A Loop Quantum Gravity Inspired Model for Matter Consistent with Spacetime Atoms

Risto Raitio *
02230 Espoo, Finland

April 20, 2016

Abstract

I propose a statistical mechanical model for black holes with the partition function sum taken over punctures of a spin network on a two-surface as given by Loop Quantum Gravity. Searching a unified quantum structure for spacetime and matter, I apply this scenario to matter particles using a composite model for quarks and leptons.

PACS 04.70.-s

Keywords: Quantum Black Hole, Statistical Mechanics, General Relativity, Loop Quantum Gravity, Hawking radiation, Davies-Unruh Effect, Standard Model

*E-mail: risto.raitio@gmail.com

1 Introduction

In this note I continue elaborating the statistical mechanical model for black holes introduced in [1]. I redefine the partition function as a sum over black hole stretched horizon area eigenvalues as given by Loop Quantum Gravity (LQG) [2, 3]. The lowest eigenvalue allows one to construct quarks and leptons as three black hole preon bound states [4] without having the problem of standard model particle masses being of the order of Planck mass.

At the same time the possibility is opened to propose a unified picture for (i) matter based on the preon model for quarks and leptons [4] and (ii) the stretched horizon model of atoms of spacetime [5]. I emphasize that there are no phenomenological reasons to introduce the composite model. The reasons are conceptual as wil be discussed below.

This note is organized as follows. In section 2 I present the main points of the stretched horizon black hole model. Consistent quantization of spacetime and matter is discussed in section 3 in connection with a model for composite quarks and leptons. Finally in section 4 I give a brief discussion of results and conclusions. An appendix is provided for main results of the stretched horizon model phase transition. The presentation is very concise throughout.

2 LQG Area Eigenvalues and the Partition Function

I consider a micro black hole dressed by a stretched horizon, which is a membrane hovering about a Planck length outside the event horizon and which is both physical and hot.

The energy of a black hole from the point of view of an observer on its stretched horizon is called Brown-York energy [6]

$$E = \frac{ac^2}{8\pi G} A \tag{2.1}$$

where a is the constant proper acceleration of an observer on the stretched horizon and A is the area of the horizon. In LQG the area eigenvalues are

$$A = \gamma l_{\mathrm{Pl}}^2 \sum\nolimits_p \sqrt{j_p(j_p+1)} \tag{2.2}$$

where the sum is over punctures p of the spin network, l_{Pl} is the Planck length, γ is the Barbero–Immirzi parameter and the values of j_p are $0, \frac{1}{2}, 1, \frac{3}{2}, ...$

The main result of this note follows from (2.1) and (2.2). With $j_p = 0$ one gets zero energy for the lowest quantum state of a black hole. This would remove the high mass problem of the composite model [4] allowing one to build quarks and leptons as three preon states as we see in the next section.

For consistent total picture the partition function is considered briefly for a spin network with N punctures, for details see [5]

$$\begin{aligned} Z(\beta) &= \sum\nolimits_n exp(-\beta E_n) \\ &= \sum\nolimits_{n_1 n_2 \ldots n_N} exp(-\beta T_0 \sum_{p=1}^{N} \sqrt{n_p(n_p+2)} \end{aligned} \tag{2.3}$$

where $T_0 = \frac{a}{16\pi}\gamma$ and $n_p = 2j_p$, with $n_p = 0, 1, 2, ...$ The resulting $Z(\beta)$ is [5]

$$Z(\beta) = \frac{1}{y-1}\Big[1 - \Big(\frac{1}{y}\Big)^N\Big] \tag{2.4}$$

where

$$y = y(\beta) = \Big[\sum_{n=1}^{\infty} exp\big(-\beta T_0\sqrt{n(n+2)}\big)\Big]^{-1} \tag{2.5}$$

When $y = 1$ one has simply $Z(\beta) = N$. The average energy at temperature T can be now be calculated and is outlined in the Appendix.

3 Unified Model of Matter and Spacetime

Statistical methods of section 2 for spacetime offer a possibility to study the matter sector from a novel point of view to build a consistent, unified picture of matter and spacetime base on black holes with a stretched horizon. This goal would imply some internal structure at scale of the order of $l_{\rm Pl}$ for quarks and leptons. Such a model has been proposed in [4] (though there never was experimental need for it).

The basic idea in [4] is that the quarks and lepton are made of black hole maxons with charge 0 or $\frac{1}{3}$ and 'color' (i, j, k) as permutation index as follows

$$\begin{aligned} u_k &= \epsilon_{ijk} m_i^+ m_j^+ m^0 \\ \bar{d}_k &= m_k^+ m^0 m^0 \\ e &= \epsilon_{ijk} m_i^- m_j^- m_k^- \\ \nu &= \epsilon_{ijk} m_i^0 m_j^0 m_k^0 \end{aligned} \tag{3.1}$$

The construction (3.1) on maxon level is matter-antimatter symmetric and 'color' singlet, which is desirable for early cosmology.

