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The Electromagnetic Wave in a Spherical Capacitor and the Nature of Earth Magnetism

Abstract

A solution of the Maxwell equations for the electromagnetic wave in a spherical capacitor which is included in an alternating current circuit or in an constant current circuit is proposed. A hypothesis of the Earth magnetism nature is presented on the basis of this solution.

Contents

1. Introduction

The electromagnetic wave in a capacitor in an alternating current or constant current circuit is investigated in [1, 2]. In this paper, a spherical capacitor in a sinusoidal current circuit or an constant current circuit is considered. The capacitor electrodes are two spheres having the same center and radii $R_2 > R_1$. A hypothesis of the Earth magnetism nature is proposed on the basis of this solution. A model of the ball lightning was substantiated previously in a similar manner [3].

2. Solution of the Maxwell Equations in the Spherical Coordinate System

Let us first consider a spherical capacitor in a sinusoidal current circuit. Fig. 1 shows the spherical coordinate system (ρ, θ, φ). Expressions for the rotor and the divergence of vector **E** in these coordinates are given in Table 1 [4]. The following nomenclature is used below in this paper

E - electrical intensities,

H - magnetic intensities,

μ - absolute magnetic permeability,
ε - absolute dielectric constant.

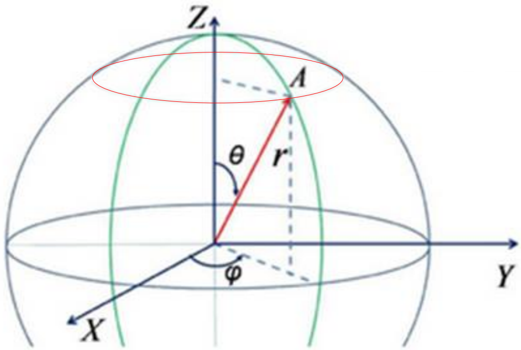

Fig. 1.

Table 1.

1	2	3
1	$\mathrm{rot}_{\rho}(E)$	$\frac{E_{\varphi}}{\rho \mathrm{tg}(\theta)} + \frac{\partial E_{\varphi}}{\rho \partial \theta} - \frac{\partial E_{\theta}}{\rho \sin(\theta) \partial \varphi}$
2	$\mathrm{rot}_{\theta}(E)$	$\frac{\partial E_{\rho}}{\rho \sin(\theta) \partial \varphi} - \frac{E_{\varphi}}{\rho} - \frac{\partial E_{\varphi}}{\partial \rho}$
3	$\mathrm{rot}_{\varphi}(E)$	$\frac{E_{\theta}}{\rho} + \frac{\partial E_{\theta}}{\partial \rho} - \frac{\partial E_{\rho}}{\rho \partial \varphi}$
4	$\mathrm{div}(E)$	$\frac{E_{\rho}}{\rho} + \frac{\partial E_{\rho}}{\partial \rho} + \frac{E_{\theta}}{\rho \mathrm{tg}(\theta)} + \frac{\partial E_{\theta}}{\rho \partial \theta} + \frac{\partial E_{\varphi}}{\rho \sin(\theta) \partial \varphi}$

With no charge on and no current between the spherical capacitor electrodes, the Maxwell equations in the spherical coordinate system take the form presented in Table 2.

Table 2.

1	2
1.	$\mathrm{rot}_{\rho} H - \frac{\varepsilon}{c} \frac{\partial E_{\rho}}{\partial t} = 0$
2.	$\mathrm{rot}_{\theta} H - \frac{\varepsilon}{c} \frac{\partial E_{\theta}}{\partial t} = 0$
3.	$\mathrm{rot}_{\varphi} H - \frac{\varepsilon}{c} \frac{\partial E_{\varphi}}{\partial t} = 0$

4.	$\mathrm{rot}_\rho E + \frac{\mu}{c}\frac{\partial H_\rho}{\partial t} = 0$
5.	$\mathrm{rot}_\theta E + \frac{\mu}{c}\frac{\partial H_\theta}{\partial t} = 0$
6.	$\mathrm{rot}_\varphi E + \frac{\mu}{c}\frac{\partial H_\varphi}{\partial t} = 0$
7.	$\mathrm{div}(E) = 0$
8.	$\mathrm{div}(H) = 0$

Below the solution will be sought for in form of functions $E,\ H$, which presented in Table. 3, where the functions of the form $E_{\varphi\rho}(\rho)$ to be calculated. It is important to note that

- these functions are independent of the argument φ;
- if $E(\theta) = \sin(\theta)$, then

$$\frac{E}{\mathrm{tg}(\theta)} + \frac{\partial E}{\partial \theta} = 2\cos(\theta). \tag{11}$$

Table 3.

