Space Axis of Time·Space Frame of Reference for Special Relativity Theory and The Core of the Theory
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Abstract
It may be hard to understand Lorentz transformation intuitively because it is derived inductively from the light speed constancy for every inertia systems. Specially the definition of space·axis (time zero line) is unclear although time·axis definition may be reasonable if the Lorentz transformation is recognized as a rotation of frame of reference on the Minkowski space. On such situation, Lorentz transformation may become understandable if the logic of space·axis definition becomes clear. And light speed constancy may not be an axiom, it should be what to be derived.

1. Introduction
Here we try to understand Lorentz Transformation from more basic definition. For it, following two systems are considered.

S system(two dimensions(ct, x)) and S’ system(two dimensions(ct’, x’)). [1]

Here both are moving relatively with velocity v.

As we consider Lorentz transformation is rotation of frame of reference on the Minkowski space, detail investigation of space axis definition is required. And the result of the process makes clear the meaning of special relativity theory.

2. Category of Systems
Lorentz transformation expresses a time·space point using two categories of frame of reference. These two categories are following.

Base System: This is static independent system
Secondary System: This is moving relatively to the Basic System and dependent system

We can also say that Lorentz transformation express Secondary System’s frame of reference on Base System’s frame of reference.

These categories are interchangeable, not be a unique feature to a system.
3. Move of time
Here exact definition of words is required.
Space distance : space dimension difference of two points
Time : For any space point, something passing. Its dimension is called time dimension.
And time is time dimension difference of two points
Time distance : space compatible distance converted from time
Time move : There are following two types time move.
  - Time passing move : At any space point, time moves on time-space graph toward
time direction according to time passing.
  - Time-space point move : A time-space point moves on time-space graph toward
time direction according to space move. (following (2))

Following has been assumed. [1]
Time (passing) moves toward time direction also toward space direction with speed c. (1)

Another definition is:
When a time-space point moves toward space direction, its started time-space point
moves same distance toward time direction. (2)
These above means:
For example, when a time-space point (0,0) moves $x$ for time $t$, the time-space point
moves to $(ct, x)$ finally. Here $ct$ is time passing move. $x$ is space move.
In this case, also the time-space point moves to $(x, ct)$ finally. $ct$ is time passing move
toward space direction on (1). $x$ is time-space point move on (2).
Here mass point at (0,0) moves $x$ toward space also the time-space point (0,0) moves
$x$ toward time for $ct$.

4. Two types of distance
On graph, value of variable is presented by distance from origin. There are two types of
the distance. These are time type and space type.
Time type presents time distance. It means time passing.
Space type presents space distance. It means space distance of two space points at a
time.

5. Requirement for time-space graph rotation
Followings are required so that time-space graph can be rotated.
  - Time axis and space axis should be same unit or same dimension.
About this, time axis have value $ct$ in order that time axis have same dimension as space.[1]

- Time axis and space axis should be same distance type.

About this, because time axis need to represent time passing, so its type cannot be changed. Then space axis should be recognized as time type. So space value $x$ means time $\frac{x}{c}$ passing.

6. Space axis definition

About Fig. 1, origin of $S$ system is $O$, origin of $S'$ system is $O'$. Space point $X$ is away $x_1$ from origin $O$ and $O'$. Time point $T$ is away $ct_1$ from origin $O$ and $O'$.

Regarding to time axis, as describe in [1], it takes (passing)time distance $ct_1$ for time to move from origin to point $T$.

While the timing, origin $O'$ moves $vt_1$ toward space direction.

This relation is $x = \frac{v}{c}ct$. This is space zero line or time axis for $S'$ system.

Same as above, it takes (passing)time distance $x_1$ for time to move from origin to space point $X$. While the timing, origin $O'$ moves $\frac{v}{c}x_1$ toward space direction also time direction. This relation is

$$ct = \frac{v}{c}x$$

$$x = \frac{c}{v}ct.$$  \hspace{1cm} (3)

This is time zero line or space axis for $S'$ system.
7. Light speed constancy

When light moves from space 0 to \( x \) for \( t \) on Base System,

For Base System

Moved distance of light: \( x = ct \) \hspace{1cm} (4)

For Secondary system

Moved distance of light: \( ct - vt = (c - v)t = (c - v)\frac{x}{c} \) \hspace{1cm} (5)

Passed time: \( (c - v)\frac{x}{c^2} \) \hspace{1cm} (6)

Light speed of \( S \) system is

\[ \frac{c}{\epsilon} = c \] \hspace{1cm} (7)

Light speed of \( S' \) system is from (5)(6)

\[ \frac{(c-v)\frac{x}{c}}{c^2} = c \] \hspace{1cm} (8)

Here moved distance depends on each system. Elapsed time also depends on each system. Then light speed which is ratio of space distance and time distance is constant or doesn’t depend on system.

8. Relativity

Lorentz transformation expresses relation of two systems on Base System.

But actually each system observes own system as it is Base System.

On Fig. 2, light flushes at the origin of \( S \) system. It moves according to time \( t_1, t_2, t_3 \ldots \)

If \( S' \) system moves with speed \( v \) relatively to \( S \) system, \( S' \) system seems to observe the light as on Fig.2.

But actually time point moves same as light do. At point P for \( S' \) system light moved \( (c-v)t_3 \).
elapsed time is $\frac{c-v}{c}t_3$.

Then when time become $t_3$ for $S'$ system, distance light moves is

$$(c-v)t_3 \times \frac{t_3}{c} = ct_3$$

(9)

This is same distance light moves for $t_3$ on $S$ system.

Fig.3 reflects this result.

For $S'$ system, light flushes at the origin and moves to all directions with speed $c$. This is just same scene as $S$ system views. This means relatively moving two systems observe same scene regarding to light propagation.

Here the assumption that $S'$ system is moving to $S$ system, is disappeared. We cannot say $S'$ system is moving nor $S$ system is moving. We can say only these are moving relatively.

9. Conclusion

On the assumption (1)(2), following are derived.

- space axis definition for Lorentz transformation on the Minkowski space
- light speed constancy
- meaning of relativity for inertia systems

Reference