Two conjectures on the primes which admit deconcatenation in two primes, involving multiples of 30

Abstract. In this paper I state the following two conjectures: (I) If p is a prime which admits deconcatenation in two primes p1 and p2, both of the form 6*k – 1, then there exist an infinity of primes q obtained concatenating q1 with q2, where q1 = 30*n – p1, q2 = 30*n – p2 and n positive integer; (II) If p is a prime which admits deconcatenation in two primes p1 and p2, both of the form 6*k + 1, then there exist an infinity of primes q obtained concatenating q1 with q2, where q1 = 30*n + p1, q2 = 30*n + p2 and n positive integer.

Conjecture 1:

If p is a prime which admits deconcatenation in two primes p1 and p2, both of the form 6*k – 1, then there exist an infinity of primes q obtained concatenating q1 with q2, where q1 = 30*n – p1, q2 = 30*n – p2 and n positive integer.

The sequence of q for p = 523 ([p1, p2] = [5, 23]:

: q = 257, for n = 1 and [q1, q2] = [25, 7];
: q = 11597, for n = 4 and [q1, q2] = [115, 97];
: q = 205187, for n = 7 and [q1, q2] = [205, 187];
(...)

The sequence of q for p = 541 ([p1, p2] = [5, 41]:

: q = 5519, for n = 2 and [q1, q2] = [55, 19];
: q = 11579, for n = 4 and [q1, q2] = [115, 79];
: q = 145109, for n = 5 and [q1, q2] = [145, 109];
(...)

The sequence of q for p = 1117 ([p1, p2] = [11, 17]:

: q = 1913, for n = 1 and [q1, q2] = [19, 13];
: q = 4943, for n = 2 and [q1, q2] = [49, 43];
: q = 109103, for n = 4 and [q1, q2] = [109, 103];
(...)

The sequence of q for p = 1123 ([p1, p2] = [11, 23]:

: q = 197, for n = 1 and [q1, q2] = [19, 7];
: q = 4937, for n = 2 and [q1, q2] = [49, 37];
: q = 349337, for n = 12 and [q1, q2] = [349, 337];

(...)

Observation:

The conjecture above seems to apply as well to Poulet numbers which admit the mentioned deconcatenation.

The sequence of q for p = 49141 ([p1, p2] = [491, 41]:

: q = 19469, for n = 17 and [q1, q2] = [19, 469];
: q = 49499, for n = 18 and [q1, q2] = [49, 499];
: q = 139589, for n = 21 and [q1, q2] = [139, 589];
(...)

The sequence of q for p = 1729 ([p1, p2] = [17, 29]:

: q = 131, for n = 1 and [q1, q2] = [13, 1];
: q = 10391, for n = 4 and [q1, q2] = [103, 91];
: q = 133121, for n = 5 and [q1, q2] = [133, 121];
: q = 163151, for n = 6 and [q1, q2] = [163, 151];
: q = 193181, for n = 7 and [q1, q2] = [193, 181];
: q = 223211, for n = 8 and [q1, q2] = [223, 211];
(...)

Note the chain of five successive primes (10391, 133121, 163151, 193181, 223211) obtained for n from 4 to 8.

Conjecture 2:

If p is a prime which admits deconcatenation in two primes p1 and p2, both of the form 6*k + 1, then there exist an infinity of primes q obtained concatenating q1 with q2, where q1 = 30*n + p1, q2 = 30*n + p2 and n positive integer.

The sequence of q for p = 719 ([p1, p2] = [7, 19]:

: q = 6779, for n = 2 and [q1, q2] = [67, 79];
: q = 127139, for n = 4 and [q1, q2] = [127, 139];
: q = 217229, for n = 7 and [q1, q2] = [217, 229];
(...)

The sequence of q for p = 743 ([p1, p2] = [7, 43]:

: q = 67103, for n = 2 and [q1, q2] = [67, 103];
: q = 127163, for n = 4 and [q1, q2] = [127, 163];
: q = 187223, for n = 6 and [q1, q2] = [187, 223];
(...)

The sequence of q for p = 137 ([p1, p2] = [13, 7]:

```
:    q = 4337,       for n = 1  and [q1, q2] = [43, 37];
:    q = 223217,     for n = 7  and [q1, q2] = [223, 217];
:    q = 253247,     for n = 8  and [q1, q2] = [253, 247];
     (...)
```

The sequence of q for p = 1319 ([p1, p2] = [13, 19]:

```
:    q = 4349,       for n = 1  and [q1, q2] = [43, 49];
:    q = 163169,     for n = 5  and [q1, q2] = [163, 169];
:    q = 223229,     for n = 7  and [q1, q2] = [223, 229];
     (...)
```