1	2
	$E_\rho = e_\rho\ E_{\rho\rho}(\rho)\cos(\theta)\sin(\omega t)$
	$E_\theta = e_\theta E_{\theta\rho}(\rho)\sin(\theta)\sin(\omega t)$
	$E_\varphi = e_\varphi E_{\varphi\rho}(\rho)\sin(\theta)\sin(\omega t)$
	$H_\rho = h_\rho\ H_{\rho\rho}(\rho)\cos(\theta)\cos(\omega t)$
	$H_\theta = h_\theta E_{\theta\rho}(\rho)\sin(\theta)\cos(\omega t)$
	$H_\varphi = h_\varphi H_{\varphi\rho}(\rho)\sin(\theta)\cos(\omega t)$

We substitute the functions $E,\ H$ from the Table 3 in Table 1 and take into account (11). Then we obtain Table 4.

Table 4.

1	2	3
1	$\mathrm{rot}_\rho(E)$	$\frac{2E_{\varphi\rho}}{\rho}\cos(\theta)\ \sin(\omega t)$

2	$\mathrm{rot}_\theta(E)$	$-\left(\frac{E_\varphi}{\rho}+\frac{\partial E_\varphi}{\partial\rho}\right)\sin(\theta)\ \sin(\omega t)$
3	$\mathrm{rot}_\varphi(E)$	$\left(\frac{E_\theta}{\rho}+\frac{\partial E_\theta}{\partial\rho}\right)\sin(\theta)\ \sin(\omega t)$
4	$\mathrm{div}(E)$	$\left(\left(\frac{E_\rho}{\rho}+\frac{\partial E_\rho}{\partial\rho}\right)+\frac{2E_{\theta\rho}}{\rho}\right)\cos(\theta)\ \sin(\omega t)$

Expressions for the rotor and divergence function H differ from those shown in the Table. 4 only in that instead of factors $\sin(\omega t)$ are factors $\cos(\omega t)$.

Substituting the expression for the curl and divergence in Maxwell's equations (see Table 2), differentiating with respect to time and reducing common factors, we obtain a new form of Maxwell's equations - see Table. 5.

Table 5.

1	**2**
1	$\frac{2E_{\varphi\rho}}{\rho}-\frac{\omega\mu}{c}H_{\rho\rho}=0$
2	$-\left(\frac{E_{\varphi\rho}}{\rho}+\frac{\partial E_{\varphi\rho}}{\partial\rho}\right)-\frac{\omega\mu}{c}H_{\theta\rho}=0$
3	$\left(\frac{E_{\theta\rho}}{\rho}+\frac{\partial E_{\theta\rho}}{\partial\rho}\right)-\frac{\omega\mu}{c}H_{\varphi\rho}=0$
4	$\left(\left(\frac{E_{\rho\rho}}{\rho}+\frac{\partial E_{\rho\rho}}{\partial\rho}\right)+\frac{2E_{\theta\rho}}{\rho}\right)=0$
5	$\frac{2H_{\varphi\rho}}{\rho}-\frac{\omega\varepsilon}{c}E_{\rho\rho}=0$
6	$-\left(\frac{H_{\varphi\rho}}{\rho}+\frac{\partial H_{\varphi\rho}}{\partial\rho}\right)-\frac{\omega\varepsilon}{c}E_{\theta\rho}=0$
7	$\left(\frac{H_{\theta\rho}}{\rho}+\frac{\partial H_{\theta\rho}}{\partial\rho}\right)-\frac{\omega\varepsilon}{c}E_{\varphi\rho}=0$
8	$\left(\left(\frac{H_{\rho\rho}}{\rho}+\frac{\partial H_{\rho\rho}}{\partial\rho}\right)+\frac{2H_{\theta\rho}}{\rho}\right)$

