Measurement theory based on the truth values provides the maximum violation of the Bell-Mermin inequality

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We investigate the violation factor of the Bell-Mermin inequality. Until now, we use an assumption that the results of measurement are ± 1. In this case, the maximum violation factor is $2^{(n-1)/2}$. The quantum predictions by n-partite Greenberger-Horne-Zeilinger (GHZ) state violate the Bell-Mermin inequality by an amount that grows exponentially with n. Recently, a new measurement theory based on the truth values is proposed. The values of measurement outcome are either $+1$ or 0. Here we use the new measurement theory. We consider multipartite GHZ state. It turns out that the Bell-Mermin inequality is violated by the amount of $2^{(n-1)/2}$. The measurement theory based on the truth values provides the maximum violation of the Bell-Mermin inequality.

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I. INTRODUCTION

The quantum theory (cf. [1–5]) gives accurate and at times remarkably accurate numerical predictions. Much experimental data has fit to the quantum predictions for long time.

On the other hand, from the incompleteness argument of Einstein, Podolsky, and Rosen (EPR) [6], a hidden-variable interpretation of the quantum theory has been an attractive topic of research [2, 3]. One is the Bell-EPR theorem [7]. This theorem says that the quantum predictions violate the inequality following from the EPR-locality condition. The condition tells that a result of measurement pertaining to one system is independent of any measurement performed simultaneously at a distance on another system.

The other is the no-hidden-variables theorem of Kochen and Specker (KS theorem) [8]. The original KS theorem says the non-existence of a real-valued function which is multiplicative and linear on commuting operators. The quantum theory does not accept the KS type of hidden-variable theory. The proof of the original KS theorem relies on intricate geometric argument. Greenberger, Horne, and Zeilinger discover [9, 10] the so-called GHZ theorem for four-partite GHZ state. And, the Bell-KS theorem becomes very simple form (see also Refs. [11–15]).

Mermin considers the Bell-EPR theorem in a multipartite state. He derives multipartite Bell inequality [16]. The quantum predictions by n-partite GHZ state violate the Bell-Mermin inequality by an amount that grows exponentially with n. And, several multipartite Bell inequalities are reported [17–25]. They also say that the quantum predictions violate local hidden-variable theories by an amount that grows exponentially with n.

It is begun to research the validity of the KS theorem by using inequalities (see Refs. [26–29]). To find such inequalities to test the validity of the KS theorem is particularly useful for experimental investigation [30]. One of authors derives an inequality [29] as tests for the validity of the KS theorem. The quantum predictions violate the inequality when the system is in an uncorrelated state. An uncorrelated state is defined in Ref. [31]. The quantum predictions by n-partite uncorrelated state violate the inequality by an amount that grows exponentially with n.

Leggett-type nonlocal hidden-variable theory [32] is experimentally investigated [33–35]. The experiments report that the quantum theory does not accept Leggett-type nonlocal hidden-variable theory. These experiments are done in four-dimensional space (two parties) in order to study nonlocality of hidden-variable theories. However there are debates for the conclusions of the experiments. See Refs. [36–38].

Recently, it is discussed that von Neumann's theory does not meet Deutsch's algorithm [39]. In von Neumann's theory, control of quantum state and observations of quantum state cannot be existential, simultaneously. In reference [40], we propose a solution of the problem. We discover the new measurement theory based on the truth values. The problem is solved if the results of measurement are either $+1$ or 0. Therefore we consider the significance of the new measurement theory based on the truth values. Especially, we investigate the relation between the new measurement theory and quantum non locality.

In this paper, we investigate the violation factor of the Bell-Mermin inequality. Until now, we use an assumption that the results of measurement are ± 1. In this case, the maximum violation factor is $2^{(n-1)/2}$. The quantum predictions by n-partite GHZ state violate the Bell-Mermin inequality by an amount that grows exponentially with n. Recently, a new measurement theory based on the truth values is proposed. The values of measurement outcome are either $+1$ or 0. Here we use the new measurement theory. We consider multipartite GHZ state. It turns out that the Bell-Mermin inequality is violated by the amount of $2^{(n-1)/2}$. The measurement theory based on the truth values provides the maximum violation of the Bell-Mermin inequality.

