Universality *of* Causality Principle

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Abstract: The causality principle is physically investigated in the framework of special relativity theory (SRT) and it is proven to be absolutely valid for subluminal, luminal and superluminal signals under any natural and/or artificial circumstances; also Einstein's thought experiment (1907), Tolman's paradox (1917), tachyonic antitelephone (1970) and Moller's thought experiment (1952) are re-examined in order to show more conclusively that the so-called causality paradoxes are in fact a pure mental construction resulted from some too-common misconceptions which mainly spring from the confusion between the concepts of (relative) velocity and (relative) speed. Thus, in light of the present work, the old and semi-persistent concern about causality violation by superluminal signals and its consequences at micro and macrophysical levels may be conceptually ruled out if one approaches the physico-mathematical formalism of SRT in an open-minded way.

Keywords: causality, paradoxes, SRT, superluminal velocities

"*Theories are only hypotheses, verified by more or less numerous facts. Those verified by the most facts are the best, but even then they are never final, never to be absolutely believed.*"

Claude Bernard (1813 –1878)

1. Introduction

Epistemologically, mathematically and physically speaking, any physical theory has its proper validity limits and its appropriate domain of applicability. The perpetual development and vivacity of Science is strongly dependent on honoring these validity limits. For example, the validity limits of classical (Newtonian) mechanics led to special relativity theory (SRT). Since the latter have light speed in vacuum as an upper limiting speed hence the superluminality does not belong to SRT-domain because, before all, SRT is crucially based on the concept of (subluminal) inertial reference frames (IRFs) that's why –if, *e.g.*, a particle's velocity were greater than or equal to the light speed in some IRF S, we could not transform from S to the rest frame of the particle because Lorentz transformation (LT) becomes unphysical ones, more precisely, when $v \geq c$ Lorentz factor, $\gamma = \left(1 - v^2/c^2\right)^{-1/2}$, becomes infinite or imaginary and as a result SRT is not related to luminal and/or superluminal IRFs, consequently, any attempt to apply SRT to superluminal motions of hypothetical or real material bodies would be waste of time [1,2]. However, if we take into account the fact that the light itself is nothing else than a physical phenomenon among infinitely many others that occurring freely in *Nature*, therefore, there is no logical reason to say that the light speed in vacuum is an upper limiting speed for all physical phenomena. This assertion is supported by the fact that according to the modern epistemological thought (largely and notably influenced by Popper and Einstein) "*Even the best of physical theories do not claim to assert an absolute truth, but rather an approximation to the truth.*"

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Strangely, more than a century since its formulation, refinement and finally its experimental and observational verification, SRT remained misunderstood by many professional physicists (fortunately not all) in spite of the fact that SRT has widely fascinated scientists, science fiction writers, scenarists artists and even futurists since this theory is often associated with the idea of traveling faster than light. In order to deny absolutely the very real possibility of the natural existence of superluminal physical phenomena, many authors of textbooks on SRT used the following phrase: "*The subtle forces of nature conspire to prevent these motions being harnessed to send signals faster than the speed of light*." as an argument. But scientifically speaking, such a phrase belonging to the theology or at least to the metaphysics rather than physics because the human *knowledge* of Nature (how it works) is ,at the same time, relative, temporary and approximate. Furthermore, until now, we are unable to predict the natural disasters such as earthquakes, tsunamis, volcanic eruptions, tornadoes, landslides, etc.

Some authors clearly refused to recognize the reality or at least the possibility of superluminal physical phenomena because according to them, SRT might be violated if there is really a physical signal faster than light speed; and they rejected or omitted to refer to a number of important theoretical and experimental works [3-9] on the topic published in *Nature*; *Science*; *Phys. Lett. Rev.*; *Phys. Lett. A*, etc. Actually, the refusal is mainly due to the fact that several authors did not read and/or scrutinize the Einstein's 1905 original paper 'On the Electrodynamics of Moving Bodies'[10], which historically marked the birth of SRT. Because in his original paper, more precisely in the 'Kinematical Part', Einstein used, at the same time, luminal, subluminal and superluminal velocities as a physico-mathematical formalism to his theory. For instance, in the first subsection entitled 'Definition of Simultaneity', he wrote: "*In agreement with experience we further assume the quantity*

$$\frac{2AB}{t'_A - t_A} = c\,,$$

to be a universal constant –the velocity of light in empty space ."

And in the second subsection entitle 'On the Relativity of Lengths and Times', he wrote: "*Let a ray of light depart from A at the time* t_A *,let it be reflected at B at the time* t_B*, and reach A again at the time* t'_A*. Taking into consideration the principle of the constancy of the velocity of light we find that*

$$t_B - t_A = \frac{r_{AB}}{c-v} \quad and \quad t'_A - t_B = \frac{r_{AB}}{c+v}\,,$$

where r_{AB} *denotes the length of the moving rod –measured in the stationary system. Observers moving with the moving rod would thus find that the two clocks were not synchronous, while observers in the stationary system would declare the clocks to be synchronous*."

As we can remark it more clearly, since in Einstein's article, v is the relative velocity between the two IRFs, thus we have $c-v<c<c+v$ and this means that, kinematically, Einstein combined three types of motion, namely subluminal, luminal and superluminal motion in order to have an operational definitions of the relativity of simultaneity and the relativity of lengths and times. In passing, normally, Einstein should not use the expressions $c-v$ and $c+v$ because according to his second

postulate: '*The velocity of light is independent of the velocity of its source.*' How then can the outward journey of a light signal to an observer moving at relative velocity v be $c+v$, on its return it travels with a velocity $c-v$? Therefore, the aforementioned expressions violated the second postulate of SRT, and the presence of $c+v$ by itself violated LT.

