Generally Covariant Quantum Theory: Examples.

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Abstract

In a previous paper of this author [1], I introduced a novel way of looking at and extending flat quantum field theory to a general curved spacetime satisfying mild geodesic conditions. The aim of this paper is to further extend the theory and clarify the construction from a physical point of view; in particular, we will study the example of a single particle propagating in a general external potential from two different points of view. The reason why we do this is mainly historical given that the interacting theory is after all well defined by means of interaction vertices and the Feynman propagator and therefore also applicable to this range of circumstances. However, it is always a pleasure to study the same question from different points of view and that is the aim of this paper.

1 Introduction.

It is a problem of general interest how to make quantum theory generally covariant, to divorce it from the observer so that objective quantities can be computed. Some people might utter that this is impossible since the observer has been build into the theory from the very start; as explained in [1, 2] this is just a matter of how one constructs the transition amplitudes and a realist ontology exists for a suitable axiomatics of a new type of quantum theory making exactly the same predictions as the old one in Minkowski spacetime. Since the theory which we shall expand and further explain in the nonrelativistic case makes no use of operators and path integrals at all, a novel ontology arises which we explained one to be of spacetime information exchange. Information which can travel superluminally but not to the (relativistic) past. Here, we define the theory in two different ways for a particle in a Newtonian cosmology with an external potential. As was explained in [2] no meaningful answers could be expected from applying the standard quantization procedure to such physical situations as the wavefunction would not determine an objective probability density for particles to be observed. Given that major parts of the general framework for the interacting theory have already been explained in [1] we start here by further defining this procedure and add an alternative construction which is inequivalent to the former in the domain where the latter applies. It is instructive to

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start with a generalized Newtonian theory as the latter is somewhat easier to comprehend.

2 A single particle in a Newtonian cosmology with an external potential.

By a Newtonian cosmology, I intend to say that spacetime is a manifold of the type $\mathbb{R} \times \Sigma$ with a preferred time function t and a family of Riemannian metrics $h_t^{\mu\nu}$ on leaves of constant t which are diffeomorphic to Σ. We have a notion of causality which is that every event of greater t lies to the future and every event of smaller t to the past; events of equal t are called simultaneous. This will be helpful once we construct the Feynman propagator. The construction of the free theory goes as follows, we design the two point function or propagator as an integral of the kind

$$W(x,y) = \int_{T^\star \mathcal{M}_x} \frac{d^4k}{(2\pi)^3} \, \delta(k^0 + \frac{|\vec{k}|^2}{2m}) \phi(x, k^a, y)$$

where k^a is determined with respect to a vierbein with fixed $e_0 = \partial_t$ and e_i undergoing local $SO(3)$ transformations; in case we want to include spin we introduce an associated $SU(2)$ spinor bundle but we shall not do that in this paper. The function $\phi(x, k^a, y)$ was determined from the coincidence limit $y \to x$ and the differential equation

$$\frac{d}{ds}\phi(x, k^a, \gamma(s)) = i(k^0 \dot{t}(s) + \dot{x}^\mu(s) k_\mu(s))\phi(x, k^a, \gamma(s))$$

where $\gamma(s) = (t(s), x^\mu(s))$ is a curve connecting x and y satisfying the geodesic equation

$$\ddot{t}(s) = 0, \quad \frac{D^{t(s)}}{ds}\dot{x}^\mu(s) = 0$$

where D^t denotes the covariant derivative with respect to h_t and $k^a(s)$ satisfies the parallel transport rule

$$\dot{k}^t(s) = 0, \quad \frac{D^{t(s)}}{ds} k^\mu(s) = 0.$$

As the reader may easily verify one obtains the result that in case h_t equals the standard Euclidean metric that

$$\phi((s,\vec{x}), k^a, (t, \vec{y})) = e^{-i\frac{|\vec{k}|^2}{2m}(t-s) + i\vec{k}.(\vec{y}-\vec{x})}.$$

In general, one shows that under parallel transport k^a undergoes an $SO(3)$ transformation preserving the constraint $k^0 + \frac{|\vec{k}|^2}{2m} = 0$ and that therefore

$$\overline{W((s,\vec{x}),(t,\vec{y}))} = W((t,\vec{y}),(s,\vec{x}))$$

and that

$$W((s,\vec{x}),(s,\vec{y}) \sim \delta(\vec{x} - \vec{y})$$

which is the Newtonian equivalent of the quantum causality condition. The Feynman propagator could be defined as $\Delta_F((s,\vec{x}),(t,\vec{y})) = W((s,\vec{x}),(t,\vec{y}))$ if

