Gravity and Static Electricity: The Same Force at the Planck Scale?

Espen Gaarder Haug*
Norwegian University of Life Sciences

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In this paper I will show that Coulomb's electrostatic force formula is exactly the same mathematically as Newton's universal gravitational force at the very bottom of the rabbit hole, that is to say at the Planck scale. There are, therefore, good reasons to expect that they are ultimately the same force. Still, the gravitational force and the electrostatic force clearly look different when we move away from the density of a Planck mass. To claim that static electricity is not ruled by the same fundamental force as gravity could be similar to claiming that kinetic energy not is related to pure energy simply because a moving mass appears to be so different from moving photons (even mathematically).

Key words: Newton's gravitational force, gravitational constant, dimensionless gravitational coupling constant, Coulomb's force, Coulomb's constant, Planck units, quantum realm, Planck mass, fine structure constant.

1 A New Perspective on the Planck Units

Haug (2016a,c) suggest that the gravitational constant should be written as a function of Planck's reduced constant[1]

$$G_p = \frac{l_p^2 c^3}{\hbar} \tag{1}$$

This way of writing Newton's gravitational constant does not change the value of the constant. If one knows the Planck length, then the gravitational constant is known, or alternatively and more practically one can calibrate the Planck length based on empirical measurements of the gravitational constant. Based on this, the Planck length is given by

$$l_p = \sqrt{\frac{\hbar G_p}{c^3}} = \sqrt{\frac{\hbar \frac{l_p^2 c^3}{\hbar}}{c^3}} = l_p \tag{2}$$

Next the Planck mass in this context results in

$$m_p = \sqrt{\frac{\hbar c}{G_p}} = \sqrt{\frac{\hbar c}{\frac{l_p^2 c^3}{\hbar}}} = \frac{\hbar}{l_p}\frac{1}{c} \tag{3}$$

Based on the quantized gravitational constant, the Planck energy can be simplified to

$$E_p = m_p c^2 = \sqrt{\frac{\hbar c}{G_p}} c^2 = \frac{\hbar}{l_p}\frac{1}{c}c^2 = \frac{\hbar}{l_p}c \tag{4}$$

Table 1 summarizes of all of the Planck units given by Haug (2016a) written in a simplified form[2]. This way of writing the Planck units does not change the value of the Planck units; it merely makes it much simpler to interpret the Planck units and their similarities and differences and to get some deeper intuition. One interesting thing to note from the table is that in the Planck form of the Planck units, one has $c^{1.5}$, $c^{2.5}$, $c^{3.5}$ and $c^{4.5}$ as well as c^4, c^5, c^7, c^8 and it is very hard to find any intuition in c powered to such

*e-mail espenhaug@mac.com. Thanks to Victoria Terces for helping me edit this manuscript.
[1] Here we use a notation more familiar to modern physics.
[2] Here we have extended the list to include Planck units linked to electromagnetism as well.

numbers. In the rewritten forms introduced in this paper, we only have c in most of the units, and c^2 for just the Planck power and Planck intensity. The rewritten forms are much easier to work with mathematically and make it easier to see relationships that have not been discussed much before. Here we will look into one such relationship, namely the potential relationship between Coulomb's electrostatic force and Newton's gravitational force.

Table 1: The table shows the standard Planck units and the units rewritten in the simpler and more intuitive form.