3. The solution of Maxwell's equations for the vacuum

First, we consider the equations for a vacuum where in the GHS system

$$\varepsilon = \mu = 1. \tag{12}$$

Then Maxwell's equations are completely symmetrical with respect to the intensities E and H. Find the sum pairs of (1-4) and (5-8). Then we get:

$$\frac{2W_{\varphi\rho}}{\rho} - \frac{\omega}{c} W_{\rho\rho} = 0, \tag{13}$$

$$\left(\frac{W_{\varphi\rho}}{\rho} + \frac{\partial W_{\varphi\rho}}{\partial \rho}\right) + \frac{\omega}{c} W_{\theta\rho} = 0, \tag{14}$$

$$\left(\frac{W_{\theta\rho}}{\rho} + \frac{\partial W_{\theta\rho}}{\partial \rho}\right) - \frac{\omega}{c} W_{\varphi\rho} = 0, \tag{15}$$

$$\left(\left(\frac{W_{\rho\rho}}{\rho} + \frac{\partial W_{\rho\rho}}{\partial \rho}\right) + \frac{2W_{\theta\rho}}{\rho}\right) = 0, \tag{16}$$

where

$$W = E + H,\ E = H = \frac{W}{2}. \tag{17}$$

The same equations (14-16) are obtained by

$$W = E\sin(\beta) + H\cos(\beta),\ |E| = |H| = \frac{|W|}{2}. \tag{17a}$$

The system of 4 equations (13-16) defines 3 unknown functions - the system is overdetermined. We show that there is a solution that satisfies all equations

Direct substitution can be seen that the equations (14, 15) has the following solution:

$$W_{\varphi\rho} = A \cdot \frac{1}{\rho} \sin\left(\frac{\omega}{c}(\rho - R)\right), \tag{18}$$

$$W_{\theta\rho} = -A \cdot \frac{1}{\rho} \cos\left(\frac{\omega}{c}(\rho - R)\right), \tag{19}$$

where $A,\ R$ - constants. Find the sum of equation (13, 16). Then we get:

$$\frac{W_{\rho\rho}}{\rho} + \frac{\partial W_{\rho\rho}}{\partial \rho} - \frac{\omega}{c} W_{\rho\rho} + \frac{2W_{\theta\rho}}{\rho} + \frac{2W_{\varphi\rho}}{\rho} = 0 \tag{20}$$

or, taking into account (18, 19),

$$\frac{W_{\rho\rho}}{\rho}+\frac{\partial W_{\rho\rho}}{\partial\rho}-\frac{\omega}{c}W_{\rho\rho}+\frac{2A}{\rho}\begin{pmatrix}\sin\left(\frac{\omega}{c}(\rho-R)\right)- \\ -\cos\left(\frac{\omega}{c}(\rho-R)\right)\end{pmatrix}=0. \qquad (21)$$

Direct substitution can be seen that equation (21) has the following solution:

$$W_{\rho\rho}=\frac{2A}{\rho}\left(\cos\left(\frac{\omega}{c}(\rho-R)\right)-\sin\left(\frac{\omega}{c}(\rho-R)\right)\right) \qquad (22)$$

or

$$W_{\rho\rho}=\frac{2\sqrt{2}A}{\rho}\sin\left(\frac{\pi}{4}-\frac{\omega}{c}(\rho-R)\right) \qquad (22)$$

Thus, the solution of Maxwell's equations for the spherical vacuum capacitor has the form of equations (18, 19, 22, 17a) or

$$H_{\rho\rho}\cos(\beta)=E_{\rho\rho}\sin(\beta)=\frac{\sqrt{2}A}{\rho}\sin\left(\frac{\pi}{4}-\frac{\omega}{c}(\rho-R)\right) \qquad (23)$$

$$H_{\varphi\rho}\cos(\beta)=E_{\varphi\rho}\sin(\beta)=\frac{A}{2\rho}\sin\left(\frac{\omega}{c}(\rho-R)\right), \qquad (24)$$

$$H_{\theta\rho}\cos(\beta)=E_{\theta\rho}\sin(\beta)=-\frac{A}{2\rho}\cos\left(\frac{\omega}{c}(\rho-R)\right). \qquad (25)$$

To find all these functions, it suffices to know the values of constants A, R. This solution means that **an electromagnetic wave does exist in the spherical capacitor in a sinusoidal current circuit.**

