In special relativity, this article presents a relativistic dynamics of massive and non-massive particles which can be applied in any inertial reference frame.

Introduction

In special relativity, the total position $\vec{r}$ of a (massive or non-massive) particle is always zero.

$$\vec{r} = 0$$

The total position $\vec{r}$ of a (massive or non-massive) particle is defined by the kinetic position $\hat{r}$ and the dynamic position $\check{r}$ as follows:

$$\hat{r} - \check{r} = 0$$

The kinetic position $\hat{r}$ of a (massive or non-massive) particle is given by:

$$\hat{r} = \frac{1}{\mu} \int m v \, dt$$

where $\mu$ is an arbitrary (universal) constant, $m$ is the relativistic mass of the particle, $v$ is the velocity of the particle and $t$ is time.

The dynamic position $\check{r}$ of a (massive or non-massive) particle is given by:

$$\check{r} = \frac{1}{\mu} \int \int F \, dt \, dt$$

where $\mu$ is the arbitrary (universal) constant, $F$ is the net force acting on the particle and $t$ is time.
The relativistic mass \( (m) \) of a massive particle is given by:

\[
m = \frac{m_o}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

where \( (m_o) \) is the rest mass of the massive particle, \( (v) \) is the speed of the massive particle and \( (c) \) is the speed of light in vacuum.

The relativistic mass \( (m) \) of a non-massive particle is given by:

\[
m = \frac{h \nu}{c^2}
\]

where \( (h) \) is the Planck constant, \( (\nu) \) is the frequency of the non-massive particle and \( (c) \) is the speed of light in vacuum.

Now, the total position \( (\bar{r}) \) of a (massive or non-massive) particle can also be expressed as follows:

\[
\frac{1}{\mu} \left[ \int m \mathbf{v} \, dt - \int \int \mathbf{F} \, dt \, dt \right] = 0
\]

Differentiating the above equation with respect to time, yields:

\[
\frac{1}{\mu} \left[ m \mathbf{v} - \int \mathbf{F} \, dt \right] = 0
\]

Differentiating again with respect to time, we have:

\[
\frac{1}{\mu} \left[ m \mathbf{a} + \frac{dm}{dt} \mathbf{v} - \mathbf{F} \right] = 0
\]

Multiplying by \( (\mu) \) and rearranging, we finally obtain:

\[
\mathbf{F} = m \mathbf{a} + \frac{dm}{dt} \mathbf{v}
\]

This equation (similar to Newton’s second law for \( v \ll c \)) will be used in the next section of this article.
The Relativistic Dynamics

If we consider a (massive or non-massive) particle with relativistic mass \( m \) then the linear momentum \( \mathbf{P} \) of the particle, the angular momentum \( \mathbf{L} \) of the particle, the net force \( \mathbf{F} \) acting on the particle, the work \( W \) done by the net force acting on the particle, and the kinetic energy \( K \) of the particle, for an inertial reference frame, are given by:

\[
\mathbf{P} = m \mathbf{v}
\]

\[
\mathbf{L} = \mathbf{P} \times \mathbf{r}
\]

\[
\mathbf{F} = \frac{d\mathbf{P}}{dt} = m \mathbf{a} + \frac{d\mathbf{m}}{dt} \mathbf{v}
\]

\[
W = \int_{1}^{2} \mathbf{F} \cdot d\mathbf{r} = \int_{1}^{2} \frac{d\mathbf{P}}{dt} \cdot d\mathbf{r} = \Delta K
\]

\[
K = m c^2
\]

where \( (\mathbf{r}, \mathbf{v}, \mathbf{a}) \) are the position, the velocity and the acceleration of the particle relative to the inertial reference frame and \( (c) \) is the speed of light in vacuum. The kinetic energy \( (K_o) \) of a massive particle at rest is \( (m_o c^2) \)

Bibliography


**W. Pauli**, Theory of Relativity.

**A. French**, Special Relativity.
Appendix

System of Equations

\[ \begin{align*}
[1] & \quad \frac{1}{\mu} \left[ \int P \, dt - \int \int F \, dt \, dt \right] = 0 \\
[2] & \quad \frac{1}{\mu} \left[ P - \int F \, dt \right] = 0 \\
[3] & \quad \frac{1}{\mu} \left[ \frac{dP}{dt} - F \right] = 0 \\
[4] & \quad \frac{1}{\mu} \left[ P - \int F \, dt \right] \times r = 0 \\
[5] & \quad \frac{1}{\mu} \left[ \frac{dP}{dt} - F \right] \times r = 0 \\
[6] & \quad \frac{1}{\mu} \left[ \int \frac{dP}{dt} \cdot dr - \int F \cdot dr \right] = 0
\end{align*} \]