Meaning of Twin Paradox and Special Relativity Theory
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Abstract
About the twin paradox of special relativity theory, there are some resolutions. But these might not be the best fit resolution considering the core concept of special relativity theory. Here we will approach the concept of special relativity thinking the resolution of twin paradox.

1. Introduction
Typical scenario of twin paradox solution is:
1) Time and space are integrated respectively from starting through returning to meeting again,
2) The paradox is recognized resolved by the fact time and space is respectively equal for both of twin when they meet again.
This may admit paradox situation during its travel. If so, this means paradox is not resolved completely.
Here we reconsider this paradox and reasonable resolution.

2. View from s system and s’ system
We consider about following two systems
s system(2dimensions(ct, x)) and s’ system(2dimensions(ct’, x’)). [1]
Here both are moving relatively with velocity v.
This situation can be shown as Minkowsky graph. (Fig.1)

On Fig.2, A is time t_A’ point in s’ system. P,Q is time t_A’ line. This is simultaneous line in s’ system.
Also B is time t_B’ point in s’ system. R,S is time t_B’ line. This is simultaneous line in s’ system.
Then on the view of $s'$ system, time passes from $\overline{PQ}$ ($t'_A$) line to $\overline{RS}$ ($t'_B$) line at every location.

On the view of $s$ system, it is $T$ position when $t'_A$ if it is measured on location $O$, it is $U$ position when $t'_B$. Then elapse time $\overline{AB}$ for $s'$ system is corresponding to $\overline{TU}$ for $s$ system.

This means:

On the view of $s'$ system, time is passing everywhere simultaneously while clock of $s$ system is moving from $O$ to $U$, for example.

The difference of time duration of $s$ system and $s'$ system is based on the difference of frame of reference. The reason of it is ‘Time Clock (CT) moves in space with velocity $c'$’. [1]

Lorentz equation is

\[ ct' = \frac{ct - \frac{vx}{\sqrt{1 - \frac{v^2}{c^2}}}}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

(1)

\[ x' = \frac{-vt + x}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

(2)

Here we set

$t'$ value of point $A$: $t'_A$,
$t'$ value of point $B$: $t'_B$,
$t$ value of point $T$: $t_T$,
$t$ value of point $U$: $t_U$,

time duration $\overline{AB} = ct'$
time duration $\overline{TU} = ct$

From (1),
\[ ct_A' = \frac{ct_T + 0}{\sqrt{1 - \frac{v^2}{c^2}}} \]  
\[ ct_B' = \frac{ct_T + 0}{\sqrt{1 - \frac{v^2}{c^2}}} \]  
\[ ct_A' - ct_B' = \frac{ct_T - ct_U}{\sqrt{1 - \frac{v^2}{c^2}}} \]  
\[ cl' = \frac{c}{\sqrt{1 - \frac{v^2}{c^2}}} \]  

(6) has been called time dilatation of moving object.

3. Time-space distance

On Fig.2, for example, \( \overline{AB} \) is time-space distance and real existence for \( s' \) system. Also \( \overline{AB} \) is time-space distance and real existence for \( s \) system. \( \overline{TU} \) is a projection of \( \overline{AB} \) as time duration for \( s \) system. Here real existence of time-space distance is only one or common for every inertia system. On the other hand, every inertia systems have own projection as time distance or space distance.

4. \( s \) system and \( s' \) system

There is no inertia system which has priority. Every inertia systems are equivalent. But when multiple systems are described at once, there are following two categories of system as Minkowsky graph in this report.

- Staying system: I myself and total staying universe
- Moving system: moving parts and assembly of these

In this report, if staying system is \( s \) system, moving system is \( s' \) system.

In \( s \) system, I myself am with staying space, and time is passing simultaneously within whole spaces. \( s' \) system is moving in \( s \) system, and it has own frame of reference.

Of course, because both systems are equivalent, each system category is exchangeable. But while one scenario is described, each category should be kept.

5. Twin Paradox

Elapse time difference for \( s \) system (\( \overline{TU} \)) and \( s' \) system (\( \overline{AB} \)) is on difference of view. But real existence time-space is common.

Then there is always no paradox situation and it becomes completely same situation when both system becomes same inertia system (same view) except moved distance even though both don’t meet again, for example, \( v \) becomes zero gradually keeping balance.
6. Conclusion

Regarding to relatively moving frame of references, each axis has different target object, so the difference of each’s value doesn’t make paradox. View is different but real existence is unique. When two systems become have same speed, they have same frame of reference.

Reference