E-bots vs. P-bots: Cooperative Eavesdropping in (partial) Silence Technichal Report

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Notations 1. *$r_e \in \mathbb{N}$ is the eavesdropping distance of an E-bot; we refer to the area within r_e around the target as the sensitive area. Let $dist_G(v, w)$ denote the length of the shortest path between $v, w \in V$; we assume G is an undirected 4-connected grid; therefore, $dist_G$ is simply Manhattan distance. $Ring_G(p, k)$ denotes points with distance $k \in \mathbb{N}$ from point $p \in G$. p_a, p_c, p_p are the capture probabilities by P-bots that are allocated to the roles* Area Patrol , Circumference Patrol *and* Pursuit *respectively, and we assume $0 < p_a \leq p_c$. P-bots focus on protecting the target point t. We strictly assume all data is flushed immediately to the sink after the E-bot escapes from the sensitive area. η is the amount of E-bots, p_d is the transmission-detection probability and $R : \mathbb{N} \to (0, 1]$ is a nonincreasing function which is the reward given to E-bots for a data item which reaches the sink x rounds after it was eavesdropped. We use $R_n = \sum_{i=1}^{n} R(i)$ to denote the reward given for the n latest consecutively-eavesdropped units. For an E-bot that uses the crawling E-bot strategy, and spends l rounds within the sensitive area each time it enters, we denote with $u(l)$ the expected reward it gains before being captured. $C(l)$ denotes the probability that it will be captured before exiting the sensitive area. We use $l_{escape} = \arg\max_{l \in \mathbb{N}} u(l)$. Let x be the reward given for data that reached the sink exclusively by flush. $E_{escape}(x, p_a, p_c, R)$, abbreviated to $E_{escape}(x)$), is the lower bound on the expected captured E-bots before then. Let k be the reward given for leaked data units that reached the sink exclusively from inside the sensitive area (by transmission). $E_{stay}(k, p_a, p_p, p_d, R)$, abbreviated to $E_{stay}(k)$, denotes the lower bound on the expected captured E-bots by the time they received the reward. If in all the transmissions the E-bots transmitted n units simultaneously, $E^n_{stay}(k, p_a, p_p, p_d, R)$, abbreviated to $E^n_{stay}(k)$, denotes the same.*

Lemma 1. *1. $C(x)$ may be bounded as follows:*

$$C(x) \geq \begin{cases} p_c & x = 1 \\ (1 - (1 - p_c)^2 (1 - p_a)^{x-2}) & o.w. \end{cases}$$

2. $E_{escape}(x, p_a, p_c, R) \geq \frac{x}{R_{l_{escape}}} \cdot \frac{C(l_{escape})}{1 - C(l_{escape})}$

Proof. 1. Consider an E-bot that uses the crawling strategy exclusively. Let l be the length of a particular visit in the sensitive area, which is also the number of data items collected - if the E-bot is not captured. The accumulated data units are necessarily unique, since that E-bot collects data only when no other E-bot is active. Hence, the $\frac{1}{C(l)}$ is the expected number of rounds the E-bot repeats the process until it is captured (Binomial distribution), and the expected reward is: $u(l) \equiv \frac{R_l}{C(l)} - R_l = \frac{cl}{C(l)} - cl$.

- For $l = 1$: the E-bot necessarily visited and immediately escaped a point in $Ring_G(t, r_e)$. Upon escaping, flushing the data do not increase capture probability, and therefore $C(1) = p_c$. Note that if an E-bot remains in $Ring_G(t, r_e)$ for $l = 2$, it risks losing the data it accumulated in the first round, and therefore such a strategy provides no benefit.
- For $l > 2$: the E-bot has the opportunity to occupy points in $Ring_G(t, 0 < i < r_e)$ (excluding the first and last rounds), thus reducing the capture probability for some of the rounds. Therefore: $C(l) = 1 - (1 - p_c)^2(1 - p_a)^{l-2}$.

