Violation of no-signaling constraint by distinguishing local quantum measurement with numerous eight-particle GHZ states

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ABSTRACT

The distinction of quantum measurements is one of the fundamentally important problems in quantum information science. In this paper we present a novel protocol for distinguishing local quantum measurement (DLQM) with multi-particle entanglement systems. It is shown that, for two spacelike separated parties, the local discrimination of two different kinds of measurement can be completed via numerous eight-particle GHZ entangled states and selective projective measurements without help of classical information. This means that no-signaling constraint can be violated by the DLQM.

Keywords: No-signaling constraint, Distinguishing local quantum measurement, Multi-particle GHZ entangled states, Selective projective measurement

1. Introduction

Quantum entanglement is one of the striking features of quantum mechanics [1]. The nonlocal nature of entanglement is the essential resource for many quantum information tasks including teleportation [2] and superdense coding [3]. However, although entanglement appears to allow particles which are separated in space to influence one another instantaneously [4], it has been pointed that this cannot be used to signal without help of classical communication [5-8], *i. e*. the no-signaling constraint [9] holds that one cannot exploit quantum entanglement to transmit classical information across spacelike intervals.

On the other hand, it is well-known that measurement is a central tenet of quantum mechanics. The problem of discrimination between quantum measurements has been recently considered in quantum information tasks [10-13]. Ji *et al*. [10] have proposed simple schemes that can perfectly idetify projective measurement apparatuses secretly chosen from a finite set. Entanglement is used in this schemes both to make possible the perfect identification and to improve the efficiency significantly. Fiurasek and Micuda [11] have studied optimal discrimination between two projective quantum measurements on a single qubit. Ziman *et al* . [12] have investigated the unambiguous comparison of unknown quantum measurements represented by nondegenerate sharp positive operator valued measures (POVM). One can notice that, in above works [10-13] of discriminating quantum measurement, employing classical communication is necessary.

For two spacelike separated parties, existing researches [5-9] have pointed that since no-signaling constraint, the local discrimination of quantum operations cannot be completed. In the last decade, the correctness of no-signaling constraint has been frequently discussed and proved (*e. g*. [14-19]). It is easy found that, however, in these discussions and demonstrations, only a single two- or multi-particle entangled state has been employed. By a careful analysis, it may be found that, if multiple multi-particle entangled states and a kind of special measurement (called selective projective measurement) are employed, the local discrimination of quantum measurements can be realized without assistance of classical communication. In this work, we first present a protocol for distinguishing local quantum measurement (DLQM) via selective projective measurement with numerous eight-particle GHZ entangled states. It is shown that, in this protocol, if both two observers (Charlie and Dick) agreed in advance that one of them (*e.g*. Charlie) should measure her qubits before an appointed time (it is equivalent that, after her measurement, Alice only announced publicly that she had completed the measurement, and did not declare the result of her measurement), the local discrimination of two different kinds of measurement can be realized by using a series of single-particle correlative measuring basis without help of classical communication. This means that the DLQM protocol may be not restricted by the no-signaling.

The structure of the paper is the following one: In section 2 we review local quantum measurement with a single entangled state. Section 3 describes two different kinds of quantum measurement. In section 4, a novel protocol for discrimination of local quantum measurements with numerous eight-particle GHZ states is presented. Finally, discussion and conclusion are given in section 5.

2. DLQM with a single entangled state

To present our protocol more clearly, let us first review existing scheme of DLQM (*e. g*. [19]). Assume that Alice and Bob share bipartite quantum system described by a known state ρ. They can make local measurements, with elements

$$\sum_i A_i^\dagger A_i = I \quad , \quad \sum_j B_j^\dagger B_j = I \tag{1}$$

on the subsystems A and B respectively, where A_i and B_j are the "detector operators" associated to the elements of a POVM for the observation of results μ_A by Alice and ν_B by Bob. If Bob is not informed that Alice got outcome μ_A, the mean value that he gets any observable ν_B is

$$\begin{aligned}\langle \nu_B \rangle &= tr_{AB}\left\{\sum_i A_i \rho A_i^\dagger \nu_B\right\} = tr_A\left\{\sum_i A_i^\dagger A_i tr_B\left\{\rho \nu_B\right\}\right\} \\ &= tr_B\left\{tr_A\left\{\rho\right\}\nu_B\right\} \quad .\end{aligned} \tag{2}$$

Since the result of Eq. (2) does not depend on Alice's operators, Bob cannot decide what measurements Alice did without her help. As described above, it is clearly shown that the DLQM with a single entangled state must obey the no-signaling constraint.