The maxon mass scale is the Planck scale and (3.1) would be superheavy particles. To get the standard model particles the large mass difference has to be explained. It is accomplished from (2.2) by setting $j_p = 0$, which leads by (2.1) to zero mass (remnant) of the hole. This $j_p = 0$ maxon may interact with the Higgs field and gain light mass from it.

I assume that the quarks and leptons are bound states of maxons with the Higgs, or some new gauge interaction, mediating the binding. Missing at the moment are calculational methods for the bound states. Numerical methods can usually be developed at some level of accuracy. The question of existence of free single maxons requires a detailed model for the binding. It may be a new particle or it can also be assumed that some kind of confinement is operating.

The gauge bosons and the Higgs would be elementary (but their composite nature is not ruled out). The three generations would be due to a gravitational mechanism of the stretched horizon or a new symmetry.

In early universe at high temperature it seems obvious that standard model quarks and leptons would not form at all. Instead all matter would be in black holes interacting gravitationally and electromagnetically. Quarks and leptons would be formed later when the temperature decreases substantially. Electroweak and QCD interaction come to play rather late. This would be subject of a separate study.

4 Conclusions

There are at present a number competing candidate theories for quantum gravity like string theory, loop quantum gravity, causal dynamical triangulation, and others. The model of sections 2 and 3 goes deep into the structure of the physical universe and can be considered a promising candidate for a unified scheme of everything. In this scenario LQG dynamics and the horizon properties of black holes are a promising origin of spacetime, gravity and matter.

Appendix

A The Phase Transition

I give below a brief sketch of the stretched horizon model first order phase transition though not directly needed for the present note. All properties of the model are derived in [5].

The average energy at temperature $T = 1/\beta$ can be calculated from the partition function (2.3)

$$E(\beta) = -\frac{\partial}{\partial\beta} \ln Z(\beta) \tag{A.1}$$

of the black hole which yields

$$E(\beta) = \Big(\frac{1}{y-1} - \frac{N}{y^N - 1}\frac{1}{y}\Big)\frac{dy}{d\beta} \tag{A.2}$$

In LQG it is assumed that the number of punctures on the stretched horizon is very large, say about 10^{122}. Therefore for $y > 1$ (A.2) simplifies to

$$E(\beta) = \frac{1}{y-1}\frac{dy}{d\beta} \tag{A.3}$$

For $y < 1$, y^N approaches zero for large N and one gets

$$E(\beta) = \frac{N}{y}\frac{dy}{d\beta} \tag{A.4}$$

There is a jump in energy of the hole when $y = 1$. Since y depends on temperature acoording to (2.5) on sees that the hole undergoes a phase transition at the critical temperature T_C defined by the solutions of

$$\sum_{n=1}^{\infty} exp\Big(-\frac{T_0}{T_C}\sqrt{n(n+2)}\Big) = 1 \tag{A.5}$$

Below the critical temperature T_C the punctures of the stretched horizon are in vacuum and there is no black hole. Above T_C the punctures get "excited" and provide the possibility of falling back to vacuum with Hawking radiation being emitted simultaneously.

From $T_0 = \frac{a}{16\pi}\gamma$ and $x = T_0/T_C \approxeq 0.508$ (obtained numerically) and choosing $\gamma = 8x \approxeq 4.06$ one gets

$$T_C = \frac{a}{2\pi} \tag{A.6}$$

which is the Davies-Unruh temperature felt by an observer on the stretched horizon with acceleration $a = constant$.

In Schwarzschild spacetime the line element includes the function $f(r) = 1 - \frac{2M}{r}$ and the acceleration is

$$\begin{aligned} a =& \frac{1}{2} f^{-1/2}\frac{df}{dr} \\ =& \Big(1 - \frac{2M}{r}\Big)^{-1/2}\frac{M}{r^2} \end{aligned} \tag{A.7}$$

Right outside the horizon $r = 2M$ and (A.6) and (A.7) the lowest temperature of the hole as seen by the observer is

$$T_C = \left(1 - \frac{2M}{r}\right)^{-1/2} \frac{1}{8\pi M} \tag{A.8}$$

This temperature corresponds according to the Tolman relation a far away observer temperature

$$T_\infty = \sqrt{f}\, T_C = \frac{1}{8\pi M} \tag{A.9}$$

which is the Hawking temperature.

References

[1] Raitio, R. (2016) A Statistical Model of Spacetime, Black Holes and Matter. Open Access Library Journal, 3: e2487. http://dx.doi.org/10.4236/oalib.1102487.

[2] Chiou, D-W. (2015) Int. J. Mod. Phys. D24, 1530005 [1412.4362].

[3] Rovelli, C. (2011) Zakopane lectures on loop gravity, arXiv:1102.3660.

[4] Raitio, R. (1980) A Model of Lepton and Quark Structure. Physica Scripta, 22, 197. http://dx.doi.org/10.1088/0031-8949/22/3/002.

[5] Mäkelä, J. (2016) Phase Transition in Loop Quantum Gravity, arXiv:1604.01393.

[6] Brown, J. and York Jr., J. (1993) Quasilocal Energy and Conserved Charges Derived from the Gravitational Action. Physical Review D47, 1407. http://dx.doi.org/10.1103/PhysRevD.47.1407