4. Electric and magnetic intensities

Let us consider a point T with coordinates φ, θ on a sphere of radius ρ .Vectors E_φ and E_θ, going from this point are in plane P, tangent to this sphere at point $T(\varphi,\ \theta)$ - see Fig. 3. These vectors are perpendicular to each other. Hence, at each point $(\varphi,\ \theta)$ the sum vector

$$\vec{H}_{\varphi\theta}=\vec{H}_\varphi+\vec{H}_\theta \qquad (27)$$

is in plane P and has an angle of $(\psi+\beta)$ to a meridian line (where φ - constant). As it follows from (24, 25) and the Table. 3, the module of this vector and the angle ψ defined by the following formulas:

$$\left|\vec{H}_{\varphi\rho}\right| = \frac{A}{2\rho}\sin(\theta)\cos(\beta) \tag{28}$$

$$\cos(\psi) = \frac{H_{\varphi\rho}}{\left|\vec{H}_{\varphi\rho}\right|} = \frac{A}{2\rho}\sin\left(\frac{\omega}{c}(\rho - R)\right) \Big/ \frac{A}{2\rho} = \sin\left(\frac{\omega}{c}(\rho - R)\right)$$

or

$$\psi = \frac{\pi}{2} - \frac{\omega}{c}(\rho - R). \tag{29}$$

We also note for further that the module of vector $\vec{H}_\rho$, as follows from (23) and Table. 3, defined by the following formula:

$$\left|\vec{H}_\rho\right| = \frac{\sqrt{2}A}{\rho}\sin\left(\frac{\pi}{4} - \frac{\omega}{c}(\rho - R)\right)\cos(\theta)\cos(\beta) \tag{30}$$

Similarly, the same relationships exist for the vectors $\vec{E}_\varphi$ and $\vec{E}_\theta$ (only necessary $\cos(\beta)$ are replaced by $\sin(\beta)$). The angle between these vectors in the plane P is straight.

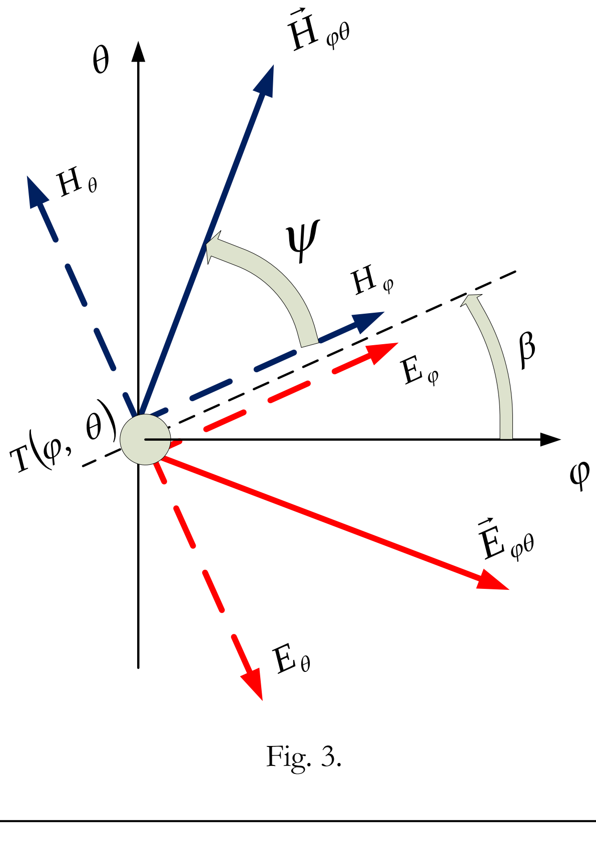

Fig. 3.

In Fig. 3a shows the vectors $\vec{H}_{\varphi\theta}$ and $\vec{E}_{\varphi\theta}$ lying in the plane P, and vectors $\vec{H}_{\rho}$ and $\vec{E}_{\rho}$ lying on a radius.