Units:	**"Normal"-form:**	**Simplified-form:**
Gravitational constant	$G \approx 6.67408 \times 10^{-11}$	$G_p = \frac{l_p^2 c^3}{\hbar}$
Planck length	$l_p = \sqrt{\frac{\hbar G_p}{c^3}}$	$l_p = l_p$
Planck time	$t_p = \sqrt{\frac{\hbar G_p}{c^5}}$	$t_p = \frac{l_p}{c}$
Planck mass	$m_p = \sqrt{\frac{\hbar c}{G_p}}$	$m_p = \frac{\hbar}{l_p}\frac{1}{c}$
Planck energy	$E_p = \sqrt{\frac{\hbar c^5}{G_p}}$	$E_p = \frac{\hbar}{l_p}c$
Relationship mass and energy	$E_p = m_p c^2$	$\frac{\hbar}{l_p}c = \frac{\hbar}{l_p}\frac{1}{c}c^2$
Reduced Compton wavelength	$\bar{\lambda}_p = \frac{\hbar}{m_p c}$	$\bar{\lambda}_p = l_p$
Planck area	$l_p^2 = \frac{\hbar G_p}{c^3}$	$l_p^2 = l_p^2$
Planck volume	$l_p^3 = \sqrt{\frac{\hbar^3 G_p^3}{c^9}}$	$l_p^3 = l_p^3$
Planck force	$F_p = \frac{c^4}{G_p}$	$F_p = \frac{\hbar}{l_p}\frac{c}{l_p}$
Planck power	$P_p = \frac{c^5}{G_p}$	$P_p = \frac{\hbar}{l_p}\frac{c^2}{l_p}$
Planck mass density	$\rho_p = \frac{c^5}{\hbar G_p^2}$	$\rho_p = \frac{\hbar}{l_p}\frac{1}{c l_p^3}$
Planck energy density	$\rho_p^E = \frac{c^7}{\hbar G_p^2}$	$\rho_p^E = \frac{\hbar}{l_p}\frac{c}{l_p^3}$
Planck intensity	$I_p = \frac{c^8}{\hbar G^2}$	$I_p = \frac{\hbar}{l_p}\frac{c^2}{l_p^3}$
Planck frequency	$\omega_p = \sqrt{\frac{c^5}{\hbar G}}$	$\omega_p = \frac{c}{l_p}$
Planck pressure	$p_p = \frac{c^7}{\hbar G^2}$	$p_p = \frac{\hbar}{l_p}\frac{c}{l_p^3}$
Coulomb's constant	$k_e = c^2 \times 10^{-7} \approx 8.99 \times 10^9$	$k_p = c^2 \times 10^{-7}$
Planck charge	$q_p = \sqrt{4\pi\epsilon_0 \hbar c} = \frac{e}{\sqrt{\alpha}}$	$q_p = \sqrt{\frac{\hbar}{c}}\sqrt{10^7}$
Planck current	$I_p = \sqrt{\frac{4\pi\epsilon_0 c^6}{G}}$	$I_p = \frac{c}{l_p}\sqrt{\frac{\hbar}{c}}\sqrt{10^7}$
Planck voltage	$V_p = \sqrt{\frac{c^4}{4\pi\epsilon_0 G}}$	$V_p = \frac{c}{l_p}\sqrt{c\hbar}\sqrt{10^{-7}}$
Planck impedance	$Z_p = \frac{1}{4\pi\epsilon_0 c}$	$Z_p = c \times 10^{-7}$

2 The Same Force?

In 1686, Isaac Newton published his law of the gravitational force, given by,

$$F_G = G\frac{m_1 m_2}{r^2} \tag{5}$$

where $G \approx 6.674 \times 10^{-11}$ is Newton's gravitational constant and m_1 and m_2 are two masses, and r is the distance between the centers of the masses. In 1784, almost hundred years after Newton published the gravitational force formula, Charles Augstin de Coulomb described the force interacting between static electrically charged particles as

$$F_C = k_e\frac{q_1 q_2}{r^2} \tag{6}$$

where $k_e = c^2 \times 10^{-7} \approx 8.99 \times 10^9$ is Coulomb's constant and q_1 and q_2 are the two charges and r is the distance between the center of the masses. Coulomb's force and Newton's gravitational force look remarkably similar from a purely functional form. They both follow the so-called inverse square law, but they are considered to be two different forces by modern physics. The Coulomb constant k_e and the Newton gravitational constant G have very different values, where Coulomb's formula require charges as inputs and Newtons formula requires masses. However, when we first rewrite the formulas in the quantized forms based on the Planck units given in Table 1, we can see that they are exactly the same force, at least mathematically. Newton's law of gravitation can be rewritten as

$$
\begin{aligned}
F_G &= G_p \frac{m_1 m_2}{r^2} \\
F_G &= G_p \frac{N_1 m_p N_2 m_p}{r^2} \\
F_G &= G_p \frac{N_1 \frac{\hbar}{l_p}\frac{1}{c} N_2 \frac{\hbar}{l_p}\frac{1}{c}}{r^2} \\
F_G &= \frac{l_p^2 c^3}{\hbar} \frac{N_1 \frac{\hbar}{l_p}\frac{1}{c} N_2 \frac{\hbar}{l_p}\frac{1}{c}}{r^2} \\
F_G &= N_1 N_2 \frac{\hbar c}{r^2}
\end{aligned} \tag{7}
$$