2. Causality Principle and its Misconception

Philosophically, the notion of causality has been debated over the centuries but physically remains one of the most valuable kinds of knowledge since it tells what can or should be done to obtain a desired consequence or to avoid an undesirable effect. Thus, in this sense, causality concerns relationships where a change in one variable necessarily results in a change in another variable. Generally, there are three conditions for causality: covariation, temporal order (chronological succession), and 'third variable'[2] or 'missing variable' . The latter comprises some plausible alternative interpretations for the observed causal relationship. It is useful to be careful in the use of the term *causality* in physics. Properly speaking, the hypothesized cause and the hypothesized effect are each temporally transient processes. For example, force is a useful concept for the explanation of acceleration, but force is not by itself a cause. More is needed. For example, a temporally transient process might be characterized by a definite change of velocity at a definite time. Such a process can be regarded as a cause. Conceptually, causality is not intrinsically implied in equations of motions, but postulated as an additional constraint that needs to be satisfied (*i.e.,* a cause always precedes its effect). This important constraint has mathematical implications such as the well-known Kramers-Kroning (dispertion) relations. From the above considerations, we can assert that in physics, the *causality principle* may be expressed as follows:

Entity *A* (*cause*) must precede entity *B* (*effect*) in time and space.

The universality of causality principle implies, among other things, that the causality pronounced in the usual sense of cause-effect (causal) relationship would be absolutely valid for subluminal, luminal and superluminal signals under any natural and/or artificial circumstances. Moreover when we would study the cause-effect relationship *via* the relative motion, we must bear in mind the universality of relativity principle and the equivalence of all the IRFs. With the help of these considerations, we can easily make the difference between what is physically acceptable and what is physically unacceptable. Now, the natural question may be asked as follows: How do we establish a cause-effect (causal) relationship?

In order to answer adequately to this questions, let us assume two IRFs S and S' in relative translational motion at the constant velocity of magnitude $v<c$ with respect to each other. The two IFRs are supposed in standard configuration. Let Δt be the temporal interval of an event *A* (*cause*) relative to S and $\Delta t'$ the temporal interval of another event *B* (*effect*) relative to S'. The necessary and sufficient conditions for cause-effect (causal) relationship are:

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[2] Just because we show there is a relationship does not mean it is a causal one. It is possible that there is some other variable or factor that is causing the outcome. This is sometimes referred to as the "third variable" or "missing variable" problem.

$$\text{If } \Delta t > 0 \text{ relative to } S \Rightarrow \Delta t' > 0 \text{ relative to } S',$$

or

$$\text{If } \Delta t < 0 \text{ relative to } S \Rightarrow \Delta t' < 0 \text{ relative to } S'.$$

It turned out that the said necessary and sufficient conditions for cause-effect (causal) relationship are in reality temporal order (chronological succession) of the two events with respect to both IRFs.

It is commonly imagined that SRT forbids travel faster than the light speed or the propagation of signals at superluminal speed. However, SRT does not explicitly contain this assumption. The Einstein's original paper, merely states that the light speed in vacuum is constant in all IRFs. The idea of a speed limit comes from SRT-prediction, the object's relativistic energy reaching the infinite value when the object's velocity reaches the light speed. Einstein himself was clear on this matter because, in order to separate SRT from superluminality, he had repeatedly stated the following in his papers: "*For velocities greater than that of light our deliberations become meaningless; we shall, however, find in what follows, that the velocity of light in our theory plays the part, physically, of an infinitely great velocity.*" [10]. Note, however, the occurrence of the expression '*in our theory*' this means that vacuum light speed is, in fact, seen as an upper limiting speed only in SRT-context because of LT. Obviously, one can legitimately consider Einstein's claim as a sort of self-contradiction, since he used the expression $c+v$. So how did physicists in general come to believe that the causality principle would be violated if signals could travel at superluminal speed? It is due to the very old common *misconception* about the causality principle itself and its possible violation by (hypothetical) superluminal signals, which historically goes back to Einstein's note published in 1907 [11]. However, in previous paper [1], we have scrutinized and proven that the so-called Einstein's causality is actually an illusion, a sort of mathematical fiction, for the reason that we can get $\Delta t > 0$ relative to S and $\Delta t' < 0$ relative to S' –even if the hypothetical signal propagating with subluminal velocity, and in [2] we have shown that "*the superluminal signals do not violate the causality principle but they can shorten the luminal vacuum time span between cause and effect.*"

The theoretical, observational and experimental evidence of the (apparent) superluminal motions at micro and macroscopic scales allows us to suggest that in *Nature* there are three kinematical levels (KLs), namely, subluminal-KL, luminal-KL and superluminal-KL in which the physical phenomena may manifest at subluminal, luminal and superluminal velocities, respectively. Also, each KL should be characterized by its own group of spatio-temporal transformations. For example, subluminal-KL is characterized by the Galilean group for subrelativistic velocities $(v << c)$ and by the Lorentz group for relativistic velocities $(v < c)$, luminal-KL and superluminal-KL would be characterized, respectively, by luminal and a superluminal group for luminal $(v = c)$ and superluminal speeds $(v > c)$. From all this, we arrive, again, at the following result regarding causality. If causality is really a universal principle, it would be valid in all the KLs. Consequently, in such a case, we can say that there are in fact three kinds of causality, *viz.*, subluminal causality, luminal causality and superluminal causality, and each kind is characterized by its proper circumstances.