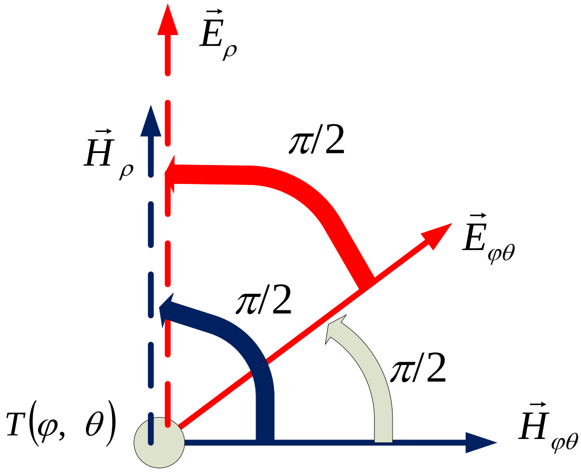

Fig. 3a.

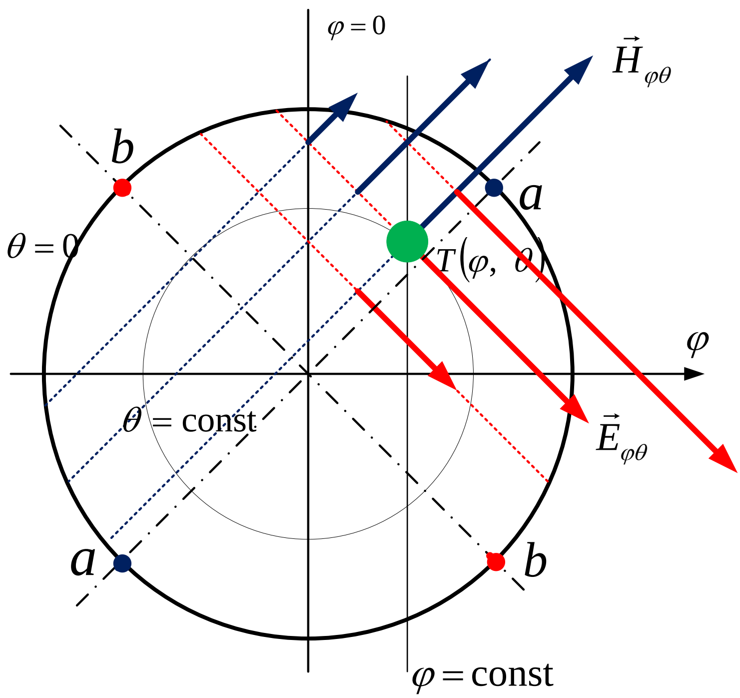

Fig. 4.

Fig. 4 shows the projection of the "Northern hemisphere" with the "Equator" where $\theta = 0$, and the "principal meridian", where $\varphi = 0$. There are highlighted in this figure point $T(\varphi,\ \theta)$, a circle with $\theta = \text{const}$ and a meridian with $\varphi = \text{const}$ both passing through this point. Vectors $\vec{E}_{\varphi\theta}$ and $\vec{H}_{\varphi\theta}$ originating at this point correspond to vectors of the same name shown in Fig. 3. Other vectors parallel to vectors in Fig. 3 are also presented. All these vectors lie on loxodromes (i.e. lines crossing all the meridians at the same angle). Specifically, all the vectors $\vec{E}_{\varphi\theta}$ are found on E-loxodromes with an angle of $(\pi/2 - \psi + \beta)$ and all the vectors $\vec{H}_{\varphi\theta}$ on H-loxodromes with an angle of $(\psi + \beta)$. The modules of vectors $\vec{E}_{\varphi\theta}$ and $\vec{H}_{\varphi\theta}$ vary in proportion to $\sin(\theta)$. These vectors are zero at $\theta = (0;\ \pi)$.

All the loxdromes pass through two poles of this sphere. To aid the visualization, an H-loxodrome is presented in Fig. 5. Fig. 4 shows the diameters aa and bb cconnecting the poles of H-loxodromes and E-loxodromes. These diameters will be called the magnetic axis and the electrical axis, respectively. The points of intersection of these axes with the external sphere will be called the magnetic pole and the electrical pole, respectively. It should be noted that these axes and, in general, vectors $\vec{E}_{\varphi\theta}$ and $\vec{H}_{\varphi\theta}$ are perpendicular.