3. Two different kinds of quantum projective measurement

Suppose that an eight-particle GHZ state is shared by Charlie and Dick,

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|00000000\rangle + |11111111\rangle\right)_{C_1C_2C_3C_4C_5C_6C_7D} \quad , \tag{3}$$

here particles C_1, C_2, …, C_7 are in the possession of Charlie and D belongs to Dick. Assume that Charlie and Dick agreed in advance that Charlie should measure his particles before an appointed time. Now, let Charlie measure the state $|\Psi\rangle$ by using two different kinds of measurement. In the first kind of measurement, Charlie makes common projective measurements (CPMs) on his particles C_1, C_2, …, and C_7 in the measurement basis $\{|x^+\rangle, |x^-\rangle\}$, where $|x^+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)$, $|x^-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)$, successively. One can see that, after measurements of Charlie, 128 possible final collapsed states of the qubit D will always be $\frac{1}{8\sqrt{2}}|x^+\rangle_D$ or $\frac{1}{8\sqrt{2}}|x^-\rangle_D$. Now we turn to the second kind of measurement. To realize the DLQM, Charlie will utilize a novel kind of projective measurements, which we refer to as selective projective measurements (SPMs), with a series of single-particle correlative measuring basis, on his particles . Firstly, Charlie measures his particle C_1 in the state $|\Psi\rangle$ in the basis $\{|\tau\rangle, |\tau^\perp\rangle\}$, where $|\tau\rangle = \alpha|0\rangle + \beta|1\rangle$, $|\tau^\perp\rangle = \beta|0\rangle - \alpha|1\rangle$, α and β are real, $\alpha^2 + \beta^2 = 1$, and let $\alpha = \sqrt{6}/3$, $\beta = \sqrt{3}/3$. If measurement outcome of Charlie is $|\tau\rangle_{C_1}$, the state of qubits C_2, C_3, …, C_7 and D will evolve as

$$|\psi_1\rangle = \frac{1}{\sqrt{2}}\left(\alpha|0000000\rangle + \beta|1111111\rangle\right)_{C_2C_3C_4C_5C_6C_7D}, \tag{4}$$

he can in turn measure the particles C_2, C_3, …, C_7 in the basis $\{|x^+\rangle, |x^-\rangle\}$. After that, the particle D will always be in the state $\frac{1}{8}|\lambda^+\rangle_D$ or $\frac{1}{8}|\lambda^-\rangle_D$, here $|\lambda^+\rangle = \frac{1}{\sqrt{2}}(\alpha|0\rangle + \beta|1\rangle)$ and $|\lambda^-\rangle = \frac{1}{\sqrt{2}}(\alpha|0\rangle - \beta|1\rangle)$. If measurement result of Charlie is $|\tau^\perp\rangle_{C_1}$, the particles C_2, C_3, …, C_7 and D will be in the state of

$$\left|\psi_1{}'\right\rangle = \frac{1}{\sqrt{2}}\left(\beta\left|0000000\right\rangle - \alpha\left|1111111\right\rangle\right)_{C_2C_3C_4C_5C_6C_7D} \quad . \tag{5}$$

Then Charlie measures his particle C_2 in the measurement basis $\left\{\left|\xi_1\right\rangle, \left|\xi_1^{\perp}\right\rangle\right\}$, which may be expressed as

$$\left|\xi_1\right\rangle = \frac{1}{T_1}\left(\frac{\alpha}{\beta}\left|0\right\rangle + \frac{\beta}{\alpha}\left|1\right\rangle\right), \qquad \left|\xi_1^{\perp}\right\rangle = \frac{1}{T_1}\left(\frac{\beta}{\alpha}\left|0\right\rangle - \frac{\alpha}{\beta}\left|1\right\rangle\right), \tag{6}$$

