General Relativity as Multifractal Analogue of the Standard Model

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Abstract

We have recently shown that the Standard Model of particle physics (SM) may be configured as a *multifractal set,* with all field components acting as primary generators of this set. The goal of this brief note is to point out that a multifractal description of General Relativity (GR) is also possible, starting from the definition of the metric tensor and relativistic interval. This finding paves the way to an unforeseen homeomorphism between the structure of SM and classical gravity.

Key words: Standard Model, General Relativity, Multifractal Set, Generalized Dimension.

1. Multifractals: a concise overview

As it is known, the *box-counting dimension* defines the main scaling property of fractal structures and is a measure of their self-similarity. Multifractals are global mixtures of fractal structures, each characterized by its local box-counting dimension. Self-similarity of multifractals is accordingly defined in terms of a *multifractal spectrum* describing the overall distribution of dimensions. In the language of chaos and complexity theory, multifractal analysis is the study of *invariant sets* and is a powerful tool for the characterization of generic *dynamical systems*.

In the recursive construction of multifractal sets from $i = 1, 2..., N$ local scales r_i with probabilities p_i, the definition of the box-counting dimension leads to [4]

$$\sum_{i=1}^{N} p_i^q r_i^{\tau(q)} = 1 \tag{1a}$$

in which

$$\sum_{i=1}^{N} p_i = 1 \tag{1b}$$

Here, q and $\tau(q)$ are two arbitrary exponents and the latter is typically presented as

$$\tau(q) = (1-q)D_q \tag{2}$$

where D_q plays the role of a *generalized dimension*.

2. GR as topological analogue of SM

Consider now the field makeup of the SM, formed by 16 *independent* "flavors": two massive gauge bosons (W, Z), gluon (g), the Higgs scalar (H), neutrinos, charged leptons and quarks. The SM structure can be conveniently organized in the 4×4 matrix

$$SM = \begin{pmatrix} g & \nu_e & \nu_\mu & \nu_\tau \\ W & e & \mu & \tau \\ Z & u & c & b \\ H & d & s & t \end{pmatrix} \tag{3}$$

Note that photon (γ) is absent from (3) as it is built from the underlying components of the electroweak sector, whereby $\gamma = \gamma(W_\mu^3, B_\mu)$ and $B_\mu = B_\mu(W_\mu^3, Z)$ [1].

It was shown in [2-3] that, near the electroweak scale M_{EW}, the spectrum of particle masses m_i entering the SM satisfies the "closure" relation

$$\sum_{i=1}^{16} \left(\frac{m_i}{M_{EW}}\right)^2 = 1 \tag{4}$$

Note that (3) shares the same formal structure with the metric tensor of GR, that is,

$$GR = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix} \tag{5}$$

where there are only 10 independent entries under the standard assumption $g_{\mu\nu} = g_{\nu\mu}$. Starting from the GR definitions of interval and proper time leads to ($c = 1$)

$$\sum_{\mu=0}^{3} \sum_{\nu=0}^{3} g_{\mu\nu} \frac{dx^{\mu}}{d\tau} \frac{dx^{\nu}}{d\tau} = 1 \tag{6}$$

subject to the constraint

$$\sum_{\nu=0}^{3} g^{\mu\nu} g_{\nu\rho} = \delta^{\mu}_{\rho} = \begin{cases} 1, & \mu = \rho \\ 0, & \mu \neq \rho \end{cases} \tag{7}$$

Comparing (1), (4) and (6) reveals the following mapping

$$\begin{gathered} GR : (p_i \Rightarrow g^{\mu\nu} g_{\nu\rho},\ q = 1/2\ ,\ D_q = 4,\ \tau(q) = 2) \\ SM : (p_i \Rightarrow 1,\ q = 0,\ D_q = \tau(q) = 2) \end{gathered} \tag{8}$$

It is interesting to note that the SM dimension $D_0 = 2$ coincides with the fractal dimension of quantum mechanical paths [5-6], while the GR dimension $D_{1/2} = 4$ recovers the four-dimensionality of geodesic paths in classical spacetime.

A couple of conclusions may be drawn from (8):

- GR may be viewed as topological analogue of the SM, defined by a unitary exponent q and a dimension that is twice the SM dimension (that is, $D_{1/2} = 2D_0$).
- The spectrum of particle *mass scales* (m_i / M_{EW}) and the *four-vector of differential coordinates* ($dx^\mu / d\tau$) form the basis for the multifractal description of SM and GR, respectively.

References

[1] see e.g., Maggiore, M. "*A Modern Introduction to Quantum Field Theory*", Oxford University Press, 2005.

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