Scaling and Distance of the Frame of Reference for Lorentz Transformation of Special Relativity Theory
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Abstract
Lorentz transformation handles an oblique frame of reference. Regarding to the oblique system, investigation about scaling and distance is required for clear understanding how the Lorentz transformation can be derived.

1. Introduction
There is a trial to derive Lorentz transformation from basic assumption. [1] In the process, moving system becomes an oblique system on orthogonal system. To complete the process, investigation of scaling of each dimensions and definition of distance for the system is required.

2. Distance of oblique system
Regarding to the Lorentz transformation, frame of reference of moving system is as Fig.1. [1]

![Diagram of oblique frame system](image)

Fig. 1

Here \( x' \) value of point P is \( \overline{PB} \) because \( ct' \)-axis is \( x' = 0 \) line and \( \overline{PB} \) is distance from the line. But \( ct' \) value of P and B is different. This means \( \overline{PB} \) are \( x' \) distance of different time. Space distance should be measured simultaneously and if so, space distance of P should be \( \overline{PA} \) or \( \overline{CO} \).

On similar reason, actual elapse time \( (ct') \) of P should be \( \overline{OA} \) not \( \overline{DP} \). Then

\[
ct' = \frac{\overline{OA}}{\sin \alpha} = \frac{\overline{DP} \cos \theta - x \sin \theta}{\sin \alpha} \tag{1}
\]

\[
x' = \frac{\overline{OC}}{\sin \alpha} = \frac{-ct \sin \theta + x \cos \theta}{\sin \alpha} \tag{2}
\]
3. Experience
If time $t$ passes for space $V$ (length $l$ in the case of two dimensions), integration of $V$ from time 0 to $t$ is defined experience($\text{exp}$). This is $\text{exp} = \int_0^t V dt$
Ten thousand years on Sun is something different from $10 \mu s$ for an atom.
Experience is that something and its value indicate its huge difference in this case.

4. Scaling
In the case of orthogonal system, (Fig.1),
\[ \text{exp} = l \times t \] (3)
In the case of oblique system, (Fig.2),
\[ \text{exp} = l \times t \sin \alpha = l \sqrt{\sin \alpha} \times t \sqrt{\sin \alpha} \] (4)

![Fig. 2](http://example.com/fig2)

![Fig. 3](http://example.com/fig3)

Comparing (3) (4), distance value of oblique system is multiple $\sqrt{\sin \alpha}$ comparing to orthogonal system’s.
This $\sqrt{\sin \alpha}$ is scaling factor for oblique system also for orthogonal system ($\sqrt{\sin \alpha} = 1$).

5. Conclusion
Considering above investigation of scaling and distance, (1) (2) become
\[
ct' = \frac{\text{ct} \cos \theta - x \sin \theta}{\sin \alpha} \sqrt{\sin \alpha} = \frac{ct \cos \theta - x \sin \theta}{\sqrt{\sin \alpha}} \] (5)
\[
x' = \frac{-ct \sin \theta + x \cos \theta}{\sin \alpha} \sqrt{\sin \alpha} = \frac{-ct \sin \theta + x \cos \theta}{\sqrt{\sin \alpha}} \] (6)

This consequence could be a part of process to derive Lorentz transformation from the basic assumption.

Reference