where $T_1 = \left[\left(\alpha/\beta\right)^2 + \left(\beta/\alpha\right)^2\right]^{1/2}$. Corresponding to outcome of Charlie's measurement $\left|\xi_1\right\rangle_{C_2}$ or $\left|\xi_1^{\perp}\right\rangle_{C_2}$, the state of particles C_3, C_4, … , C_7 and D will evolve as $\left|\psi_2\right\rangle$ or $\left|\psi_2{}'\right\rangle$, which can be expressed as

$$\left|\psi_2\right\rangle = \frac{1}{\sqrt{2}T_1}\left(\alpha\left|000000\right\rangle - \beta\left|111111\right\rangle\right)_{C_3C_4C_5C_6C_7D} \quad ,$$

$$\left|\psi_2{}'\right\rangle = \frac{1}{\sqrt{2}T_1}\left(\frac{\beta^2}{\alpha}\left|000000\right\rangle + \frac{\alpha^2}{\beta}\left|111111\right\rangle\right)_{C_3C_4C_5C_6C_7D} \quad . \tag{7}$$

As described above, we can easy find that the goal of the SPMs is as much as possible to make the particle D collapsed into the state $\frac{1}{G}\left|\lambda^{+}\right\rangle$ or $\frac{1}{G}\left|\lambda^{-}\right\rangle$ after all, where G is a constant or a coefficient related to α and β. By the formulae deducing, 128 possible final collapsed states of the particle D after Charlie's measurements are given in Appendix A. The relation of the outcomes of Charlie's measurement and the possible final collapsed states of the particle D may be written as

$$\left|\tau\right\rangle_{C_1} \quad \rightarrow \quad \left|\varphi_1^{\pm}\right\rangle_D = \frac{1}{8}\left|\lambda^{\pm}\right\rangle_D \qquad \text{(64 terms)}$$

$$\left|\xi_1\right\rangle_{C_2} \quad \rightarrow \quad \left|\varphi_2^{\pm}\right\rangle_D = \frac{1}{4\sqrt{2T_1}}\left|\lambda^{\pm}\right\rangle_D \qquad \text{(32 terms)}$$

$$\left|\xi_2\right\rangle_{C_3} \quad \rightarrow \quad \left|\varphi_3^{\pm}\right\rangle_D = \frac{1}{4H_2}\left|\lambda^{\pm}\right\rangle_D \qquad \text{(16 terms)}$$

$$\left|\xi_3\right\rangle_{C_4} \quad \rightarrow \quad \left|\varphi_4^{\pm}\right\rangle_D = \frac{1}{2\sqrt{2}H_3}\left|\lambda^{\pm}\right\rangle_D \qquad \text{(8 terms)}$$

$$\left|\xi_4\right\rangle_{C_5} \quad \rightarrow \quad \left|\varphi_5^{\pm}\right\rangle_D = \frac{1}{2H_4}\left|\lambda^{\pm}\right\rangle_D \qquad \text{(4 terms)}$$

$$\left|\xi_5\right\rangle_{C_6} \quad \rightarrow \quad \left|\varphi_6^{\pm}\right\rangle_D = \frac{1}{\sqrt{2}H_5}\left|\lambda^{\pm}\right\rangle_D \qquad \text{(2 terms)}$$

$$\left|\xi_6\right\rangle_{C_7} \quad \rightarrow \quad \left|\varphi_7^+\right\rangle_D = \frac{1}{H_6}\left|\lambda^+\right\rangle_D \qquad \text{(1 term)}$$

$$\left|\xi_6^\perp\right\rangle_{C_7} \quad \rightarrow \quad \left|\varphi_7^-\right\rangle_D = F\left|\eta\right\rangle_D, \qquad \text{(1 term)} \qquad (12)$$

where $F = \frac{\sqrt{\alpha^{254}+\beta^{254}}}{\sqrt{2}H_6\alpha^{63}\beta^{63}}$, $\left|\eta\right\rangle_D$ is a normalized state, which is given by

$$\left|\eta\right\rangle_D = \frac{1}{\sqrt{\alpha^{254}+\beta^{254}}}\left(\beta^{127}\left|0\right\rangle - \alpha^{127}\left|1\right\rangle\right)_D \ . \qquad (13)$$