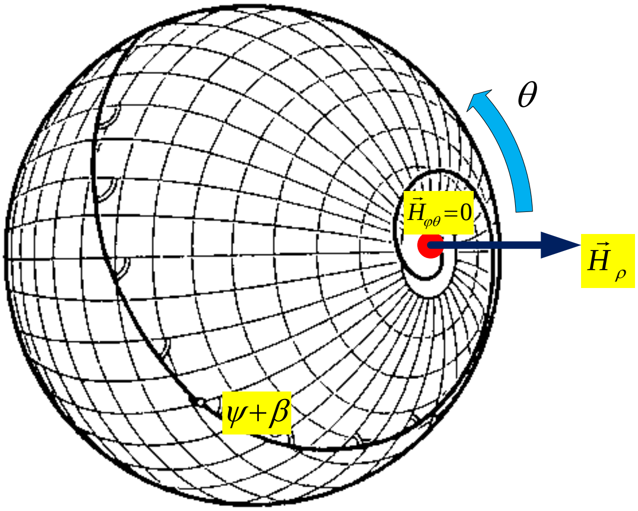

Fig. 5.

Therefore, in a spherical capacitor we can consider only one vector of the electrical field intensities $\vec{E}_{\varphi\theta}$ and only one vector of the magnetic field intensities $\vec{H}_{\varphi\theta}$. As these vectors lie on the sphere, they will be called spherical vectors.

Let us now consider the vectors of the radial field intensities components $\vec{E}_\rho$ and $\vec{H}_\rho$. They are independent of $\cos(\theta)$ - see Table 3. Hence, there are only radial field intensities components at point where the spherical components are zero. The point highlighted in Fig. 5 is the magnetic pole with $\theta = 0$ and $\vec{H}_{\varphi\theta} = 0$.

5. An Electromagnetic Wave in a Charged Spherical Capacitor

A solution of the Maxwell equations for a parallel-plate capacitor being charged [2] stems from a solution of these equations for a parallel-plate capacitor in a sinusoidal current circuit [1]. In this paper the method described in [1] will be used in solving the Maxwell equations for a spherical capacitor being charged.

Let us consider the field intensities in the form of functions presented in Table 6. These functions differ from functions of Table 3 only by the type of time dependence: in Table 3, E and H functions depend on time as $\sin(\omega t)$, $\cos(\omega t)$, respectively, while in Table 6, E and H functions depend on time as $(1-\exp(\omega t))$, $(\exp(\omega t)-1)$, respectively.

Although the indicated substitution, the solution of Maxwell's equations remain unchanged.

Table 6.

1	2
	$E_\rho = e_\rho\, E_{\rho\rho}(\rho)\cos(\theta)(1-\exp(\omega t))$
	$E_\theta = e_\theta E_{\theta\rho}(\rho)\sin(\theta)(1-\exp(\omega t))$
	$E_\varphi = e_\varphi E_{\varphi\rho}(\rho)\sin(\theta)(1-\exp(\omega t))$
	$H_\rho = h_\rho\, H_{\rho\rho}(\rho)\cos(\theta)(\exp(\omega t)-1)$
	$H_\theta = h_\theta E_{\theta\rho}(\rho)\sin(\theta)(\exp(\omega t)-1)$
	$H_\varphi = h_\varphi H_{\varphi\rho}(\rho)\sin(\theta)(\exp(\omega t)-1)$

Bias Current

$$J_z = \frac{d}{dt} E_z = -\omega \cdot e_\rho v(\rho) \cdot \exp(\omega t) \tag{31}$$

Fig. 6 presents intensities components and their time derivatives as well as the bias current as a function of time for $\omega = -300$: H_z is shown with a solid line, E_z with a dashed line, and J_z with dotted line. It is evident that with $t \Rightarrow \infty$ the amplitudes of all intensities components tend to a constant together, while the current amplitude approaches zero. This corresponds to the capacitor charging via a fixed resistor.

When the capacitor becomes fully charged, the current stops to flow. However, the stationary flow of the electromagnetic energy is maintained according to [2].