2.1. Causality paradoxes

By definition, *paradox* is a situation or statement which seems impossible or is difficult to understand because it contains two opposite facts or characteristics. It seems unlikely that Nature really has paradoxes, but it seems more likely that only misguided human beings can have paradoxes and the so-called causality paradoxes are actually a pure mental construction resulted from some too-common misconceptions which mainly spring from the confusion between the concepts of (relative) velocity and (relative) speed. However, as we will see, the mental creation of causality paradoxes cannot preclude the real or hypothetical existence of superluminal signals. For instance, in spite of the fact that SRT itself has more than a dozen of paradoxes, nevertheless *it* is always robust as a physical theory valid in its proper domain of applicability.

Among the several paradoxes of causality, we can cite, *e.g*., tachyonic antitelephone which is a hypothetical device in theoretical physics that could be used to send signals into one's own past. Einstein in 1907 presented a thought experiment of superluminal signals can lead to a paradox of causality, which was also described by Einstein and Arnold Sommerfeld in 1910 as a means "to telegraph into the past". The same thought experiment was described by Richard Chace Tolman in 1917; thus, it is also known as Tolman's paradox [12]. However, as it was already shown for the case of Einstein [1], all these paradoxes arising from some misconceptions which mostly caused by the confusion between the concepts of (relative) velocity and (relative) speed when the mentioned authors examined the causality *via* the relative motion. Also the confusion between mathematics and physics has a hand in the creation of paradoxes. It is easy to prove that all these paradoxes are useless if we take into account the difference between mathematics and physics.

First, Mathematics is not Physics, and Physics is not Mathematics. The inhabitants of the mathematical world are purely abstract objects characterized by an absolute freedom. However, the inhabitants of the physical world are purely concrete objects –in the theoretical sense and/or in the experimental/observational sense –and are characterized by very relative and restricted freedom. When applied outside its original context, mathematics should play the role of an accurate language and useful tool, and gradually should lose its abstraction. Let us illustrate these considerations by the following equation:

$$\mathbf{r} = \tfrac{1}{2}\boldsymbol{a}t^2 + \mathbf{v}_0 t + \mathbf{r}_0\,,\ t \geq 0\ . \tag{1}$$

This equation applies to a particle moving linearly, in three dimensions in a straight line with constant acceleration. Since the position, velocity, and acceleration are collinear (parallel, and lie on the same line) –only the magnitudes of these vectors are necessary, and because the motion is along a straight line, the problem effectively reduces from three dimensions to one and Eq.(1) simply becomes

$$x(t) \equiv x = \tfrac{1}{2}at^2 + v_0 t + x_0\,,\ t \geq 0\ . \tag{2}$$

Now, let us determine the numerical values of the duration and the magnitude of the final velocity of the particle's motion for the case when $x = 9\,\mathrm{m}$, $x_0 = 3\,\mathrm{m}$, $a = 2\,\mathrm{ms^{-2}}$ and $v_0 = 1\,\mathrm{ms^{-1}}$.

In order to make easy the calculations, we must, firstly, determine the expression of the final velocity by differentiating Eq.(2) with respect to time, to get

$$v = at + v_0 \,. \tag{3}$$

After substitution into Eq.(2) and some algebraic manipulations, we find the following numerical values $t = 2$ or $t = -3$. So after finishing the first process of calculations and by taking into account the difference between *pure* mathematical solutions and *pure* physical solutions, we do have a good deal of freedom in deciding which solutions are relevant to the problem before us. Since according to Eqs.(1) and (2), we have $t \geq 0$ thus the solution $t = -3$ is physically excluded and the required numerical value is $t = 2\,\mathrm{s}$. Finally, direct substitution into Eq.(3) yields the magnitude of the final velocity $v = 5\,\mathrm{ms}^{-1}$. However, we can artificially create a paradox if we deliberately ignore the initial condition $t \geq 0$ of the problem and focus our attention on $t = -3$ as a solution, and saying, e.g., since $|-3| > 2$, thus the major part of the particle's motion occurred in past during the negative time! But a clever reader can even give us a "pseudo-real physical" sense to this negative time by arguing that since there is a huge difference between the concept of displacement (vector quantity) and the concept of distance (positive scalar quantity), hence, $t = -3$ should represent the duration of the particle's displacement in past, *i.e.*, the quantity $\left(x(t) - x_0\right)$ from Eq.(2) may be interpreted as a *displacement* in past for $t = -3$ and as a *distance* in present/future for $t = 2$.

2.2. Origin of causality paradoxes

As it was said repeatedly, paradoxes are superfluous to science in general and to physics in particular, and are a human construct this means, among other things, we may create them in any way we like. In the present subsection and the following ones, we focus our attention exclusively on the so-called Einstein's causality (1907), Tolman's paradox (1917), and Benford's and co-authors' tachyonic antitelephone (1970) because these authors *erroneously* believed to have shown the impossibility of superluminal signals. Unfortunately, numerous other authors have made the same statement without scrutinizing, *e.g.*, Einstein's and Tolman's original papers, which containing major flaws. We will see later that these major flaws are in fact caused by the confusion between the concept of (relative) velocity and the concept of (relative) speed.

i) *Velocity and Speed*: But to understand our subsequent lines of argumentation better, let us first recall ,in few words, the elementary concepts of (relative) velocity and (relative) speed. Just as displacement and distance have distinctly different meanings (despite their similarities), so do velocity and speed. *Velocity* is a physical vector quantity; both magnitude and direction are needed to define it. *Speed* is a scalar absolute value (magnitude of velocity), being a coherent derived unit whose quantity is measured in the SI (metric) system as meters *per* second (m/s). For example, "5 meters per second" is a scalar (not a vector), whereas "5 meters *per* second east" is a vector.