Thus much Charlie has completed his selective measurements. From Eq. (12), one can note that, after Charlie making the SPMs on his all particles, the states $\frac{1}{q_m H_m}\left|\lambda^\pm\right\rangle$ ($q_m = 2^{(7-m)/2}, m = 1, 2, \cdots, 7$) in all 128 collapsed states of the particle D accounted for 127, and the state $\left|\varphi_7^-\right\rangle_D$ for 1. On the other hand, by simple calculation, it is easy found that, after Charlie's measurements, the probability of the particle D being in the state $\frac{1}{q_m H_m}\left|\mu^\pm\right\rangle$ ($q_m = 2^{(7-m)/2}, m = 1, 2, \cdots, 7$) is 0.75, and in the state $\left|\eta\right\rangle_D$ is 0.25. It must be pointed out that it is just these measured results of the SPM that led to the realization of the DLQM.

As mentioned above, after Charlie making the CPMs or SPMs on his particles respectively, the final collapsed states of the particle D are obvious different. It must be emphasized that, whether Charlie's measurements are the CPMs or SPMs, since Charlie and Dick agreed in advance that Charlie should measure his particles before an appointed time, Dick can always know that the particle D must be collapsed into the state corresponded to one of Charlie's 128 results of measurement after Charlie's measurements.

4. DLQM with numerous eight-particle GHZ entangled states

The detailed procedure of our DLQM protocol can be described as follows. Assume that two spacelike separated observers, Charlie and Dick, share N eight-particle GHZ states. To ensure the following analysis becomes exact, here we take $N = 30$ [20]. Thus, the 30 eight-particle GHZ states can be given by

$$\left|\Psi^{(n)}\right\rangle = \frac{1}{\sqrt{2}}\left(\left|00000000\right\rangle + \left|11111111\right\rangle\right)_{C_1^{(n)}C_2^{(n)}C_3^{(n)}C_4^{(n)}C_5^{(n)}C_6^{(n)}C_7^{(n)}D^{(n)}}, \qquad (14)$$

where $n = 1, 2, \cdots, 30$, and the particles $C_1^{(n)}$, $C_2^{(n)}$, …, $C_7^{(n)}$ are in the possession of Charlie and $D^{(n)}$ belong to Dick. Different from previous quantum operation discrimination schemes, we assume that there is no classical channel between Charlie and Dick. In this situation, before the agreed time t, Charlie should randomly make two different kinds of measurement, CPMs or SPMs, on his particles in the state $\left|\Psi^{(n)}\right\rangle$ ($n=1,2,\cdots,30$) respectively.

(s1) If Charlie performs the CPMs on his particles, all particles $D^{(n)}$ will be collapsed into the states $\frac{1}{8\sqrt{2}}\left|x^{+}\right\rangle_{D^{(n)}}$ or $\frac{1}{8\sqrt{2}}\left|x^{-}\right\rangle_{D^{(n)}}$. At the appointed time t, Dick measures his particles $D^{(n)}$ all in the computational basis. After that, by statistics theory, the probability of all particles $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to one.

(s2) If Charlie employs SPMs to measure his particles, by mentioned above, the probability of all qubits $D^{(n)}$ in the states $\frac{1}{q_m H_m}\left|\lambda^{+}\right\rangle_D$ or $\frac{1}{q_m H_m}\left|\lambda^{-}\right\rangle_D$ ($q_m=2^{(7-m)/2}, m=1,2,\cdots,7$) is $(0.75)^{30}\approx 0.00018$, *i.e.*, the probability of at least one particle $D^{(n')}$ in the state $\left|\varphi_7^{-}\right\rangle_D$ is $1-(0.75)^{30}\approx 0.99982$. This means that, after Charlie's SPMs, at least one qubit $D^{(n')}$ will be collapsed into the state $\left|\varphi_7^{-}\right\rangle_D$. Then, at the appointed time t, Dick measures his particles $D^{(n)}$ all in the computational basis. It is easy found that, after Dick's measurements, the probability of the particles $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$ will be different from the situation Charlie made the CPMs. In order to explain this clearly, without loss of generality, let us first discuss the situation in which only one particle $D^{(n')}$ in the state $\left|\varphi_7^{-}\right\rangle_D$ after Charlie's measurements. From the state $\left|\varphi_7^{-}\right\rangle_D$ in Eq. (12), one can see that, after measurements of Dick, the probability of the particle $D^{(n')}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to w ($w=\left(\frac{x^{64}}{y^{63}}\right)^2/\left(\frac{y^{64}}{x^{63}}\right)^2\approx 6.15\times 10^{18}$), *i.e.*, the particle $D^{(n')}$ will be always collapsed to the state $|1\rangle$. As a special situation, we also assume that all the other 29 particles $D^{(n)}$ are in the states $\left|\varphi_1^{\pm}\right\rangle_D$ after Alice's measurements and then all the 29 particles are in the state $|0\rangle$ after Dick's measurements. In this case, it is easy found that the probability of the 30 particles $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of 1 to 1.655 after Dick's measurements. For general situation in which only one particle $D^{(n')}$ in the state $\left|\varphi_7^{-}\right\rangle_D$ and other 29 particles $D^{(n)}$ collapsed randomly into the states $\frac{1}{q_m H_m}\left|\lambda^{\pm}\right\rangle_D$ ($q_m = 2^{(7-m)/2}$, $m=1,2,\ldots,7$) after Charlie's measurements, one can find that the probability of the 30 particles $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to $v_{(1)}$ ($v_{(1)}$ > 1.655) after Dick's measurements.

