SPECIAL RELATIVITY WITH MASSIVE AND NON-MASSIVE PARTICLES

A. Blato



(2016) Buenos Aires

Argentina

In special relativity, this article presents a relativistic dynamics of massive and non-massive particles which can be applied in any inertial reference frame.

## Introduction

In special relativity, the total position ( $\bar{\mathbf{r}}$ ) of a (massive or non-massive) particle is always zero.

$$\bar{\mathbf{r}} \,=\, 0$$

The total position ( $\bar{\mathbf{r}}$ ) of a (massive or non-massive) particle is defined by the kinetic position ( $\hat{\mathbf{r}}$ ) and the dynamic position ( $\check{\mathbf{r}}$ ) as follows:

$$\hat{\mathbf{r}} - \check{\mathbf{r}} \,=\, 0$$

The kinetic position ( $\hat{\mathbf{r}}$ ) of a (massive or non-massive) particle is given by:

$$\hat{\mathbf{r}} \,\doteq\, \frac{1}{\mu} \int m \, \mathbf{v} \, dt$$

where ( $\mu$ ) is an arbitrary ( universal ) constant, ( $m$ ) is the relativistic mass of the particle, ( $\mathbf{v}$ ) is the velocity of the particle and ( $t$ ) is time.

The dynamic position ( $\check{\mathbf{r}}$ ) of a (massive or non-massive) particle is given by:

$$\check{\mathbf{r}} \,\doteq\, \frac{1}{\mu} \int\!\!\int \mathbf{F} \, dt \, dt$$

where ( $\mu$ ) is the arbitrary ( universal ) constant, ( $\mathbf{F}$ ) is the net force acting on the particle and ( $t$ ) is time.

The relativistic mass ( $m$ ) of a massive particle is given by:

$$m \doteq \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}$$

where ( $m_o$ ) is the rest mass of the massive particle, ( $v$ ) is the speed of the massive particle and ( $c$ ) is the speed of light in vacuum.

The relativistic mass ( $m$ ) of a non-massive particle is given by:

$$m \doteq \frac{h\,\nu}{c^2}$$

where ( $h$ ) is the Planck constant, ( $\nu$ ) is the frequency of the non-massive particle and ( $c$ ) is the speed of light in vacuum.

Now, the total position ( $\bar{\mathbf{r}}$ ) of a (massive or non-massive) particle can also be expressed as follows:

$$\frac{1}{\mu}\left[\,\int m\,\mathbf{v}\,dt \;-\int\!\!\int \mathbf{F}\,dt\,dt\,\right] = \;0$$

Differentiating the above equation with respect to time, yields:

$$\frac{1}{\mu}\left[\,m\,\mathbf{v} \;-\int \mathbf{F}\,dt\,\right] = \;0$$

Differentiating again with respect to time, we have:

$$\frac{1}{\mu}\left[\,m\,\mathbf{a} + \frac{dm}{dt}\,\mathbf{v} \;-\; \mathbf{F}\,\right] = \;0$$

Multiplying by ( $\mu$ ) and rearranging, we finally obtain:

$$\mathbf{F} \;=\; m\,\mathbf{a} + \frac{dm}{dt}\,\mathbf{v}$$

This equation ( similar to Newton's second law for $v \ll c$ ) will be used in the next section of this article.

## The Relativistic Dynamics

If we consider a (massive or non-massive) particle with relativistic mass $m$ then the linear momentum $\mathbf{P}$ of the particle, the angular momentum $\mathbf{L}$ of the particle, the net force $\mathbf{F}$ acting on the particle, the work $\mathrm{W}$ done by the net force acting on the particle, and the kinetic energy $\mathrm{K}$ of the particle, for an inertial reference frame, are given by:

$$\mathbf{P} \doteq m\,\mathbf{v}$$

$$\mathbf{L} \doteq \mathbf{P} \times \mathbf{r}$$

$$\mathbf{F} = \frac{d\mathbf{P}}{dt} = m\,\mathbf{a} + \frac{dm}{dt}\,\mathbf{v}$$

$$\mathrm{W} \doteq \int_1^2 \mathbf{F} \cdot d\mathbf{r} = \int_1^2 \frac{d\mathbf{P}}{dt} \cdot d\mathbf{r} = \Delta\,\mathrm{K}$$

$$\mathrm{K} \doteq m\,c^2$$

where $(\,\mathbf{r},\ \mathbf{v},\ \mathbf{a}\,)$ are the position, the velocity and the acceleration of the particle relative to the inertial reference frame and $(\,c\,)$ is the speed of light in vacuum. The kinetic energy $(\,\mathrm{K}_o\,)$ of a massive particle at rest is $(\,m_o\,c^2\,)$

## Bibliography


**A. Einstein**, Relativity: The Special and General Theory.

**E. Mach**, The Science of Mechanics.

**W. Pauli**, Theory of Relativity.

**A. French**, Special Relativity.

## Appendix

## System of Equations

$$
\begin{tikzcd}
 & \boxed{[1]} \arrow[d, "dt"] & \\
\boxed{[4]} \arrow[d, "dt"] & \boxed{[2]} \arrow[l, "\times \mathbf{r}"'] \arrow[d, "dt"] & \\
\boxed{[5]} & \boxed{[3]} \arrow[l, "\times \mathbf{r}"'] \arrow[r, "\int d\mathbf{r}"] & \boxed{[6]}
\end{tikzcd}
$$

$$[1] \qquad \frac{1}{\mu}\left[\int \mathbf{P}\,dt - \int\!\!\int \mathbf{F}\,dt\,dt\right] = 0$$

$$[2] \qquad \frac{1}{\mu}\left[\mathbf{P} - \int \mathbf{F}\,dt\right] = 0$$

$$[3] \qquad \frac{1}{\mu}\left[\frac{d\mathbf{P}}{dt} - \mathbf{F}\right] = 0$$

$$[4] \qquad \frac{1}{\mu}\left[\mathbf{P} - \int \mathbf{F}\,dt\right] \times \mathbf{r} = 0$$

$$[5] \qquad \frac{1}{\mu}\left[\frac{d\mathbf{P}}{dt} - \mathbf{F}\right] \times \mathbf{r} = 0$$

$$[6] \qquad \frac{1}{\mu}\left[\int \frac{d\mathbf{P}}{dt} \cdot d\mathbf{r} - \int \mathbf{F} \cdot d\mathbf{r}\right] = 0$$