ii) *Relativistic (relative) Velocities*: Also it is judged necessary to recall the relativistic transformations of (relative) velocities since the cited authors used the relative motion to demonstrate the alleged causality violation by hypothetical superluminal signals . To this end, let us consider two IRFs S and S', which are in relative uniform translational motion at subluminal velocity $\mathbf{v}$ of the magnitude v with respect to each other along their common $x|x'$–axis. Also, the two origins O and O' coincide at the moment $t = t' = 0$. The two IRFs are connected by LTs as follows:

$$S \to S' : \begin{cases} x' = \gamma(x - vt) \\ y' = y \\ z' = z \\ t' = \gamma\left(t - vx/c^2\right) \end{cases} , \qquad S' \to S : \begin{cases} x = \gamma(x + vt') \\ y = y' \\ z = z' \\ t = \gamma\left(t' + vx'/c^2\right) \end{cases} , \tag{4}$$

where

$$\gamma = \left(1 - v^2/c^2\right)^{-1/2}, \qquad v < c .$$

Let us call the vector $\mathbf{u}\left(u_x, u_y, u_z\right)$ of the magnitude u the relative velocity of a material point in IRF S and let us consider a second S' in straight-line uniform motion at subluminal velocity of the magnitude v relative to S along the x–axis. In S' the same material point is characterized by the relative velocity $\mathbf{u}'\left(u'_{x'}, u'_{y'}, u'_{z'}\right)$ of the magnitude u'. The two frames S and S' are in standard configuration and related by LTs. Thus, a direct differentiation of LTs (4), gives the required relativistic transformations of relative velocities:

$$\mathbf{u}' : \begin{cases} u'_{x'} = (u_x - v)\left[1 - u_x v/c^2\right]^{-1} \\ u'_{y'} = u_y \gamma^{-1}\left[1 - u_x v/c^2\right]^{-1} \\ u'_{z'} = u_z \gamma^{-1}\left[1 - u_x v/c^2\right]^{-1} \end{cases} , \qquad \mathbf{u} : \begin{cases} u_x = (u'_{x'} + v)\left[1 + u'_{x'} v/c^2\right]^{-1} \\ u_y = u'_{y'} \gamma^{-1}\left[1 + u'_{x'} v/c^2\right]^{-1} \\ u'_z = u'_{z'} \gamma^{-1}\left[1 + u'_{x'} v/c^2\right]^{-1} \end{cases} . \tag{5}$$

The magnitude of each relative velocity gives the relative speed of the material point with respect to S and S':

$$\|\mathbf{u}'\| \equiv u' = \sqrt{u'^2_{x'} + u'^2_{y'} + u'^2_{z'}} , \qquad \|\mathbf{u}\| \equiv u = \sqrt{u_x^2 + u_y^2 + u_z^2} . \tag{6}$$

Consider now the important particular case, that is, when the material point moves in S along the x–axis , we obtain from (5) the well-known relativistic addition law of relative velocities:

$$\mathbf{u}' : \begin{cases} u'_{x'} = (u_x - v)\left[1 - u_x v/c^2\right]^{-1} \\ u'_{y'} = 0 \\ u'_{z'} = 0 \end{cases} , \qquad \mathbf{u} : \begin{cases} u_x = (u'_{x'} + v)\left[1 + u'_{x'} v/c^2\right]^{-1} \\ u_y = 0 \\ u_z = 0 \end{cases} . \tag{7}$$

And from (6), we get the relative speed of the material point with respect to S and S':

$$u' = \sqrt{u'^2_{x'} + 0 + 0} = |u'_{x'}| , \quad u = \sqrt{u_x^2 + 0 + 0} = |u_x| . \tag{8}$$

Discussion: Now, we are arriving at the source of the common misconception which mostly occurred from the confusion between the concepts of relative *velocity* and relative *speed*, and unfortunately the same confusion is the origin of causality paradoxes. As we can remark it more clearly from (7a), that is, the first component that defines the relativistic relative velocity $u'_{x'} = (u_x - v)\left[1 - u_x v/c^2\right]^{-1}$ may be negative (if $u_x < v$), null (if $u_x = v$) or positive (if $u_x > v$), however, it is magnitude (8a) is always positive or null, *i.e*., the relativistic relative speed should be of the form

$$\left|(u_x - v)\left[1 - u_x v/c^2\right]^{-1}\right| \geq 0. \tag{9}$$

Thus, when we would study the causality (cause-effect relationship) *via* the relative motion, we must use the relative speeds to evaluate the temporal intervals Δt and $\Delta t'$ with respect to S and S'. Unfortunately, Einstein, Tolman and many modern authors used the relative velocities instead of the relative speeds as we will see. But before, let us again illustrate the above considerations through the following simple example: A boat is rowed directly upriver at 2.5 m/s relative to the water. Stationary observer on the shore sees that the boat is moving at only 0.5 m/s relative to the shore. –What is the relative velocity (both magnitude and direction) of the river? Is it moving with or against the boat?

$V_{b/o} = V_{b/w} + V_{w/o}$

$V_{w/o} = V_{b/o} - V_{b/w}$

$= 0.5$ m/s – 2.5 m/s

$= -2$ m/s, it is moving against the boat.

Further, if, for instance, we want to know the time took by the river to travel a certain given distance, say, $D = 10$ m. In this case we must use the relative speed of the river, and consequently we find $t = D/|V_{w/o}| = 5$ s. But if we have, for example, used the relative velocity instead of the relative speed, we will have a negative time. that's exactly what happened with the abovementioned authors.