(s3) Now let us discuss the situation in which there are two particles $D^{(n')}$ and $D^{(n'')}$ in the state $\left|\varphi_7^-\right\rangle$ after Charlie's measurements. Similar to the above mentioned, it is easy found that the probability of the 30 particles $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to $v_{(2)}$ ($v_{(2)}$ ≥ 3.43) after Dick's measurements.

(s4) For the situations in which more particles $D^{(1)}$, $D^{(2)}$, …, $D^{(j)}$ ($j=3,4,\cdots,30$) collapsed into the state $\left|\varphi_7^-\right\rangle_D$ after Charlie's measurements, the probability of the 30 particles $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to $v_{(j)}$ ($v_{(j)}$ > $v_{(2)}$, $j=3,4,\cdots,30$) after Dick's measurements. As described above, after measurements of Charlie, in the situation in which at least one particle $D^{(n')}$ in the state $\left|\varphi_7^-\right\rangle_D$, the probability of the 30 particles $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$ will be in the ratio of one to V ($V \geq 1.655$) (we call V the discriminated parameter) after Dick's measurements, here $V \in \left\{v_{(i)} : \mathrm{i}=1,2,\cdots,30\right\}$.

As mentioned above, according to these results, Dick can distinguish that the measurements used by Charlie are CPMs or SPMs. Thus, the DLQM is realized successfully without help of classical information. This means that our DLQM protocol may be not restricted by the no-signaling constraint.

5. Discussion and conclusion

Before conclusion, we make some discussion. (i) It should be noted that, in the present DLQM protocol, Dick did not obtain Charlie's quantum information, *i.e.*, if Charlie's measurements are SPMs, Dick couldn't have learned the coefficients α and β in the measuring basis performed by Charlie since he is not informed that Charlie got result of measurement. In fact, Dick doesn't need to know Charlie's quantum information (*e.g.* the coefficients α and β). As mentioned above, after his measurements, Dick can determine that the measurements performed by Charlie are CPMs or SPMs only according to the probability of his qubits $D^{(n)}$ in the state $|0\rangle$ or $|1\rangle$. That is to say, in our DLQM protocol, the entanglement can be used for transmission of information (*e.g.* the classical messages 0 and 1 can be represented by CPMs and SPMs respectively) without assistance of classical communication. (ii) It must be pointed that, in our protocol, it is essential that eight-particle GHZ states are applied. It is easy found that if l-particle GHZ states ($l < 8$) are employed, the DLQM will not be completed. For example, if 30 seven- or six-particle GHZ states are used, from (s2) in section 4 one can see that, the discriminated parameter V will be 0.83 or 0.41. In this case, the CPMs and SPMs cannot be distinguished. On the other hand, to ensure the discriminated parameter $V \geq 1.655$, one can only use 15 seven-particle or 7 (7.5) six-particle GHZ states. However, in these cases, the exact of measurement results will not be guaranteed. (iii) If only a single eight-particlre GHZ state was employed in our protocol, the DLQM cannot be realized, as described in section 2. It is just because of that numerous eight-particle GHZ states and the SPMs have been used, our DLQM protocol can be completed successfully. (iv) We should emphasize that our work has been completed in the framework of standard quantum mechanics.

