Legendre Polynomials , Chebyshev Polynomials , C_n Polynomials , Number Pi

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abstract

In this note we show some formulas related with: Legendre Polynomials $P_n(x)$, Chebyshev Polynomials $T_n(x)$, Polynomials $C_n(x)$, and Number Pi

Número π, Polinomios de Legendre $P_n(x)$, Polinomios de Chebyshev $T_n(x)$, Polinomios $C_n(x)$

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Resumen

Se muestran fórmulas que involucran la constante Pi , los polinomios de Legendre $P_n(x)$, los polinomios de Chebyshev $T_n(x)$ y los polinomios $C_n(x)$.

1. Introducción

La constante Pi se define por:

$$\pi = 4\sum_{n=0}^{\infty}(-1)^n(2n+1)^{-1} \tag{1}$$

Los polinomios de Legendre se definen por:

$$P_n(x)=\frac{1}{2^n n!}\frac{d^n}{dx^n}\left(x^2-1\right)^n \quad ,x\in[-1,1],n\in\mathbb{N}\cup\{0\} \tag{2}$$

Los polinomios de Chebyshev se definen por:

$$T_n(x)=\cos\left(n\cos^{-1}(x)\right)=x^n-\binom{n}{2}x^{n-2}\left(1-x^2\right)+\binom{n}{4}x^{n-4}\left(1-x^2\right)^2-\ldots \tag{3}$$

$$n\in\mathbb{N}\cup\{0\},x\in[-1,1]$$

Los polinomios $C_n(x)$ se definen por:

$$C_n(x)=\frac{1}{4x\sqrt{x^2-1}}\left(\left(x-\sqrt{x^2-1}\right)^n\left(1-2x^2+2x\sqrt{x^2-1}\right)\right.$$
$$\left.-\left(x+\sqrt{x^2-1}\right)^n\left(1-2x^2-2x\sqrt{x^2-1}\right)+2\sqrt{x^2-1}\,sen\left(\frac{n\pi}{2}\right)\right) \tag{4}$$

$$n \in \mathbb{N} \cup \{0\}, x \in [-1,1]$$

2. Fórmulas

$$\frac{\pi}{6} + 2x \sum_{n=0}^{\infty} \frac{C_n(x)}{(n+2)\left(\sqrt{3}\right)^{n+2}} = \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{P_n(x) P_m(x)}{(n+m+1)\left(\sqrt{3}\right)^{n+m+1}} \tag{5}$$

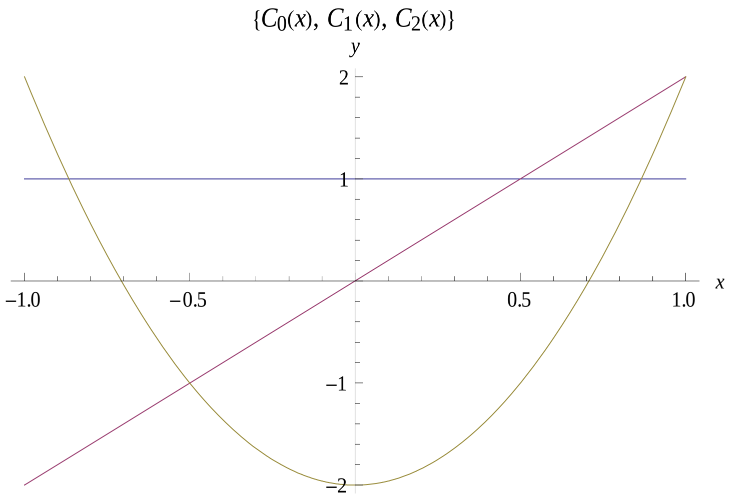
$$\frac{\pi}{6} + \frac{2x}{3} \sum_{n=0}^{\infty} \frac{(n+5) C_n(x)}{(n+2)(n+4)\left(\sqrt{3}\right)^{n+2}} = \sum_{n=0}^{\infty} \frac{T_n(x)}{(n+1)\left(\sqrt{3}\right)^{n+1}} \tag{6}$$

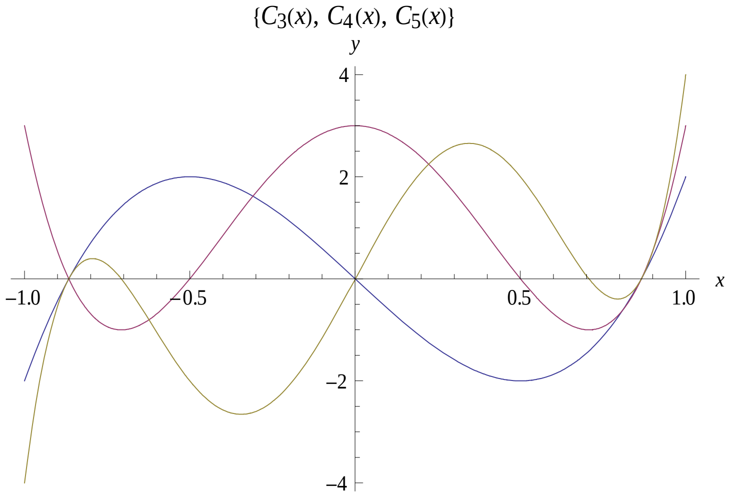
En (5) y(6) $x \in [-1,1]$.