2.3. Re-examination of Einstein's thought experiment on superluminal signals (1907)

Historically, in 1907, Einstein proposed the thought experiment in order to demonstrate the impossibility of superluminal signals after his discussion with Wilhelm Wien the occurrence of velocities exceeding the light speed in dispersive and absorptive media. In [1], we have disproved Einstein's allegation by means of two counterexamples. At the present, our main aim is to reconsider Einstein's thought experiment to show the already mentioned misconception and confusion. To this end, let us rewrite Einstein’s proof [11]:

“From the addition theorem of velocities results the further interesting consequence, that no action can exist which can be utilized for arbitrary signaling and which has a propagation velocity greater than light speed in vacuum. In fact, suppose a material strip extended along the x-axis of S, relative to which a certain action can be propagated with the velocity W (*as judged from the material strip*)*, and let observers who are at rest relative to S be situated both at the point* $x = 0$ (*point A*) *and at the point* $x = \lambda$ (*point B*)*. Let the observer at A send signals to the observer at B*

by means of the aforementioned action, through the material strip, which is not at rest but moves with the velocity v $(v<c)$ *in the direction of the negative x-axis. The signal is then, according to the first of equations* (3), *carried from A to B with the velocity* $(W-v)\left[1-\left(W\,v/c^2\right)\right]^{-1}$ *. The time T required for this is therefore*

$$T=\lambda\left(1-\frac{W\,v}{c^2}\right)(W-v)^{-1} \quad .$$

The speed v can take on any value smaller than c. If therefore, as we have assumed, $W>c$*, we can always choose* v *so that* $T<0$*. This result signifies that we must consider as possible a transmission mechanism that allows the intended action to precede the cause. Although from a purely logical point of view this result does not contain, in my opinion, any contradiction, yet it clashes so much with the character of our whole experience, that the impossibility of the assumption* $W>c$ *appears thereby to be sufficiently proven."*

Einstein wrote this note in 1907 (see Ref. [11]); that is more than a century ago - with the express purpose of showing the clash between causality and superluminal signals. However, close inspection of Einstein's proof reveals one important thing that has escaped many authors' attention: according to above thought experiment, the material strip played the role of IRF S' which was in uniform relative motion at velocity of magnitude v *in the direction of the negative* x-axis of *S*, this means Einstein studied the causality (cause-effect relationship) by means of the relative motion that's why he employed the addition theorem of velocities. Now, let us focus our attention on the signal velocity relative to *S* ,*i.e.*, $W'=(W-v)\left[1-\left(W\,v/c^2\right)\right]^{-1}$ which is in fact the first component of $\mathbf{u}'$ in (7a) for the case $u'_{x'}=W'>c$ and $u_x=W>c$. Therefore, W' is in reality the relative velocity of signal not its relative speed that should be defined as $\left|(W-v)\left[1-\left(W\,v/c^2\right)\right]^{-1}\right|\geq 0$. Thus Einstein confused between the concept of (relative) velocity and the concept of (relative) speed. Again, If we put Einstein's treatment under closer scrutiny, especially the assertion "*If therefore, as we have assumed,* $W>c$*, we can always choose v so that* $T<0$", we find that for Einstein, the inequality $T<0$ is a criterion or sufficient condition to claim that the effect precedes the cause and consequently causality is violated by superluminal signals. But Einstein disregarded the fact that we can get $T<0$ even for the case of subluminal signals because since the beginning he used the relative velocity (which may be negative, null or positive) instead of the relative speed, which is always positive or null.

–*Proof* : Let us follow the Einstein's thought experiment, but this time we use subluminal velocities instead superluminal velocities. With this aim, Supposing the signal, under consideration, was propagated at subluminal velocity $W<c$. The signal was then carried from *A* to *B* with the velocity $(W-v)\left[1-\left(W\,v/c^2\right)\right]^{-1}$. The time *T* required for this is therefore

$$T=\lambda\left(1-\frac{W\,v}{c^2}\right)(W-v)^{-1} \quad .$$

Since, as we have assumed, $v < c$ and $W < c$, thus *mathematically* we can always choose v so that $T < 0$. For this purpose, let $\forall k, k' \in \left]1, +\infty\right[\subset \mathbf{R} : k > k'$ so that $W = c/k$ and $v = c/k'$. Thus after substitution into the above relation, we get

$$T = \left(1 - \frac{1}{kk'}\right)\left(\frac{1}{k} - \frac{1}{k'}\right)^{-1} \frac{\lambda}{c} < 0. \qquad \text{(i)}$$

In view of the fact that there is an infinite set of couples (k, k') satisfying the above conditions and inequality (i), for this reason, we can affirm that by assuming $W < v < c$, we can always choose v so that $T < 0$. Therefore, this *counterexample* shows us that the inequality $T < 0$ cannot hold the status of a criterion or sufficient condition to prove the violation of causality because if, since the beginning, Einstein used the relative speed instead of the relative velocity he could obtain $T > 0$ for the case $W > c$. Einstein published his 'thought experiment' in the 1907 issue of *Jahrbuch der Radioaktivität und Elektronik*, so the questions to be asked are the following: why did the reviewers fail to point out the above counterexample? Why did the modern authors fail to draw attention to the fact that one can obtain $T < 0$ *even* for the case $W < v < c$?

2.4. Re-examination of Tolman's paradox (1917)

It is best to recall that Einstein in 1907 presented a thought experiment of superluminal signals can lead to a causality paradox, which was also described by Einstein and Sommerfeld in 1910 as a means "to telegraph into the past". The same thought experiment was described by Tolman in 1917; thus, it is also known as Tolman's paradox. Now, our central purpose is to re-examine Tolman's thought experiment in order to disprove his claim. To this end, let us rewrite Tolman's proof:

" … *The question naturally arises whether velocities which are greater than that of light could ever possibly be obtained in any way. This problem can be attacked in an extremely interesting manner. Consider two points A and B on the X axis of the system S, and suppose that some impulse originates at A, travels to B with the velocity u and at B produces some observable phenomenon, the starting of the impulse at A and the resulting phenomenon at B thus being connected by the relation of 'cause' and 'effect'. The time elapsing between the cause and its effect as measured in the units of system S will evidently be*

$$\Delta t = t_B - t_A = \frac{x_B - x_A}{u}\ , \qquad \text{(ii)}$$

where x_A *and* x_B *are the coordinates of the two points A and B. Now in another system S', which has the velocity V with respect to S, the time elapsing between cause and effect would evidently be*

$$\Delta t' = t'_B - t'_A = \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\left(t_B - \frac{V}{c^2}x_B\right) - \frac{1}{\sqrt{1 - \frac{V^2}{c^2}}}\left(t_A - \frac{V}{c^2}x_A\right),$$

Where we have substituted for t'_A *and* t'_B *in accordance with equation* (12). *Simplifying and introducing equation* (28) *we obtain*

$$\Delta t' = \frac{1 - \frac{uV}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \Delta t \,. \tag{iii}$$

Let us suppose now that there are no limits to the possible magnitude of the velocities u and V, and in particular that the causal impulse can travel from A to B with a velocity u greater than that of light. It is evident that we could then take a velocity u greater enough so that $\frac{uV}{c^2}$ *would be greater than unity and* $\Delta t'$ *would become negative. In other words, for an observer in system S' the effect which occurs at B would precede in time its cause which originates at A. such a condition of affairs might not be a logical impossibility; nevertheless its extraordinary nature might incline us to believe that no causal impulse can travel with a velocity greater than that of light.*"

It is clear from the above passage that Tolman had imitated Einstein to be precise he tried to study the causality (cause-effect relationship) *via* the relative motion. However, it seems since the beginning he confused between the concept of the relative *velocity* and the concept of relative *speed*. This confusion is well reflected by the formula (ii). Why? Since x_A and x_B are the coordinates of the two points *A* and *B*, thus the expression $x_B - x_A$ is simply a *displacement* between two points not a distance $|x_B - x_A|$ between two fixed points. Like the concept of (relative) velocity, the displacement may be negative, null or positive, however, the distance is always positive or null, *i.e.*,

$$|x_B - x_A| \geq 0 \,. \tag{10}$$

Therefore, *u* is not the relative speed *but* the relative velocity consequently Δt may be negative, null or positive and this implies that $\Delta t'$ in (iii) may be negative, null or positive independently of the relative velocity *u*.

Also, as we can remark it, Tolman used LT and, of course, the relativistic transformations of relative velocities. Furthermore, contrary to Einstein who respected the physical context of his 'thought experiment' by using the explicit expression of the relative velocity of his hypothetical superluminal signal with respect to IRF *S*', Tolman did not respect the physical context of his 'thought experiment' and it seems he deliberately omitted to write, as Einstein, the explicit expressions of the relative velocity with respect to *S* and *S*' since he proposed his 'thought experiment' to study the causality (cause-effect relationship) by the use of the relative motion. Why we said deliberately? Because, as we have pointed out above, he utilized LT and the relativistic transformations of relative velocities. This omission led Tolman to confound mathematics with physics and leaving the physical context of his 'thought experiment'. That's why he said: "*Let us suppose now that there are no limits to the possible magnitude of the velocities u and V* …" In such a case formula (iii) becomes physically meaningless because the denominator could be imaginary if there are no limits to the possible magnitude of *V*, and also he said: "*It is evident that we could then* *take a velocity u greater enough so that* $\frac{uV}{c^2}$ *would be greater than unity and* $\Delta t'$ *would become negative.*" This claim is founded on pure mathematical context by considering *u* as an absolute velocity instead to be a relative velocity because, as we said, if he respected the physical context of his 'thought experiment' like Einstein, he would arrive at another result. However, if we respect the physical context of Tolman's thought experiment, we can find, in natural manner, that the quantity $\frac{uV}{c^2}$ remains always smaller than unity even for the (relative) superluminal velocities and consequently $\Delta t'$ remains always positive.

–Proof : Let us write the explicit expressions of the relative velocities of the hypothetical impulse with respect to *S* and *S'*, that is, by applying LT to the physical context of Tolman's thought experiment, we obtain

$$u = \frac{u' + V}{1 + \frac{u'V}{c^2}} \quad , \tag{iv}$$

$$u' = \frac{u - V}{1 - \frac{uV}{c^2}} \quad . \tag{v}$$

Now, returning to (iii) and replacing *u* with its explicit expression (iv), we find

$$\Delta t' = \frac{1 - \frac{uV}{c^2}}{\sqrt{1 - \frac{V^2}{c^2}}} \Delta t = \gamma \left[1 - \left(\frac{u' + V}{1 + \frac{u'V}{c^2}} \right) \frac{V}{c^2} \right] \Delta t \quad . \tag{vi}$$

Let us focus our attention on the expression $\frac{uV}{c^2}$ which becomes according to (vi):

$$\left(\frac{u' + V}{1 + \frac{u'V}{c^2}} \right) \frac{V}{c^2} = \left(\frac{u'V + V^2}{u'V + c^2} \right). \tag{vii}$$

It is quite clear from (vii), *i.e.*, in view of the fact that $V < c$ thus the quantity $\left(\frac{u'V + V^2}{u'V + c^2} \right)$ is always smaller than unity *whatever* the values of u' consequently $\Delta t'$ remains always positive when $u' < c$, $u' = c$ and/or $u' > c$. In passing, it is worthwhile to note that in spite of the fact that, conceptually, SRT was not established to investigate superluminal physical phenomena because its formalism is not compatible with superluminal IRFs *but* it seems there is a certain exception concerning the superluminal signals which, as we have seen, can be investigated in the framework of SRT.