In conclusion, we have proposed a theoretical protocol for local discrimination of two different kinds of measurement by using selective measurement and numerous eight-particle GHZ states. To realize the protocol, a series of single-particle correlative measuring basis has been employed. It is shown that, in this protocol, if both two observers agreed in advance that one of them (*e.g.* Charlie) should measure his particles before an appointed time, DLQM can be realized successfully without assistance of classical information. This means that the no-signaling constraint can be violated by the DLQM. Compared with previous DLQM scheme [19], the advantage of the present DLQM protocol is that it does not restricted by the no-signaling. So far there has been experiment implementing the eight-particle GHZ state [21], hence, we hope our work can be experimentally realized in the near future and stimulate further research on quantum communication and quantum information processing.

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Appendix A

From Eqs. (6) and (7), Charlie can measure his particles according to the result of his own measurement. If result of Charlie's measurement is $|\xi_1\rangle_{C_2}$ in state (6), he should measure his particles C_3, …, C_7 in state $|\psi_2\rangle$ under the basis $\{|x^+\rangle, |x^-\rangle\}$, successively. After that, the particle D will always be in the state

$$|\varphi_2^+\rangle = \frac{1}{4\sqrt{2}T_1}|\lambda^+\rangle_D \quad \text{or} \quad |\varphi_2^-\rangle = \frac{1}{4\sqrt{2}T_1}|\lambda^-\rangle_D . \tag{A1}$$

If Charlie's measured outcome is $|\xi_1^\perp\rangle_{C_2}$, he can measure his particle C_3 in state $|\psi_2{}'\rangle$ under the basis $\{|\xi_2\rangle, |\xi_2^\perp\rangle\}$, which is given by

$$\left|\xi_2\right\rangle = \frac{1}{T_2}\left(\frac{\alpha^2}{\beta^2}\left|0\right\rangle + \frac{\beta^2}{\alpha^2}\left|1\right\rangle\right),$$

$$\left|\xi_2^{\perp}\right\rangle = \frac{1}{T_2}\left(\frac{\beta^2}{\alpha^2}\left|0\right\rangle - \frac{\alpha^2}{\beta^2}\left|1\right\rangle\right). \tag{A2}$$

where $T_2 = \left[\left(\alpha/\beta\right)^4 + \left(\beta/\alpha\right)^4\right]^{1/2}$. If Charlie's result of measurement is $\left|\xi_2\right\rangle_{C_3}$, the particles $C_{4,}$ …, C_7 and D will be collapsed into the state $\left|\psi_3\right\rangle$, which is given by

$$\left|\psi_3\right\rangle = \frac{1}{\sqrt{2}H_2}\left(\alpha\left|00000\right\rangle - \beta\left|11111\right\rangle\right)_{C_4C_5C_6C_7D} \quad , \tag{A3}$$

where $H_2 = T_1T_2$. Then Charlie can in turn measure his particles C_4, …, C_7 in the basis $\left\{\left|x^+\right\rangle, \left|x^-\right\rangle\right\}$, and particle D will be collapsed into the state

$$\left|\varphi_3^+\right\rangle_D = \frac{1}{4H_2}\left|\lambda^+\right\rangle_D \quad \text{or} \quad \left|\varphi_3^-\right\rangle_D = \frac{1}{4H_2}\left|\lambda^-\right\rangle_D \quad . \tag{A4}$$

If Charlie's outcome of measurement is $\left|\xi_2^{\perp}\right\rangle_{C_3}$, the state of particles C_4, …, C_7 and D will evolve as

$$\left|\psi_3'\right\rangle = \frac{1}{\sqrt{2}H_2}\left(\frac{\beta^4}{\alpha^3}\left|00000\right\rangle - \frac{\alpha^4}{\beta^3}\left|11111\right\rangle\right)_{C_4C_5C_6C_7D} \quad . \tag{A5}$$