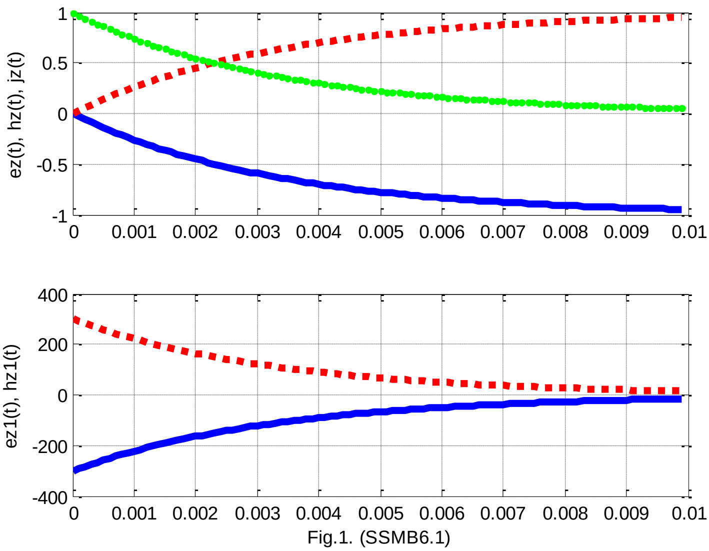

Fig. 6

Thus, the solution of the Maxwell equations for a capacitor being charged and the solution for a capacitor in a sinusoidal current circuit differ only in that the former includes exponential time functions while the latter contains sinusoidal time functions.

The electromagnetic wave structure remains the same - see Section 3. It is evident from Section 3 that there is an electromagnetic wave in a spherical capacitor with only spherical vectors $\vec{E}_{\varphi\theta}$, $\vec{H}_{\varphi\theta}$ and radial vectors $\vec{E}_\rho$, $\vec{H}_\rho$. Fig. 7 shows the mathematical *mm*, the magnetic *aa*, and the electrical bb axes of the capacitor. These axes are perpendicular to each other.

Thus, we can say that the spherical capacitor is a device equivalent to the magnet and simultaneously electrets which are perpendicular to each other.

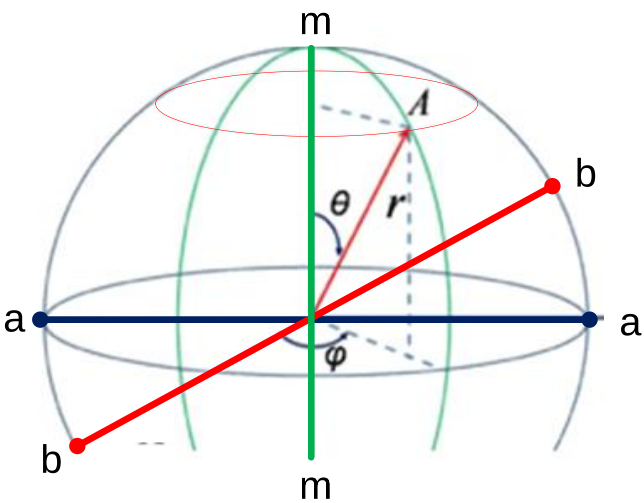

Fig. 7.

6. The Magnetic and the Electrical Field of the Earth

It is known that the Earth electrical field can be considered as a field "between spherical capacitor electrodes" [5]. These electrodes are the Earth surface having a negative charge and the ionosphere having a positive charge. The charge of these electrodes is maintained by continuous atmospheric thunderstorm activities.

It is also known that there is the Earth magnetic field. However, in this case no generally accepted explanation of the source of this field is available. "The problem of the origin and retaining of the field has not been solved as yet." [7].

It flows from the above mentioned that **the Earth electrical field is responsible for the Earth magnetic field.**

Let us consider this problem in more details.

The vector field $\vec{H}_{\varphi\theta}$ in a diametral plane passing through the magnetic axis is shown in Fig. 8. Here, $\left|\vec{H}_{\varphi\theta}\right| = 0.7;\ \rho = 1.$ The vector field $\vec{H}_\rho$ in a diametral plane passing through the magnetic axis is shown in Fig. 9. Here, $\left|\vec{H}_\rho\right| = 0.4;\ \rho = 1.$ The vector field $\vec{H} = \vec{H}_{\varphi\theta} + \vec{H}_\rho$ in a diametral plane passing through the magnetic axis is shown in Fig. 10. Here, $\left|\vec{H}_{\varphi\theta}\right| = 0.3;\ \left|\vec{H}_\rho\right| = 0.2;\ \rho = 1.$