3. Los Polinomios $C_n(x)$

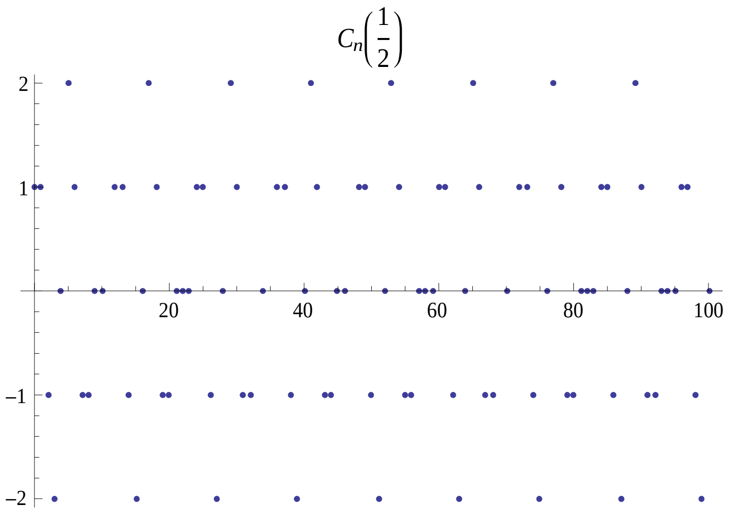
$$C_0(x) = 1, C_1(x) = 2x, C_2(x) = 4x^2 - 2, C_3(x) = 8x^3 - 6x \tag{7}$$

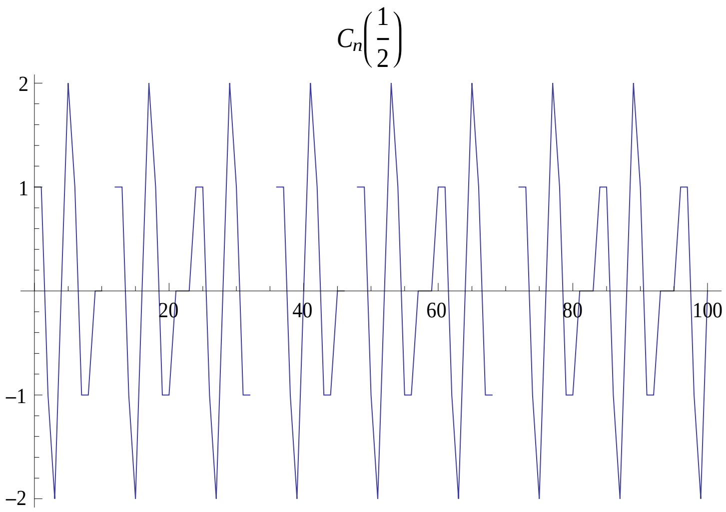
$$C_4(x) = 16x^4 - 16x^2 + 3, C_5(x) = 32x^5 - 40x^3 + 12, \ldots \tag{8}$$



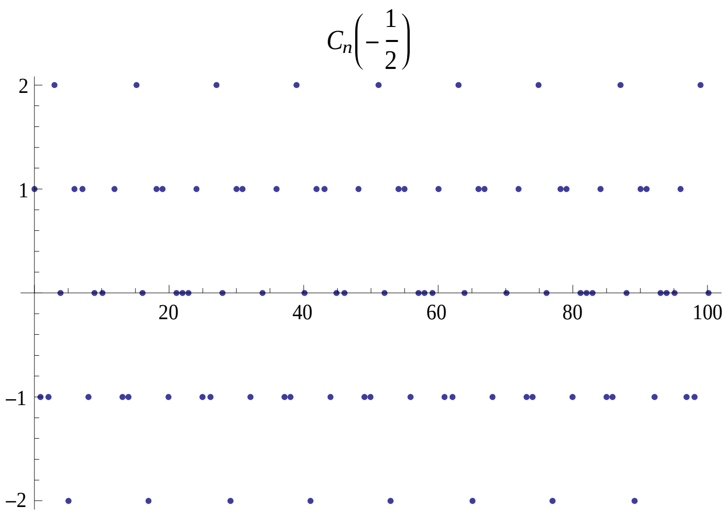


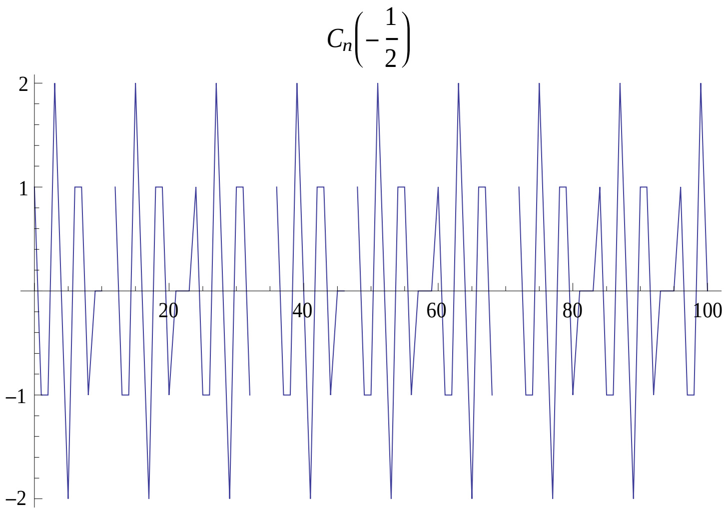
4. La Sucesión $C_n\left(\frac{1}{2}\right)$





5. La Sucesión $C_n\left(-\frac{1}{2}\right)$





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