Then Charlie can measure his particle C_4 in the basis

$$\left|\xi_3\right\rangle = \frac{1}{T_3}\left(\frac{\alpha^4}{\beta^4}\left|0\right\rangle + \frac{\beta^4}{\alpha^4}\left|1\right\rangle\right) \quad ,$$

$$\left|\xi_3^{\perp}\right\rangle = \frac{1}{T_3}\left(\frac{\beta^4}{\alpha^4}\left|0\right\rangle - \frac{\alpha^4}{\beta^4}\left|1\right\rangle\right) \quad , \tag{A6}$$

where $T_3 = \left[\left(\frac{\alpha}{\beta}\right)^8 + \left(\frac{\beta}{\alpha}\right)^8\right]^{1/2}$. If Charlie's result of measurement is $\left|\xi_3\right\rangle_{C_4}$, the particles C_5, C_6, C_7 and D will be in the state of

$$\left|\psi_4\right\rangle = \frac{1}{\sqrt{2}H_3}\left(\alpha\left|0000\right\rangle - \beta\left|1111\right\rangle\right)_{C_5C_6C_7D} \quad , \tag{A7}$$

where $H_3 = T_1T_2T_3$. Charlie should measure his particles C_5, C_6 and C_7 in the basis

$\left\{\left|x^{+}\right\rangle,\left|x^{-}\right\rangle\right\}$, then particle D will be in the state

$$\left|\varphi_4^{+}\right\rangle_D=\frac{1}{2\sqrt{2}H_3}\left|\lambda^{+}\right\rangle_D \quad \text{or} \quad \left|\varphi_4^{-}\right\rangle_D=\frac{1}{2\sqrt{2}H_3}\left|\lambda^{-}\right\rangle_D. \tag{A8}$$

If Charlie's outcome of measurement is $\left|\xi_3^{\perp}\right\rangle_{C_4}$, the state of particles C_5, C_6, C_7 and D will be transferred as

$$\left|\psi_4{}'\right\rangle=\frac{1}{\sqrt{2}H_3}\left(\frac{\beta^8}{\alpha^7}\left|0000\right\rangle+\frac{\alpha^8}{\beta^7}\left|1111\right\rangle\right)_{C_5C_6C_7D}. \tag{A9}$$

Charlie can measure his particle C_5 under the basis

$$\left|\xi_4\right\rangle=\frac{1}{T_4}\left(\frac{\alpha^8}{\beta^8}\left|0\right\rangle+\frac{\beta^8}{\alpha^8}\left|1\right\rangle\right),$$

$$\left|\xi_4^{\perp}\right\rangle=\frac{1}{T_4}\left(\frac{\beta^8}{\alpha^8}\left|0\right\rangle-\frac{\alpha^8}{\beta^8}\left|1\right\rangle\right), \tag{A10}$$

where $T_4=\left[\left(\frac{\alpha}{\beta}\right)^{16}+\left(\frac{\beta}{\alpha}\right)^{16}\right]^{1/2}$. If Charlie's result of measurement is $\left|\xi_4\right\rangle_{C_5}$, the state of particles C_6, C_7 and D will evolve as

$$\left|\psi_5\right\rangle=\frac{1}{\sqrt{2}H_4}\left(\alpha\left|000\right\rangle+\beta\left|111\right\rangle\right)_{C_6C_7D}, \tag{A11}$$

where $H_4=T_1T_2T_3T_4$. Then Charlie measures his particles C_6 and C_7 in the basis $\left\{\left|x^{+}\right\rangle,\left|x^{-}\right\rangle\right\}$, and particle D will be collapsed into the state

$$\left|\varphi_5^{+}\right\rangle_D=\frac{1}{2H_4}\left|\lambda^{+}\right\rangle_D \quad \text{or} \quad \left|\varphi_5^{-}\right\rangle_D=\frac{1}{2H_4}\left|\lambda^{-}\right\rangle_D. \tag{A12}$$

If Charlie's result of measurement is $\left|\xi_4^{\perp}\right\rangle_{C_5}$, the particles C_6, C_7 and D will be in the state

$$\left|\psi_5{}'\right\rangle=\frac{1}{\sqrt{2}H_4}\left(\frac{\beta^{16}}{\alpha^{15}}\left|000\right\rangle-\frac{\alpha^{16}}{\beta^{15}}\left|111\right\rangle\right)_{C_6C_7D}. \tag{A13}$$