$t > s$ and $W((t, \vec{y}), (s, \vec{x}))$ otherwise. The reader notices that since the propagator vanishes at simultaneous distances, there is no way of arriving at a spin statistics result in the Newtonian context; indeed, the latter is a purely relativistic feature and this should be appreciated as such. Therefore, our definition of the Feynman propagator is rather ad hoc as it could equally well contain a minus sign when interchanging y with x. There are, however, some important lessons to be drawn from the Newtonian framework; define an n-particle bosonic IN-state $|s, \vec{x}_1, \ldots, \vec{x}_n\rangle$ and a corresponding n-particle bosonic OUT-state $|t, \vec{y}_1, \ldots, \vec{y}_n\rangle$ then the $2n$-point function

$$\langle t, \vec{y}_1, \ldots, \vec{y}_n | s, \vec{x}_1, \ldots, \vec{x}_n\rangle$$

is defined as

$$\sum_{\sigma \in S_n} \prod_{i=1}^{n} W((s, \vec{x}_i), (t, \vec{y}_{\sigma(i)}))$$

and for Fermions the appropriate signature of the permutation should be taken into account. In order to describe realistic interaction theories, we should include spin degrees of freedom by means of the Pauli matrices; however, we will not do this and content ourselves with the equivalent of relativistic ϕ^4 theory. Here, a "Feynman diagram" is a multi-graph with interaction four vertices lying in spacetime between the hypersurfaces $s < t$ and initial and final vertices given by $s, \vec{x}_i$ and $t, \vec{y}_i$ respectively. We exclude interaction vertices with a loop (edge from the vertex to itself) since those are ill defined due to the delta singularity in the propagator. Each diagram D has a symmetry factor $s(D)$ given by the number or symmetries of the multigraph (keeping the ends fixed) and we demand each interaction vertex to be connected to an IN or OUT boundary vertex; hence our amplitude reads

$$\langle t, \vec{y}_1, \ldots, \vec{y}_m | s, \vec{x}_1, \ldots, \vec{x}_n\rangle = \sum_D \frac{(-i\lambda)^V}{(4!)^V (V!) s(D)} \left(\prod_{j=1}^{V} \int_{[s,t]\times\Sigma} dt_j d\vec{x}_j \sqrt{h_{t_j}(\vec{x}_j)} \right) \prod_E \Delta_F(E)$$

where E stands for the edges in the multigraph and V stands for the number of internal vertices. Here, $\Delta_F(E)$ has an obvious meaning due to the time ordering in the definition of the Feynman propagator. This constitutes the *definition* of the theory and we notice that it is precisely the same as in the relativistic case except for the finite integration range to which we shall come back later on when discussing the relativistic theory in further depth. The interacting theory for a single particle in an external potential $V(r, \vec{x})$ gets a similar definition which coincides fully with the standard Feynman path integral framework and it is given by

$$\langle t, \vec{y} | s, \vec{x}\rangle = \sum_{n=0}^{\infty} (-i)^n \int_s^t dt_n d\vec{x}_n \sqrt{h_{t_n}(\vec{x}_n)} \int_s^{t_n} dt_{n-1} \ldots \int_s^{t_2} dt_1 d\vec{x}_1 \sqrt{h_{t_1}(\vec{x}_1)}$$

$$W((s, \vec{x}), x_1) \prod_{j=1}^{n} W(x_j, x_{j+1}) V(x_j)$$

where $x_{n+1} = (t, \vec{y})$. The definition differs from the previous one in the sense that now all paths are oriented straight towards the future and no bending is allowed. In case of the flat Newtonian theory, one recognizes that the Schrodinger equation with respect to t and $\vec{y}$ is satisfied. Similar constructions can be made for a general number of particles and classical interacting fields such as an electromagnetic gauge potential. While this certainly is the correct prescription for the interacting theory, one may wonder, out of shear curiosity, whether it would be possible to generalize our construction of the two point function directly in order to include an external potential. This would certainly clarify the meaning of our construction. The answer is *no* as we will show now; consider any potential $V(\vec{x})$, then our two point function should read

$$W(x,y) = \int_{T^\star \mathcal{M}_x} d^4k \, \delta(k^0 + \frac{|\vec{k}|^2}{2m} + V(\vec{x}))\phi(x,k^a,y)$$

where in this case the new constraint has to be preserved during transport over the geodesics. It is not even sure that the geodesics constitute a good choice but we will keep them anyway; then the natural transport equations working in any dimension read

$$\dot{k}^t(s) = 0, \;\; \frac{D^{t(s)}}{ds} k^\mu(s) = -\frac{2mk^\mu(s)}{h_{t(s)\,\alpha\beta} k^\alpha(s) k^\beta(s)} \dot{\gamma}^\nu(s) \partial_\nu V.$$

We do not need to further posit the evident Schrodinger equation for $\phi(x,k^a,\gamma(t))$ as the problem really resides in the pole structure near $\vec{k} = 0$ of the transporter equation. The simple example of a one dimensional harmonic oscillator shows that the k^1 values can become purely imaginary which should be forbidden. We now come to the treatment of the relativistic theory.

