (1)$$

where $p, r, s \geq 1$ are integers, and $p + r + s = n \geq 3$, where

$$T_1, T_2, \dots, T_p;\ I_1, I_2, \dots, I_r;\ F_1, F_2, \dots, F_s \quad (2)$$

are respectively sub-membership degrees, sub-indeterminacy degrees, and sub-nonmembership degrees of element x with respect to the n-valued refined neutrosophic set A.

Therefore, one has n (sub)components.

Let's consider all of them being crisp numbers in the interval $[0, 1]$.

2 General case.

Now, in general, let's consider n crisp-components (variables):

$$y_1, y_2, \dots, y_n \in [0, 1]. \quad (3)$$

If all of them are 100% independent two by two, then their sum:

$$0 \leq y_1 + y_2 + \ \dots + y_n \leq n. \quad (4)$$

But if all of them are 100% dependent (totally interconnected), then

$$0 \leq y_1 + y_2 + \ \dots + y_n \leq 1. \quad (5)$$

When some of them are partially dependent and partially independent, then

$$y_1 + y_2 + \ \dots + y_n \in (1, n). \quad (6)$$

For example, if y_1 and y_2 are 100% dependent, then

$$0 \leq y_1 + y_2 \leq 1, \quad (7)$$

while other variables $y_3, \dots, y_n$ are 100% independent of each other and also with respect to y_1 and y_2, then

$$0 \leq y\_3 + \cdots + y\_n \leq n - 2, \quad (8)$$

thus

$$0 \leq y_1 + y_2 + y_3 + \cdots + y_n \leq n - 1. \quad (9)$$

3 Fuzzy Set.

Let T and F be the membership and respectively the nonmembership of an element $x(T, F)$ with respect to a fuzzy set A, where T, F are crisp numbers in $[0, 1]$.

If T and F are 100% dependent of each other, then one has as in classical fuzzy set theory

$$0 \leq T + F \leq 1. \quad (10)$$

But if T and F are 100% independent of each other (that we define now for the first time in the domain of fuzzy setand logic), then

$$0 \leq T + F \leq 2. \quad (11)$$

We consider that the sum $T + F = 1$ if the information about the components is complete, and $T + F < 1$ if the information about the components is incomplete.

Similarly, $T + F = 2$ for complete information, and $T + F < 2$ for incomplete information.

For complete information on T and F, one has $T + F \in [1, 2]$.

4 Degree of Dependence and Degree of Independence for two Components.

In general (see [1], 2006, pp. 91-92), the sum of two components x and y that vary in the unitary interval [0, 1] is:

$$0 \leq x + y \leq 2 - d^{\circ}(x, y), \tag{12}$$

where $d^{\circ}(x, y)$ is the *degree of dependence* between x and y.

Therefore $2 - d^{\circ}(x, y)$ is the *degree of independence* between x and y.

Of course, $d^{\circ}(x, y) \in [0, 1]$, and it is zero when x and y are 100% independent, and 1 when x and y are 100% dependent.

In general, if T and F are $d\%$ dependent [and consequently $(100 - d)\%$ independent], then

$$0 \leq T + F \leq 2 - d/100. \tag{13}$$

5 Example of Fuzzy Set with Partially Dependent and Partially Independent Components.

As an example, if T and F are 75% (= 0.75) dependent, then

$$0 \leq T + F \leq 2 - 0.75 = 1.25. \tag{14}$$

6 Neutrosophic Set

Neutrosophic set is a general framework for unification of many existing sets, such as fuzzy set (especially intuitionistic fuzzy set), paraconsistent set, intuitionistic set, etc. The main idea of NS is to characterize each value statement in a 3D-Neutrosophic Space, where each dimension of the space represents respectively the membership/truth (T), the nonmembership/falsehood (F), and the indeterminacy with respect to membership/nonmembership (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$ with not necessarily any connection between them.

For software engineering proposals the classical unit interval [0, 1] is used.

For single valued neutrosophic set, the sum of the components (T+I+F) is (see [1], p. 91):

$$0 \leq T+I+F \leq 3, \tag{15}$$

when all three components are independent;

$$0 \leq T+I+F \leq 2, \tag{16}$$

when two components are dependent, while the third one is independent from them;

$$0 \leq T+I+F \leq 1, \tag{17}$$

when all three components are dependent.

When three or two of the components T, I, F are independent, one leaves room for incomplete information (sum < 1), paraconsistent and contradictory information (sum > 1), or complete information (sum = 1).