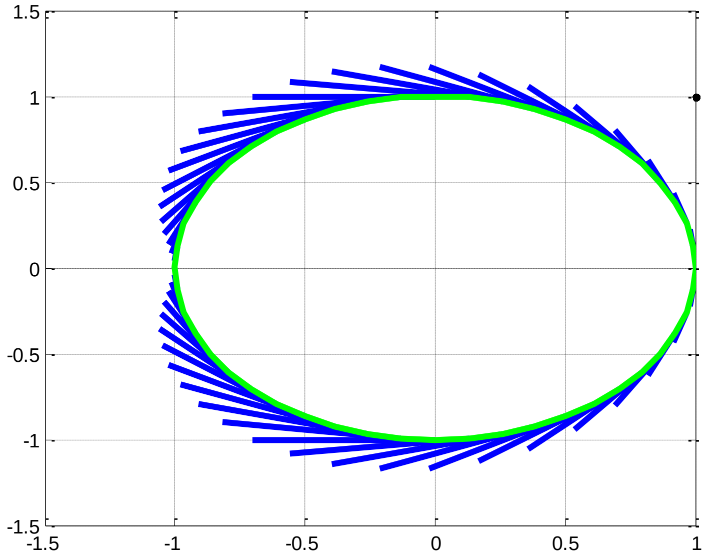

FIG. 8. (Sfera.88)

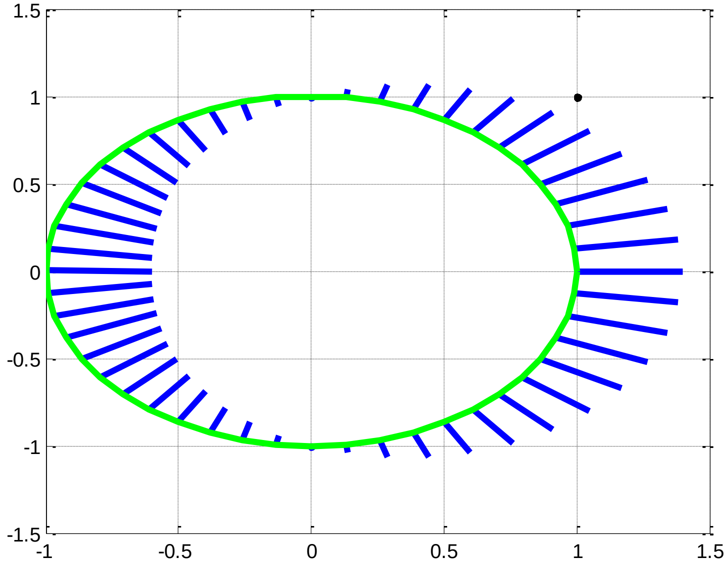

FIG. 9. (Sfera.88)

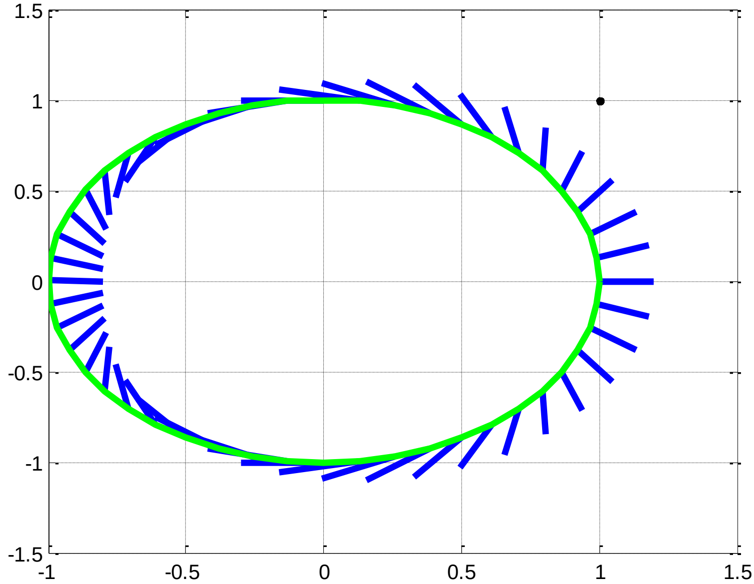

FIG. 10. (Sfera.88)

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