A Note on The Reduced Mass

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Abstract
In this note we are rewriting the reduced mass formula into a form that potentially gives more intuition on what is truly behind the reduced mass.

Keywords: Reduced mass, Planck mass, Compton wavelength.

The Reduced Mass in a Slightly Different Form

The reduced mass in modern physics is normally given by

\[
\mu = \frac{1}{\frac{1}{m_1} + \frac{1}{m_2}}
\]

\[
\mu = \frac{m_1 m_2}{m_1 + m_2}
\]

Haug (2016) has shown that the Planck mass can be rewritten as

\[
M_p = \sqrt{\frac{\hbar c}{G_p}} = \frac{\hbar}{l_p c}
\]

(2)

Basically any (uniform) mass can be written as

\[
M = \frac{\hbar}{\lambda c}
\]

(3)

where \(\lambda\) is the reduced Compton wavelength of the mass of interest. For example, for a Planck mass the reduced Compton wavelength is \(l_p\), for an electron it is \(\lambda_e\), and for a proton it is \(\lambda_p\). This form of notation for the mass of elementary particles does not change their values (weight) etc., it simply gives deeper insight and makes it easier to understand the subatomic world. The difference between different “elementary” masses is basically the Compton wavelength. Based on this we can rewrite the reduced mass as

\[
\mu = \frac{1}{\frac{1}{\lambda_A} + \frac{1}{\lambda_B}}
\]

\[
\mu = \frac{\hbar}{\lambda_A + \lambda_B}
\]

\[
\mu = \frac{\hbar}{\frac{\lambda_A^2}{\lambda_A^2 + \lambda_B^2}}
\]

\[
\mu = \frac{\hbar}{\frac{1}{\lambda_A} + \frac{1}{\lambda_B}}
\]

(4)

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\(^\ddagger\)See also Haug (2014) where a similar mass formula is derived directly from ancient atomism.
From this perspective the reduced mass is simply a “new” mass, where the Compton wavelengths of the two masses are added together. Because the mass will increase, the shorter the Compton wavelength is, then the reduced mass must be smaller than any of the two masses from which we “constructed” the reduced mass. As an example, we can consider the reduced mass of an electron and a proton; this is given by

$$\mu = \frac{\hbar}{\lambda_e + \lambda_p} \frac{1}{c}$$  \hspace{1cm} (5)

In the special case where $m_1 = m_2$, we must have $\lambda_A = \lambda_B$ and we get the well-known result

$$\mu = \frac{\hbar}{\lambda + \lambda} \frac{1}{c}$$

$$\mu = \frac{\hbar}{2\lambda} \frac{1}{c}$$

$$\mu = \frac{1}{2} m_1 = \frac{1}{2} m_2$$  \hspace{1cm} (6)

Based on this we can for example re-write the “notation” of Schrödinger equation for a hydrogen atom

$$E\psi = -\frac{\hbar^2}{2\mu} \Delta^2 \psi - \frac{e^2}{4\pi \varepsilon_0 r} \psi$$

$$E\psi = -\frac{\hbar^2}{\lambda_e + \lambda_p} \frac{1}{c} \Delta^2 \psi - \left( \frac{\sqrt{\pi} \sqrt{\alpha \sqrt{10^7}}}{4\pi \frac{1}{c^2} 4.4 \times 10^{-10} r} \right)^2 \psi$$

$$E\psi = -\frac{1}{2} \hbar c (\lambda_e + \lambda_p) \Delta^2 \psi - \frac{e^2}{r} \psi$$

$$E\psi = -\frac{1}{2} \hbar c (\lambda_e + \lambda_p) \Delta^2 \psi - \frac{\hbar}{r} \cos \psi$$  \hspace{1cm} (7)

References