2.4. Re-examination of tachyonic antitelephone (1970)

In 1970, Benford *et al.*, published an article entitled '*The Tachyonic Antitelephone*' [13]. In this article, the authors believed to have shown, by their hypothetical device, the impossibility of the existence of tachyons because the existence of such superluminal particles would have allowed sending information into the past. In order to understand the authors' lines of thought let us rewrite the abstract and second paragraph in the first column, first page, respectively:

"*The problem of detecting faster-than-light particles is reconsidered in relation to Tolman's paradox. It is shown that some of the experiments already under way or contemplated must either yield negative results or give to causal contradictions.* "

"*In 1917 Tolman present an argument (Tolman's paradox) showing that if faster-than-light signals can be propagated, then communication with the past is possible. That is, they would comprise an 'antitelephone'.* "

An attentive reading of the above abstract and paragraph allows us to say that, *firstly*, the authors' work was, at the same time, inspired *by* Tolman's paradox and exclusively based *on* it. Since we have already proven that the Tolman's paradox is in fact a pure misapprehension, therefore, the authors' so-called 'tachyonic antitelephone' is only another illusion. *Secondly*, to prove the alleged causal contradictions (page 2), the authors used LT, however, LT is not compatible with superluminal particles because LT by itself is not consistent with superluminal IRFs.

Moller's thought experiment (1952)

Until now we have only re-examined the one-way thought experiment ,*i.e.*, when the hypothetical superluminal signal sent in one-direction. At present we want to reconsider Moller's thought experiment which is based on the idea of sending a superluminal signal in two-way, explicitly, when the signal is sent back to the sender. This thought experiment has been proposed , in 1952, by Moller as an example in his book entitled '*The Theory of Relativity*', page 52. The central objective of Moller is to demonstrate that in Nature no signals can exist which propagate with a superluminal velocity relative to any subluminal IRF. Regrettably, as we will see soon, Moller's major flaw is similar to that of Tolman. He was victim of the confusion between the concept of relative *velocity* and the concept of relative *speed*, and also victim of the confusion between mathematics and physics. Like before, in order to be credible, let us rewrite Moller's proof:

"*If* $\mathbf{u}$, *and also* $\mathbf{u}'$, *are parallel to the x-axis, we get from* (45) *the relativistic addition theorem for velocities*

$$u' = \frac{u - v}{1 - \dfrac{uv}{c^2}} \;, \qquad\qquad u = \frac{u' + v}{1 + \dfrac{u'v}{c^2}} \;. \tag{viii}$$

For $u' = c$ *it gives also* $u = c$.

From the Lorentz transformations it follows directly that no systems of inertia S' can exist for which $v > c$, *since the equations* (24) *as well as the expressions for the Lorentz contraction and the retardation of clocks would become imaginary in this case. But it can be shown, furthermore, that particles (or, more generally, signals) cannot move with a velocity greater than c relative to any* *inertial system, since this would lead to absurd results. Let us assume for a moment that we were able to emit signals with a velocity greater than the velocity of light. At the time t = t' =* 0,*where the two systems of coordinates S and S' in Fig.*8 *coincide, we could then send a signal from the common origins O, O' along the negative* x'*-axis with a constant velocity* $u' > c$ *relative to S'. At the time* $t_1' > 0$, *this signal would arrive at a point P on the negative* x'*-axis with the coordinate* $x_p' = u't_1'$. *The space-time coordinates of this event in S are, according to* (24'),

$$\left.\begin{aligned} x_p &= \gamma\left(x_p' + vt_1'\right) = -\gamma t_1'\left(u' - v\right) < 0 \\ t_1 &= \gamma\left(t_1' + \frac{vx_p'}{c^2}\right) = \gamma t_1'\left(1 - \frac{vu'}{c^2}\right) \end{aligned}\right\}. \tag{ix}$$

Immediately after its arrival in P the signal is supposed to be sent back to O with a velocity $w > c$ *relative to S. The motion of the signal is thus described by the equation*

$$x = w\left(t - t_1\right) + x_p. \tag{x}$$

This signal will arrive at the origin O of S at a time t_2 *which is obtained from this equation by putting* $x = 0$, *thus at the time*

$$t_2 = t_1 - \frac{x_p}{w} = \gamma t_1'\left[1 - \frac{u'v}{c^2} + \frac{\left(u' - v\right)}{w}\right]. \tag{xi}$$

If we now choose u' *and* w *so that*

$$u' > \frac{c^2}{v}, \qquad w > \frac{u' - v}{\frac{u'v}{c^2} - 1}, \tag{xii}$$

we could obtain that

$$t_2 < 0, \tag{xiii}$$

i.e., that, at the return of the signal to O, the clock at O records a number which is smaller than that recorded by the same clock at the moment of departure of the signal. Obviously this is impossible, and therefore we can infer that in nature no signals can exist which move with a velocity greater than the velocity of light relative to any system of inertia."

As anyone can remark it, since the beginning, Moller want to study the causality violation by the hypothetical superluminal signal sent in tow-direction, thus he expected to investigate the cause-effect relationship by the use of the relative motion. Bu unfortunately, contrary to Einstein who, as we know, respected the physical context of his 'thought experiment' by making use of the explicit expression of the relative velocity of his superluminal signal relative to IRF. Like Tolman, Moller intentionally *refused* to write the explicit expressions of the signal (relative) velocity with respect to *S* and *S*'. Once again, why we said intentionally? Because he already wrote the explicit expressions of the relative velocities (viii) called by him *the relativistic addition theorem for velocities*. However, there is no such thing for u' *relative to S'* and for w *relative to S.* This intentionality led Moller to confound mathematics with physics in order to escape the physical context of his own 'thought experiment'. That's why he said: "*If we now choose* u' *and* w *so that* (xii) *we could obtain that* (xiii)" because, firstly, he confused between the concept of relative velocity and the concept of relative speed and, secondly, he intentionally neglected the fact that we can get $t_2 < 0$ even for the case of subluminal signals as we will see immediately.