Charlie can measure his particle C_6 under the basis $\left\{\left|\xi_5\right\rangle,\left|\xi_5^{\perp}\right\rangle\right\}$, which is given by

$$\left|\xi_5\right\rangle=\frac{1}{T_5}\left(\frac{\alpha^{16}}{\beta^{16}}\left|0\right\rangle+\frac{\beta^{16}}{\alpha^{16}}\left|1\right\rangle\right),$$

$$\left|\xi_5^{\perp}\right\rangle = \frac{1}{T_5}\left(\frac{\beta^{16}}{\alpha^{16}}|0\rangle - \frac{\alpha^{16}}{\beta^{16}}|1\rangle\right), \tag{A14}$$

where $T_5 = \left[\left(\frac{\alpha}{\beta}\right)^{32} + \left(\frac{\beta}{\alpha}\right)^{32}\right]^{1/2}$. If Charlie's outcome of measurement is $\left|\xi_5\right\rangle_{C_6}$, the particles C_7 and D will be collapsed into the state

$$\left|\psi_6\right\rangle = \frac{1}{\sqrt{2}H_5}\left(\alpha|00\rangle - \beta|11\rangle\right)_{C_7D} \quad , \tag{A15}$$

where $H_5 = T_1T_2T_3T_4T_5$. Then Charlie measures his particle C_7 under the basis $\left\{\left|x^+\right\rangle, \left|x^-\right\rangle\right\}$, and particle D will be in the state of

$$\left|\varphi_6^+\right\rangle = \frac{1}{\sqrt{2}H_5}\left|\lambda^+\right\rangle \quad \text{or} \quad \left|\varphi_6^-\right\rangle = \frac{1}{\sqrt{2}H_5}\left|\lambda^-\right\rangle \quad . \tag{A16}$$

If Charlie's measured result is $\left|\xi_5^{\perp}\right\rangle_{C_6}$, the state of the particles C_7 and D will evolve as

$$\left|\psi_6{}'\right\rangle = \frac{1}{\sqrt{2}H_5}\left(\frac{\beta^{32}}{\alpha^{31}}|00\rangle + \frac{\alpha^{32}}{\beta^{31}}|11\rangle\right)_{C_7D} \quad , \tag{A17}$$

then he can measure the particle C_7 in the basis

$$\left|\xi_6\right\rangle = \frac{1}{T_6}\left(\frac{\alpha^{32}}{\beta^{32}}|0\rangle + \frac{\beta^{32}}{\alpha^{32}}|1\rangle\right),$$

$$\left|\xi_6^{\perp}\right\rangle = \frac{1}{T_6}\left(\frac{\beta^{32}}{\alpha^{32}}|0\rangle - \frac{\alpha^{32}}{\beta^{32}}|1\rangle\right), \tag{A18}$$

where $T_6 = \left[\left(\frac{\alpha}{\beta}\right)^{64} + \left(\frac{\beta}{\alpha}\right)^{64}\right]^{1/2}$. If Charlie's outcome of measurement is $\left|\xi_6\right\rangle_{C_7}$, the particle D will be in the state of

$$\left|\varphi_7^+\right\rangle_D = \frac{1}{H_6}\left|\lambda^+\right\rangle_D, \tag{A19}$$

where $H_6 = T_1T_2T_3T_4T_5T_6$. If Charlie's measured result is $\left|\xi_6^{\perp}\right\rangle_{C_7}$, the state of particle D will evolve as

$$\left|\varphi_7^-\right\rangle_D = \frac{1}{\sqrt{2}H_6}\left(\frac{\beta^{64}}{\alpha^{63}}|0\rangle - \frac{\alpha^{64}}{\beta^{63}}|1\rangle\right)_D$$

$$= F\left|\eta\right\rangle_D \quad . \tag{A20}$$

where $F=\frac{\sqrt{\alpha^{254}+\beta^{254}}}{\sqrt{2}H_6\alpha^{63}\beta^{63}}$, and $|\eta\rangle_D$ is a normalized state, which is given by

$$|\eta\rangle_D=\frac{1}{\sqrt{\alpha^{254}+\beta^{254}}}\left(\beta^{127}|0\rangle-\alpha^{127}|1\rangle\right)_D. \tag{A21}$$

Thus, 128 possible final collapsed states of the particle D are obtained.

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