Short Note n°2: Number Pi

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abstract

In this note we show some formulas related with: Number Pi

Fórmula Para Pi

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Resumen-Abstract

En esta nota mostramos una fórmula para la constante Pi:

$$\pi = 4\left(1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \cdots\right) = 3.141592\ldots$$

Introducción

Tres números reales: s, c, r , definidos como sigue:

(1) $$s = \sqrt{\frac{\left(-9+i\sqrt{111}\right)^{1/3}}{2\cdot 3^{2/3}} + \frac{2}{\left(3\left(-9+i\sqrt{111}\right)\right)^{1/3}}}$$

(2) $$c = \frac{1}{3} - \frac{2\cdot 2^{2/3}\left(1-i\sqrt{3}\right)}{3\left(-5+3i\sqrt{111}\right)^{1/3}} - \frac{\left(1+i\sqrt{3}\right)\left(-5+3i\sqrt{111}\right)^{1/3}}{6\cdot 2^{2/3}}$$

(3) $$r = \sqrt{\frac{1}{3} - \frac{4\cdot 2^{2/3}\left(1-i\sqrt{3}\right)}{3\left(5+3i\sqrt{111}\right)^{1/3}} - \frac{\left(1+i\sqrt{3}\right)\left(5+3i\sqrt{111}\right)^{1/3}}{3\cdot 2^{2/3}}}$$

Los valores aproximados de s, c, r , son:

(4) $$\begin{cases} s = 0.9151860113022505\ldots \\ c = 0.4030317167626847\ldots \\ r = 0.4403822958233339\ldots \end{cases}$$

Los números s, c, r , satisfacen las siguientes ecuaciones polinomiales:

(5) $$4s^6 - 4s^2 + 1 = 0$$

(6) $$2c^3 - 2c^2 - 2c + 1 = 0$$

(7) $$r^6 - r^4 - 5r^2 + 1 = 0$$

Algunas fórmulas interesantes son:

(8) $$s=\sqrt{\frac{1}{\sqrt{3}}+\sqrt{\frac{8-3\sqrt{3}}{36+12\sqrt{3}\sqrt{\frac{8-3\sqrt{3}}{36+12\sqrt{3}\sqrt{\frac{8-3\sqrt{3}}{36+12\sqrt{3}\ldots.}}}}}}}$$

(9) $$c=\frac{1}{3}+\frac{5}{72}+\frac{3}{4}\left(\frac{5}{72}+\frac{3}{4}\left(\frac{5}{72}+\cdots\right)^3\right)^3$$

(10) $$r=\sqrt{-1+\sqrt{\frac{4}{4-\sqrt{\frac{4}{4-\sqrt{\frac{4}{4-\cdots}}}}}}}$$

(11) $$s=\sqrt{\left(\sqrt[3]{\frac{1}{4}+\sqrt[3]{\frac{1}{4}+\sqrt[3]{\frac{1}{4}+\cdots}}}\right)-\left(\frac{1}{4}+\left(\frac{1}{4}+\left(\frac{1}{4}+\cdots\right)^3\right)^3\right)}$$

(12) $$s^2+c^2=1$$

(13) $$r\,s-c=0$$

Fórmula Para Pi

(14)

$$\pi=8\sum_{n=0}^{\infty}\frac{1}{2n+1}r^{2n+1}\sin\left((2n+1)\cot^{-1}r\right)=$$

$$8\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{c}{s}\right)^{2n+1}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{2k+1}c^{2n-2k}s^{2k+1}=$$

$$8\sum_{n=0}^{\infty}\frac{1}{2n+1}\left(\frac{\sqrt{1-s^2}}{s}\right)^{2n+1}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{2k+1}(1-s^2)^{n-k}s^{2k+1}=$$

$$8\sum_{n=0}^{\infty}\frac{1}{2n+1}\frac{c^{2n+1}}{(1-c^2)^n}\sum_{k=0}^{n}(-1)^k\binom{2n+1}{2k+1}c^{2n-2k}(1-c^2)^k$$

Referencias

[1] Valdebenito, E.:, Pi Handbook , manuscript , unpublished , 1989 , (20000 formulas).