where N_1 and N_2 are the numbers of Planck masses in mass one and mass two respectively. In the special case when we simply have two Planck masses and where $r = l_p$, we simply get

$$F_G = \frac{\hbar}{l_p}\frac{c}{l_p} \tag{8}$$

I will claim the first part $\frac{\hbar}{l_p}$ is just a "scaling coefficient" and that the essence in gravity truly is $\frac{c}{l_p}$, which can be seen as number of hits per second. However, one does not need to assume this interpretation of the gravitational force to go further in this paper. Coulomb's law rewritten in Planck form is given by

$$
\begin{aligned}
F_C &= k_e \frac{q_1 q_2}{r^2} \\
F_C &= k_e \frac{N_1 q_p N_2 q_p}{r^2} \\
F_C &= c^2 \times 10^{-7} \frac{N_1 \sqrt{\frac{\hbar}{c}}\sqrt{10^7} N_2 \sqrt{\frac{\hbar}{c}}\sqrt{10^7}}{r^2} \\
F_C &= N_1 N_2 \frac{\hbar c}{r^2}
\end{aligned} \tag{9}
$$

From the derivations above, it seems that the Newton and Coulomb constants, G and k_e, have no deeper meaning other than to manipulate their input into the correct formula for the same force. We could just as well have come up with another formula based on the total rest mass energy of the objects in question, for example, and then introduced yet another constant to turn these two rest mass energies into the gravitational force. This is no surprise, as the insight into the quantum realm and the relationship between energy and matter was much more limited back in Newton and Coulomb's time. Naturally we also have

$$
\begin{aligned}
\frac{F_C}{F_G} &= \frac{k_e \frac{N_1 q_p N_2 q_p}{r^2}}{G_p \frac{N_1 m_p N_2 m_p}{r^2}} \\
\frac{F_C}{F_G} &= \frac{k_e N_1 q_p N_2 q_p}{G_p N_1 m_p N_2 m_p} \\
\frac{F_C}{F_G} &= \frac{N_1 N_2 \frac{\hbar c}{r^2}}{N_1 N_2 \frac{\hbar c}{r^2}} = 1
\end{aligned} \tag{10}
$$

That is to say the gravitational force and the Coulomb's electrostatic force are ultimately the same formula at the Planck scale and they naturally also have the same strength.

3 Is The Charge Scaling Factor an Artifact?

If you look closely at Table 1, you will observe that the Planck units related to electricity, unlike all the other Planck units, are scaled by a numerical value. For example the Planck charge is multiplied by $\sqrt{10^7}$

$$q_p = \sqrt{\frac{\hbar}{c}}\sqrt{10^7} \tag{11}$$

We are not sure, but we suspect this could be another "artifact" due to slightly incorrectly scaled inputs. This would also be no surprise, as the charge notation was designed long before we started to understand the quantum realm. We might consider the question of whether we would ever observe independent charges in reality or not. If we always need two so-called opposite charges to observe static electricity, then these two charges are always multiplied by the Coulomb's constant k_e. The Coulomb's constant is simply $c^2 \times 10^{-7}$. The 10^{-7} term in the Coulomb's constant seems to be present to correct for the 10^7 scaling "missterm" in the charges. Would it not be simpler and more logical to always operate with charges scaled by 10^{-7} and have Coulomb's constant scale by 10^7? In other words, remove these numerical scaling constants all together. One could just as well claim that the Planck charge is $\sqrt{\frac{\hbar}{c}}$ rather than $\sqrt{\frac{\hbar}{c}}\sqrt{10^7}$ and that the rescaled Coulomb's constant simply should be written as c^2.

