Primes obtained concatenating 30p with 30q then adding or subtracting 1, where p and q=p+6 primes

Abstract. In this paper I state the following three conjectures: let \([p, q]\) be a pair of sexy primes \((q = p + 6)\); then: (I) there exist an infinity of primes obtained concatenating 30*p with 30*q and adding 1 to the resulted number; example: for \([p, q] = [23, 29]\), the number 690871 is prime; (II) there exist an infinity of primes obtained concatenating 30*p with 30*q and subtracting 1 from the resulted number; example: for \([p, q] = [23, 29]\), the number 690869 is prime; (III) there exist an infinity of pairs of twin primes obtained concatenating 30*p with 30*q and adding/subtracting 1 from the resulted number; example: for \([p, q] = [101, 107]\), the numbers 30303209 and 30303211 are primes.

The sequence of primes \(p\) such that \(q = p + 6\) is also prime: (A023201 in OEIS)

\[
\]

Conjecture I:

There exist an infinity of primes \(r\) obtained concatenating 30*p with 30*q and adding 1 to the resulted number, where \([p, q]\) is a pair of sexy primes; example: for \([p, q] = [23, 29]\), the number 690871 is prime.

The sequence of primes \(r\):

\[
210391, 510691, 690871, 11101291, 15901771, 30303211, 47104891, 57905971, 66906871, 93309511, 99301011, 1383014011, 1509015271, 1803018211, 1923019411 (\ldots),
\]

obtained for \([p, q] = [7, 13], [17, 23], [23, 29], [37, 43], [53, 59], [101, 107], [157, 163], [193, 199], [223, 229], [311, 317], [331, 337], [461, 467], [503, 509], [601, 607], [641, 647]\ldots

Conjecture II:

There exist an infinity of primes \(r\) obtained concatenating 30*p with 30*q and subtracting 1 from the
resulted number, where \([p, q]\) is a pair of sexy primes; example: for \([p, q] = [23, 29]\), the number 690869 is prime.

The sequence of primes \(r\): 

\[
\begin{align*}
150329, & \quad 330509, \quad 690869, \quad 12301409, \quad 30303209, \\
32103389, & \quad 50105189, \quad 66906869, \quad 68106989, \quad 69907169, \\
75307709, & \quad 78908069, \quad 81308309, \quad 1671016889, \\
1761017789, & \quad 1803018209, \quad 1821018389, \quad 1923019409 \\
(...), & \\
\end{align*}
\]

obtained for \([p, q] = [5, 11], [11, 17], [23, 29], [41, 47], [101, 107], [107, 113], [167, 173], [223, 229], [227, 233], [233, 239], [251, 257], [263, 269], [271, 277], [557, 563], [587, 593], [601, 607], [607, 613], [641, 647]...

Conjecture III:

There exist an infinity of pairs of twin primes obtained concatenating \(30*p\) with \(30*q\) and adding/subtracting 1 from the resulted number, where \([p, q]\) is a pair of sexy primes; example: for \([p, q] = [101, 107]\), the numbers 30303211 and 30303209 are primes.

The sequence of such pairs of twin primes:

\[
\begin{align*}
[690869, 690871], & \quad [30303209, 30303211], [66906869, 66906869], [1803018209, 1803018211], [1923019409, 1923019411], \\
\end{align*}
\]

obtained for \([p, q] = [23, 29], [101, 107], [223, 229], [601, 607], [641, 647]...

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