

Primes obtained concatenating $30p$ with $30q$ then adding or subtracting 1, where p and $q=p+6$ primes

Abstract. In this paper I state the following three conjectures: let $[p, q]$ be a pair of sexy primes ($q = p + 6$); then: (I) there exist an infinity of primes obtained concatenating $30*p$ with $30*q$ and adding 1 to the resulted number; example: for $[p, q] = [23, 29]$, the number 690871 is prime; (II) there exist an infinity of primes obtained concatenating $30*p$ with $30*q$ and subtracting 1 from the resulted number; example: for $[p, q] = [23, 29]$, the number 690869 is prime; (III) there exist an infinity of pairs of twin primes obtained concatenating $30*p$ with $30*q$ and adding/subtracting 1 from the resulted number; example: for $[p, q] = [101, 107]$, the numbers 30303209 and 30303211 are primes.

The sequence of primes p such that $q = p + 6$ is also prime:
(A023201 in OEIS)

: 5, 7, 11, 13, 17, 23, 31, 37, 41, 47, 53, 61, 67,
73, 83, 97, 101, 103, 107, 131, 151, 157, 167, 173,
191, 193, 223, 227, 233, 251, 257, 263, 271, 277,
307, 311, 331, 347, 353, 367, 373, 383, 433, 443,
457, 461, 503, 541, 557, 563, 571, 587, 593, 601,
607, 613, 641, 647 (...)

Conjecture I:

There exist an infinity of primes r obtained concatenating $30*p$ with $30*q$ and adding 1 to the resulted number, where $[p, q]$ is a pair of sexy primes; example: for $[p, q] = [23, 29]$, the number 690871 is prime.

The sequence of primes r :

: 210391, 510691, 690871, 11101291, 15901771,
30303211, 47104891, 57905971, 66906871, 93309511,
993010111, 1383014011, 1509015271, 1803018211,
1923019411 (...),
obtained for $[p, q] = [7, 13], [17, 23], [23, 29],$
 $[37, 43], [53, 59], [101, 107], [157, 163], [193,$
 $199], [223, 229], [311, 317], [331, 337], [461,$
 $467], [503, 509], [601, 607], [641, 647]...$

Conjecture II:

There exist an infinity of primes r obtained concatenating $30*p$ with $30*q$ and subtracting 1 from the

resulted number, where $[p, q]$ is a pair of sexy primes; example: for $[p, q] = [23, 29]$, the number 690869 is prime.

The sequence of primes r:

: 150329, 330509, 690869, 12301409, 30303209,
32103389, 50105189, 66906869, 68106989, 69907169,
75307709, 78908069, 81308309, 1671016889,
1761017789, 1803018209, 1821018389, 1923019409
(...),
obtained for $[p, q] = [5, 11], [11, 17], [23, 29],$
 $[41, 47], [101, 107], [107, 113], [167, 173], [223,$
 $229], [227, 233], [233, 239], [251, 257], [263,$
 $269], [271, 277], [557, 563], [587, 593], [601,$
 $607], [607, 613], [641, 647]...$

Conjecture III:

There exist an infinity of pairs of twin primes obtained concatenating $30 \cdot p$ with $30 \cdot q$ and adding/subtracting 1 from the resulted number, where $[p, q]$ is a pair of sexy primes; example: for $[p, q] = [101, 107]$, the numbers 30303211 and 30303209 are primes.

The sequence of such pairs of twin primes:

: [690869, 690871], [30303209, 30303211], [66906869,
66906869], [1803018209, 1803018211], [1923019409,
1923019411]...
obtained for $[p, q] = [23, 29], [101, 107], [223,$
 $229], [601, 607], [641, 647]...$