Two conjectures on the primes which admit deconcatenation in two primes, involving multiples of 30

Abstract. In this paper I state the following two conjectures: (I) If $p$ is a prime which admits deconcatenation in two primes $p_1$ and $p_2$, both of the form $6*k - 1$, then there exist an infinity of primes $q$ obtained concatenating $q_1$ with $q_2$, where $q_1 = 30*n - p_1$, $q_2 = 30*n - p_2$ and $n$ positive integer; (II) If $p$ is a prime which admits deconcatenation in two primes $p_1$ and $p_2$, both of the form $6*k + 1$, then there exist an infinity of primes $q$ obtained concatenating $q_1$ with $q_2$, where $q_1 = 30*n + p_1$, $q_2 = 30*n + p_2$ and $n$ positive integer.

Conjecture 1:

If $p$ is a prime which admits deconcatenation in two primes $p_1$ and $p_2$, both of the form $6*k - 1$, then there exist an infinity of primes $q$ obtained concatenating $q_1$ with $q_2$, where $q_1 = 30*n - p_1$, $q_2 = 30*n - p_2$ and $n$ positive integer.

The sequence of $q$ for $p = 523$ ($[p_1, p_2] = [5, 23]$):

: $q = 257$, for $n = 1$ and $[q_1, q_2] = [25, 7]$;
: $q = 11597$, for $n = 4$ and $[q_1, q_2] = [115, 97]$;
: $q = 205187$, for $n = 7$ and $[q_1, q_2] = [205, 187]$;
(...)

The sequence of $q$ for $p = 541$ ($[p_1, p_2] = [5, 41]$):

: $q = 5519$, for $n = 2$ and $[q_1, q_2] = [55, 19]$;
: $q = 11579$, for $n = 4$ and $[q_1, q_2] = [115, 79]$;
: $q = 145109$, for $n = 5$ and $[q_1, q_2] = [145, 109]$;
(...)

The sequence of $q$ for $p = 1117$ ($[p_1, p_2] = [11, 17]$):

: $q = 1913$, for $n = 1$ and $[q_1, q_2] = [19, 13]$;
: $q = 4943$, for $n = 2$ and $[q_1, q_2] = [49, 43]$;
: $q = 109103$, for $n = 4$ and $[q_1, q_2] = [109, 103]$;
(...)

The sequence of $q$ for $p = 1123$ ($[p_1, p_2] = [11, 23]$):

: $q = 197$, for $n = 1$ and $[q_1, q_2] = [19, 7]$;
: $q = 4937$, for $n = 2$ and $[q_1, q_2] = [49, 37]$;
: $q = 349337$, for $n = 12$ and $[q_1, q_2] = [349, 337]$;
Observation:

The conjecture above seems to apply as well to Poulet numbers which admit the mentioned deconcatenation.

The sequence of $q$ for $p = 49141$ ([p1, p2] = [491, 41]):

: $q = 19469$, for $n = 17$ and $[q_1, q_2] = [19, 469]$;
: $q = 49499$, for $n = 18$ and $[q_1, q_2] = [49, 499]$;
: $q = 139589$, for $n = 21$ and $[q_1, q_2] = [139, 589]$;

(...)

The sequence of $q$ for $p = 1729$ ([p1, p2] = [17, 29]):

: $q = 131$, for $n = 1$ and $[q_1, q_2] = [13, 1]$;
: $q = 10391$, for $n = 4$ and $[q_1, q_2] = [103, 91]$;
: $q = 133121$, for $n = 5$ and $[q_1, q_2] = [133, 121]$;
: $q = 163151$, for $n = 6$ and $[q_1, q_2] = [163, 151]$;
: $q = 193181$, for $n = 7$ and $[q_1, q_2] = [193, 181]$;
: $q = 223211$, for $n = 8$ and $[q_1, q_2] = [223, 211]$;

(...)

Note the chain of five successive primes (10391, 133121, 163151, 193181, 223211) obtained for $n$ from 4 to 8.

Conjecture 2:

If $p$ is a prime which admits deconcatenation in two primes $p_1$ and $p_2$, both of the form $6k + 1$, then there exist an infinity of primes $q$ obtained concatenating $q_1$ with $q_2$, where $q_1 = 30n + p_1$, $q_2 = 30n + p_2$ and $n$ positive integer.

The sequence of $q$ for $p = 719$ ([p1, p2] = [7, 19]):

: $q = 6779$, for $n = 2$ and $[q_1, q_2] = [67, 79]$;
: $q = 127139$, for $n = 4$ and $[q_1, q_2] = [127, 139]$;
: $q = 217229$, for $n = 7$ and $[q_1, q_2] = [217, 229]$;

(...)

The sequence of $q$ for $p = 743$ ([p1, p2] = [7, 43]):

: $q = 67103$, for $n = 2$ and $[q_1, q_2] = [67, 103]$;
: $q = 127163$, for $n = 4$ and $[q_1, q_2] = [127, 163]$;
: $q = 187223$, for $n = 6$ and $[q_1, q_2] = [187, 223]$;

(...)

The sequence of $q$ for $p = 137$ ([p1, p2] = [13, 7]):
q = 4337, for n = 1 and [q₁, q₂] = [43, 37];
q = 223217, for n = 7 and [q₁, q₂] = [223, 217];
q = 253247, for n = 8 and [q₁, q₂] = [253, 247];
(...)

The sequence of q for p = 1319 ([p₁, p₂] = [13, 19]):

q = 4349, for n = 1 and [q₁, q₂] = [43, 49];
q = 163169, for n = 5 and [q₁, q₂] = [163, 169];
q = 223229, for n = 7 and [q₁, q₂] = [223, 229];
(...)