II. THE MEASUREMENT THEORY BASED ON THE TRUTH VALUES PROVIDES THE MAXIMUM VIOLATION OF THE BELL-MERMIN INEQUALITY

Let us consider n particles $j = 1, 2, \ldots, n$. Let us consider σ_x^j and σ_y^j as Pauli observables for jth particle. We insert Q as an operator

$$Q = \mathrm{Re}\left(\pm \prod_{j=1}^{n} (\sigma_x^j + i\sigma_y^j)\right). \tag{1}$$

Let us consider the following GHZ state:

$$|\Phi_n\rangle = \frac{1}{\sqrt{2}}(|+_1; +_2; \cdots; +_n\rangle + |-_1; -_2; \cdots; -_n\rangle). \tag{2}$$

We have the following quantum expected value

$$|\langle \Phi_n | Q | \Phi_n \rangle| = 2^{(n-1)} \tag{3}$$

where the local results of measurements are either $+1$ or 0. In fact, the value of the quantum expected value does not change.

We insert Q' as another operator

$$Q' = \mathrm{Re}\left(\pm e^{\frac{\pi}{4}i} \prod_{j=1}^{n} (\sigma_x^j + i\sigma_y^j)\right). \tag{4}$$

Let us consider the following GHZ state:

$$|\Psi_n\rangle = \frac{1}{\sqrt{2}}(|+_1; +_2; \cdots; +_n\rangle + e^{\phi i}|-_1; -_2; \cdots; -_n\rangle). \tag{5}$$

We have the following quantum expected value by taking ϕ to be suitable phase

$$|\langle \Psi_n | Q' | \Psi_n \rangle| = 2^{(n-1)} \tag{6}$$

where the local results of measurements are either $+1$ or 0.

On the other hand, let us consider a classical counterpart of Q and Q' when the local results of measurements are either $+1$ or 0.

Let us consider the Bell-Mermin inequality. First, let us consider n is odd. We consider C as

$$C = \mathrm{Re}\left(\pm \prod_{j=1}^{n} (v(\sigma_x^j) + iv(\sigma_y^j))\right) \tag{7}$$

where $v(\sigma_x^j) = +1, 0$ and $v(\sigma_y^j) = +1, 0$. We see

$$|C| \le 2^{(n-1)/2} \quad n = \text{odd}. \tag{8}$$

The maximum of $|C|$ is equal to the real part of a product of complex numbers each of which has magnitude of $\sqrt{2}$ and a phase of $\pm\pi/4$ or $\pm 3\pi/4$. When n is odd the product must lie along an axis at $\pm\pi/4$ to the real axis and its real part can only attain the maximum value $2^{(n-1)/2}$. Therefore, the value $|C|$ is bounded as (8).

Next, let us consider n is even. We consider C' as

$$C' = \mathrm{Re}\left(\pm e^{\frac{\pi}{4}i} \prod_{j=1}^{n} (v(\sigma_x^j) + iv(\sigma_y^j))\right) \tag{9}$$

where $v(\sigma_x^j) = +1, 0$ and $v(\sigma_y^j) = +1, 0$. We see

$$|C| \le 2^{(n-1)/2} \quad n = \text{even}. \tag{10}$$

The maximum of $|C|$ is equal to the real part of a product of complex numbers each of which has magnitude of $\sqrt{2}$ and a phase of $\pm\pi/4$ or $\pm 3\pi/4$. When n is even the product must lie along an axis at $\pm\pi/4$ to the real axis and its real part can only attain the maximum value $2^{(n-1)/2}$. Therefore, the value $|C|$ is bounded as (10).

Therefore, we have a violation of the Bell-Mermin inequality with the following factor from (8) and (10)

$$\left|\frac{\langle \Phi_n | Q | \Phi_n \rangle}{C}\right| \text{ or } \left|\frac{\langle \Psi_n | Q' | \Psi_n \rangle}{C'}\right| \tag{11}$$

that is

$$2^{(n-1)/2}. \tag{12}$$

Hence, the measurement theory based on the truth values provides the maximum violation of the Bell-Mermin inequality.

III. CONCLUSIONS

In conclusion, we have investigated the violation factor of the Bell-Mermin inequality. Until now, we have used an assumption that the results of measurement are ± 1. In this case, the maximum violation factor has been $2^{(n-1)/2}$. The quantum predictions by n-partite GHZ state have violated the Bell-Mermin inequality by an amount that grows exponentially with n. Recently, a new measurement theory based on the truth values has been proposed. The values of measurement outcome have been either $+1$ or 0. Here we have used the new measurement theory. We have considered multipartite GHZ state. It has turned out that the Bell-Mermin inequality is violated by the amount of $2^{(n-1)/2}$. The measurement theory based on the truth values has provided the maximum violation of the Bell-Mermin inequality.

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