3 Some novel details about the relativistic theory.

In this section, we treat the general definition of the relativistic multi-particle theory in full detail based upon the results obtained in [1] and section two of this paper. To fully understand the philosophical nature of the problem at hand, I refer to [2] for further details; indeed *some* reflection about what we are going to do will be needed. In the previous section, we obtained that the domain of integration was over the chunck of spacetime between the IN and OUT events; one might wish to attribute this feature to the Newtonian character of the theory, but matters are not as simple alas. To put it philosophically, at the instant s, we are living in a NOW which is given by a hypersurface of constant s and at time t this will be a hypersurface of constant t. This NOW has nothing to do with the Newtonian character of the interactions but reveals the healthy point of view that all interactions from IN to OUT cannot travel to the *realized* past of IN and nor to the future of OUT. The philosophical point of view taken here is that the realized past does not exist anymore in any sense and does not interfere with present calculations and neither does the (potential) future play any part in this. This stance is remnicent of the notion of local causality in general relativistic theories where the only data from the past which are important reside in the NOW and its first normal derivatives. We will call this pure stance to be of TYPE I, as the reader may guess one also has TYPE II and III and mixed types as well. TYPE II in this context is best explained by saying that the realized past of spacetime would be of importance in the considerations: in that case, we should let the integral go between $-\infty$ and t instead of s and t. TYPE III then means that the potential future, which is fixed in theories where the metric field is background, also is involved in determining amplitudes associated to the OUT configurations; in that case, the integral should extend between s and ∞. The type alluded to in standard quantum field theory is mixed, the integrals go between $-\infty$ and $+\infty$ as they should for an S-matrix. Still arguing in the Newtonian context, TYPE I or TYPE II seems obviously correct and we should better understand the objections against TYPE III. The former implies that, in a sense, the realized past still exists and plays in its wholeness a role in determining the present transition amplitudes; this constitutes an extension of the notion of Einstein causality in the relativistic context as information may travel superluminally. This may be and we have argued in [2] that the relativistic NOW indeed has the structure of a four dimensional spacetime with a future spacelike boundary which we might call the "psychological present" so that spacetime grows to the future as to speak. Given that the dynamics of spacetime itself might be quantum or stochastic in nature, it seems very hard, if not impossible to devise something akin to TYPE III unless the spacetime metric is almost fully determined by the past configurations. What does all of this mean in the context of relativity; either we should include a closed boundary for the transition amplitude which has a spacelike initial and final surface and a timelike tube at infinity (or even at a finite distance). The initial surface contains the IN points and the final surface the OUT points and all processes remain within the spacetime volume deliniated by the full spacetime surface. This would be the relativistic equivalent of TYPE I, note that the inclusion of the boundary has nothing to do with the choice of any observer or something alike but merely serves to say that processes don't go to the past of IN, nor to the future of OUT. In the other case, we say that spacetime has at least a future spacelike boundary on which all OUT points reside; in that case the past of IN points is taken into account and therefore we have the relativistic equivalent of TYPE II. Note that those boundaries naturally arise in contemplating the evolution of spacetime to be a growth process towards the future where the manifold evolves too and is not taken statically at all, see [2]. These considerations just show how hard it is going to be to formulate a theory of dynamical spacetime, in either quantum gravity. This finishes our present short paper as we needed still to clarify those simple things before we could adress the issues of renormalization which could depend upon the TYPE one is choosing.

4 Conclusions.

In this short paper, we have further clarified the construction in [1] and have finished the definition of the interacting theory. Here some slight, but in my opinion deep, deviations from the standard Minkowski picture could arise and we have elaborated on the meaning of the different constructions which could be assumed. The full theory is defined in a manifestly covariant way and therefore totally divorced from issues regarding the observer; we are now left with tackling the technical issues regarding renormalization and the finiteness of the theory. This is work for the future to come and we shall focus of many different aspects including modified propagators in curved spacetime.

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