If all three components T, I, F are dependent, then similarly one leaves room for incomplete information (sum < 1), or complete information (sum = 1).

The dependent components are tied together.

Three sources that provide information on T, I, and F respectively are independent if they do not communicate with each other and do not influence each other.

Therefore, max{T+I+F} is in between 1 (when the degree of independence is zero) and 3 (when the degree of independence is 1).

7 Examples of Neutrosophic Set with Partially Dependent and Partially Independent Components.

The max{T+I+F} may also get any value in (1, 3).

a) For example, suppose that T and F are 30% dependent and 70% independent (hence T + F ≤ 2-0.3 = 1.7), while I and F are 60% dependent and 40% independent (hence I + F ≤ 2-0.6 = 1.4). Then max{T + I + F} = 2.4 and occurs for T = 1, I = 0.7, F = 0.7.

b) Second example: suppose T and I are 100% dependent, but I and F are 100% independent. Therefore T + I ≤ 1 and I + F ≤ 2, then T + I + F ≤ 2.

8 More on Refined Neutrosophic Set

The Refined Neutrosophic Set [4], introduced for the first time in 2013. In this set the neutrosophic component (T) is split into the subcomponents (T_1, T_2, …, T_p) which represent types of truths (or sub-truths), the neutrosophic component (I) is split into the subcomponents (I_1, I_2, …, I_r) which represents types of indeterminacies (or sub-indeterminacies), and the neutrosophic components (F) is split into the subcomponents (F_1, F_2, …, F_s) which represent types of falsehoods (or sub-falsehoods), such that p, r, s are integers ≥ 1 and p + r + s = n ≥ 4. (18)

When n = 3, one gets the non-refined neutrosophic set. All T_j, I_k, and F_l subcomponents are subsets of [0, 1].

Let's consider the case of refined single-valued neutrosophic set, i.e. when all n subcomponents are crisp numbers in [0, 1].

Let the sum of all subcomponents be:

$$S = \sum_{1}^{p} T_j + \sum_{1}^{r} I_k + \sum_{1}^{s} F_l \tag{19}$$

When all subcomponents are independent two by two, then

$$0 \leq S \leq n. \tag{20}$$

If *m* subcomponents are 100% dependent, $2 \leq m \leq n$, no matter if they are among T_j, I_k, F_l or mixed, then

$$0 \leq S \leq n - m + 1 \quad (21)$$

and one has $S = n - m + 1$ when the information is complete, while $S < n - m + 1$ when the information is incomplete.

9 Examples of Refined Neutrosophic Set with Partially Dependent and Partially Independent Components.

Suppose T is split into T_1, T_2, T_3, and I is not split, while F is split into F_1, F_2. Hence one has:

$$\{T_1, T_2, T_3; I; F_1, F_2\}. \quad (22)$$

Therefore a total of 6 (sub)components.

a) If all 6 components are 100% independent two by two, then:

$$0 \leq T_1 + T_2 + T_3 + I + F_1 + F_2 \leq 6 \quad (23)$$

b) Suppose the subcomponets T_1, T_2, and F_1 are 100% dependent all together, while the others are totally independent two by two and independent from T_1, T_2, F_1, therefore:

$$0 \leq T_1 + T_2 + F_1 \leq 1 \quad (24)$$

whence

$$0 \leq T_1 + T_2 + T_3 + I + F_1 + F_2 \leq 6 - 3 + 1 = 4. \quad (25)$$

One gets equality to 4 when the information is complete, or strictly less than 4 when the information is incomplete.

c) Suppose in another case that T_1 and I are 20% dependent, or $d°(T_1, I) = 20\%$, while the others similarly totally independent two by two and independent from T_1 and I, hence

$$0 \leq T_1 + I \leq 2 - 0.2 = 1.8 \quad (26)$$

whence

$$0 \leq T_1 + T_2 + T_3 + I + F_1 + F_2 \leq 1.8 + 4 = 5.8, \quad (27)$$

since $0 \leq T_2 + T_3 + F_1 + F_2 \leq 4$. (28)

Similarly, to the right one has equality for complete information, and strict inequality for incomplete information.

Conclusion.

We have introduced for the first time the degree of dependence/independence between the components of fuzzy set and neutrosophic set. We have given easy examples about the range of the sum of components, and how to represent the degrees of dependence and independence of the components. Then we extended it to the refined neutrosophic set considering the degree of dependence or independence of subcomponets.

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