4 The Link to the Small Gravitational Coupling Constant

There is also an interesting link between the Coulomb force for Planck charges and the gravitational force between two electrons:

$$\begin{aligned}
\frac{F_G}{F_C} &= \frac{Gm_em_e}{k_eq_pq_p} \\
\frac{F_G}{F_C} &= \frac{\frac{l_p^2c^3}{\hbar}\frac{\hbar}{\bar{\lambda}_e}\frac{1}{c}\frac{\hbar}{\bar{\lambda}_e}\frac{1}{c}}{c^2\sqrt{\frac{\hbar}{c}}\sqrt{\frac{\hbar}{c}}} \\
\frac{F_G}{F_C} &= \frac{\frac{l_p^2c}{\hbar}\frac{\hbar}{\bar{\lambda}_e}\frac{\hbar}{\bar{\lambda}_e}}{\hbar c} \\
\frac{F_G}{F_C} &= \frac{\frac{l_p^2}{\bar{\lambda}_e^2}\hbar c}{\hbar c} \\
\frac{F_G}{F_C} &= \frac{l_p^2}{\bar{\lambda}_e^2}
\end{aligned} \tag{12}$$

The result is known as the small gravitational coupling constant:

$$\alpha_G = \frac{l_p^2}{\bar{\lambda}_e^2} = \frac{m_p}{m_e} = \frac{Gm_e^2}{\hbar c} \tag{13}$$

That the small gravitational coupling constant can be written in this form, as recently shown by Haug (2016b). The gravitational coupling constant is often described as the dimensionless gravitational constant and has been discussed in a series of papers in theoretical physics, see Silk (1977), Rozental (1980), Neto (2005) and Burrows and Ostriker (2013), for example. The dimensionless gravitational coupling constant is only dimensionless in the sense that it does not change value if we change the unit systems of the speed of light, etc. It is not dimensionless in the sense that it holds between any two masses. It could be better described as the dimensionless electron gravitational coupling constant, as it gives the gravitational relationship between two electrons relative to that of two Planck masses. For two Planck masses, the gravitational coupling constant is

$$\alpha_G = \frac{l_p^2}{l_p^2} = 1 \tag{14}$$

This later coupling constant is a more fundamental dimensionless constant that indirectly shows that the gravitational force is identical to Coulomb's force for Planck masses.

5 The Relation to the Fine Structure Constant

In modern physics, the charge of an electron is given by $e = q_p\sqrt{\alpha}$ where α is the fine structure constant. We also have

$$\alpha \quad = \quad = \frac{k_e q_p q_p \frac{\bar{\lambda}_e}{a_0}}{G m_p m_p} \approx 0.007297356 \tag{15}$$

where $a_0 = \frac{\hbar}{m_e c\alpha}$ is the Bohr radius. This can be rewritten as

$$\begin{aligned} a_0 &= \frac{\hbar}{m_e c\alpha} \\ a_0 &= \frac{\hbar}{\frac{\hbar}{\bar{\lambda}_e}\frac{1}{c}c\alpha} \\ a_0 &= \frac{\bar{\lambda}_e}{\alpha} \end{aligned} \tag{16}$$

This means that the Bohr radius is $\frac{1}{\alpha} \approx 137.04$ times the electron radius. If the electron moves back and forth across the Bohr radius (rather than around the Bohr circumference), then it has to move 137 times as long as the reduced Compton wave length of the electron, $\bar{\lambda}_e$. This could be the logical reason for the reduced expression of the electron charge force. The fine structure constant has also been interpreted as the velocity of the electron $v_e = \alpha c$. It seems that modern physics cannot fully agree on what the fine structure constant is at a deep and logical level. To say the force between charges not is the same as the gravitational force could be somewhat similar to saying kinetic energy not is energy, although I do not claim to have in-depth understanding of the fine structure constant and exactly how to interpret the elementary charge. Still, I will note that the electrostatic force is very similar to the gravitational force and could very likely be two different faces of the same underlying fundamental force.

6 Conclusion

We claim that Coulomb's electrostatic force and Newton's gravitational force are the same force, at least mathematically, at the quantum level, that is at the Planck scale. The constants in the Coulomb's force law and Newton's gravitational force law could simply be seen as a mathematical "artifacts" necessary to manipulate different types of input into the right formula for the force. We do not claim to have solved all of the questions concerning the similarities and differences between static electricity and the gravitational force, but we hope this paper can be a small contribution on the road to an even better understanding of the quantum realm. We will end on a light note by saying: May the Force be with you!

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