2. An E-bot that transmits from within the sensitive area does not increase the amount of unique accumulated data (and potentially only decreases it), and does not contribute to transmissions from outside the sensitive area. Additionally, by design of the P-bots in this strategy transmissions may not decrease the probability of the E-bot for being captured. The expected number of transmitted data from outside the sensitive area until an E-bot gets captured $u(l) = R_l(\frac{1}{C(l)} - 1)$ is maximized for $l = l_{escape}$. That is, $E_{escape}(\frac{R_{l_{escape}}}{C(l_{escape})} - R_{l_{escape}}) \geq 1$ holds, and due to the linearity of expected value $E_{escape}(l) \geq l \frac{1}{\frac{R_{l_{escape}}}{C(l_{escape})} - R_{l_{escape}}} = l \frac{C(l_{escape})}{R_{l_{escape}}(1 - C(l_{escape}))}$ follows.

$\square$

Lemma 2. $E^n_{stay}(l) \geq (n - \frac{(-1+p_a)((1-p_a)^n-1)}{p_a} + p_p)(\frac{l}{nR_n})$.

Proof. Consider an E-bot that exclusively uses the transmitting strategy. Since only one unique data unit is generated in each round, the E-bot that transmitted the oldest data unit had stayed for at least n rounds, at least one other E-bot had stayed for $n-1$, another for $n-2$ and so forth. Accordingly, the independent risk each E-bot takes is at least $1 - (1 - p_a)^n$, $1 - (1 - p_a)^{n-1}$,$\ldots$, $1 - (1 - p_a)$, which is summed up to $n - (1 - p_a)\frac{(1-p_a)^n-1}{(1-p_a)-1} = n - \frac{(-1+p_a)((1-p_a)^n-1)}{p_a}$. After the transmission of the n units, the pursuit algorithm was invoked and targeted one of the transmitting E-bots that was not yet captured. That is, after any transmission an additional risk of p_p follows for some agent. Therefore, for a reward of R_n, $n - \frac{(-1+p_a)((1-p_a)^n-1)}{p_a}$ E-bots are expected to be captured before the transmissions begin, and additional p_p immediately in the next round. Similarly to the previous lemma, due to the linearity of expected value, $E^n_{stay}(n) = \frac{(n - \frac{(-1+p_a)((1-p_a)^n-1)}{p_a} + p_p)}{R_n}$ and $E^n_{stay}(l) = (n - \frac{(-1+p_a)((1-p_a)^n-1)}{p_a} + p_p)(\frac{l}{nR_n})$

(note that we disregard the option of leaving the sensitive area while transmitting, since this is considered flushing the data). $\square$

Theorem 1. *The expected reward of the E-bots is bounded from above by:* $\frac{\eta}{\min(E_{escape}(1), E_{stay}(1))}$.

Proof. This follows directly from the previous lemmas, since in every round only a single new unique data unit is generated in each round. For any joint strategy (that involves all E-bots), consider all events in which reward is given to evaders is either by n simulteneous transmissions or by flush and the events of an evader entering/existing the sensitive area, until all E-bots are captured. If data reaches the sink by both flush and transmissions, this implies that either duplicated or old data was collected. Therefore, we compare any possible list of events to an alternative reordered list in which the same amount of data units were collected, but all transmissions occur continuously and exclusively at the first rounds, and flush operations occur continuously and exclusively at the latter rounds of the game. Data units that reached the sink more than once are considered in first time only. For the initial events, if a total of l data units were transmitted, the method described in 2 bounds from below the amount of captured E-botsis preferable (assuming the optimal amount n of simultaneous transmissions was repeatedly used for $\frac{l}{n}$ times), since if E-bots entered or exited the sensitive area this may not increase the received reward by transmission - but may increase the amount of captured E-bots. Similarly, for the latter events 1 describes a method that minimizes the expected captured E-bots, and if a total of k data units were flushed, then flushing l_{escape} units for $\frac{k}{l_{escape}}$ time is preferable. Since the lower bound of expected captured E-bots in either the crawling strategy or transmitting strategy is more effective, the theorem follows since either a reward of $\frac{l_{escape}}{E_{escape}(l_{escape})} = \frac{1}{E_{escape}(1)}$ or $\frac{(n - \frac{(-1+p_a)((1-p_a)^n-1)}{p_a} + p_p)}{R_n} = \frac{1}{E^n_{stay}(1)}$ is received before every of η E-bots is expected to be captured. □