–*First counterexample*: To disprove Moller's claim, suppose we are dealing with subluminal signal sent in two-direction, *i.e.*, $u' < c$ relative to S and $w < c$ relative to S', so that $w < u' < v < c$. Therefore, *mathematically*, we can always choose u' and w so that $t_2 < 0$. For this purpose, let $\forall k, k', k'' \in \left]1, +\infty\right[\subset \mathbf{R} : k' < k < k''$ in order that $u' = c/k, v = c/k'$ and $w = c/k''$. After substitution into the relation (xi) and some algebraic manipulations, we get

$$\frac{t_2}{\gamma t_1'} = \left[\left(1 + \frac{k''}{k}\right) - \left(\frac{k''}{k'} + \frac{1}{kk'}\right)\right] < 0. \qquad \text{(xiv)}$$

In view of the fact that there is an infinite set of triplets (k, k', k'') satisfying the above conditions and inequality (xiv), for this reason, we can affirm that by assuming $w < u' < v < c$, we can always choose u' and w so that $t_2 < 0$. Consequently, if we follow Moller's way of thinking , we can say in such a case: –Evidently this is impossible, and accordingly we can deduce that in Nature no signals can exist which propagating with a velocity smaller than that of light relative to any IRF–. Certainly, our claim is extremely far from Science in general and physics in particular. Therefore, This first *counterexample* shows us more clearly the fallacy of Moller's claim: "*Obviously this is impossible, and therefore we can infer that in nature no signals can exist which move with a velocity greater than the velocity of light relative to any system of inertia.*" because his inequality (xiii), *i.e.*, $t_2 < 0$ cannot hold the status of a criterion or sufficient condition to prove the violation of causality or to say –the superluminal signals do not exist in Nature for the reason that the human knowledge about Nature is, at the same time, relative, temporary and approximate. And as a pedagogical illustration, we have listed in Table 1 – according to the conditions of the above first counterexample – some numerical values for the inequality (xiv) .

k	k'	k''	$t_2/\gamma t_1'$
2.75	1.25	3.00	– 1/55
3.00	1.50	4.50	– 13/18
5.00	2.00	7.00	– 6/5
6.00	3.00	9.00	– 5/9
7.00	4.00	11.00	– 3/14
8.00	4.50	13.00	– 7/24

Table 1: Some numerical values for the inequality (xiv) when the signal is supposed subluminal.

Second counterexample: At this time, to refute once again Moller's claim we should maintain the superluminality of the signal proposed by Moller, and let us prove that *mathematically* we can always choose $u' > c$ and $w > c$ so that $t_2 > 0$. With this purpose, let $\forall \ell, \ell', \ell'' \in \left]1, +\infty\right[\subset \mathbf{R} : \ell'' < \ell' < \ell$ such that $u' = \ell c, v = c/\ell'$ and $w = \ell'' c$. After substitution into the relation (xi) and some algebraic manipulations, we get

$$\frac{t_2}{\gamma t_1'} = \left[\left(1+\frac{\ell}{\ell''}\right) - \left(\frac{\ell}{\ell'} + \frac{1}{\ell'\ell''}\right)\right] > 0\,. \qquad \text{(xv)}$$

Since there is an infinite set of triplets (ℓ, ℓ', ℓ'') satisfying the above conditions and inequality (xv), hence, we can assert that by assuming $u' > c$, $w > c$ and $v < c$, we can always choose u' and w so that $t_2 > 0$. And as an instructive illustration, we have listed in Table 2 – according to the conditions of the above second counterexample – some numerical values for the inequality (xv) .

ℓ''	ℓ'	ℓ	$t_2/\gamma t_1'$
1.25	2.75	3.00	111/55
1.50	3.00	4.50	43/18
2.00	5.00	7.00	3
3.00	6.00	9.00	22/9
4.00	7.00	11.00	15/7
4.50	8.00	13.00	2.236

Table 2: Some numerical values for the inequality (xv) when the signal is supposed superluminal.

3. Universality of Causality Principle

Finally we are arriving at our principal objective, namely, the investigation of the universality of causality principle. Technically speaking, we would, in general manner, investigate the *hypothetical* superluminal signals by means of the study of causality (cause-effect relationship) through the relative motion. Exactly like before, *i.e.*, the investigation should be performed in the framework of SRT. To this aim, Let *S* and *S'* be two IRFs such that *S'* is in state of relative translational motion at subluminal velocity of magnitude $v < c$ with respect to *S* along the x-axis. Suppose relatively to *S* there is an ultra-sophisticated apparatus –in state of relative rest– capable of producing three types of readable signal, namely, subluminal, luminal and superluminal signals. The signals are emitted at fixed point $A(x_A, y_A, z_A, t_A)$ in *S* and received at fixed point $B(x_B, y_B, z_B, t_B)$ in *S*. A certain hypothetical readable signal propagating from *A* to *B* at a relative velocity of magnitude $u = \sqrt{u_x^2 + u_y^2 + u_z^2}$. It is evidently that the time elapsing between the cause (emission) and its effect (reception) will be

$$\Delta t = t_B - t_A = \frac{\sqrt{(x_B - x_A)^2 + (y_B - y_A)^2 + (z_B - z_A)^2}}{\sqrt{u_x^2 + u_y^2 + u_z^2}}\,. \qquad (11)$$

Relatively to *S'*, the same hypothetical readable signal propagating at a relative velocity of magnitude $u' = \sqrt{u_{x'}'^2 + u_{y'}'^2 + u_{z'}'^2}$ and the time elapsing between emission and reception will be

$$\Delta t' = t'_B - t'_A = \frac{\sqrt{(x'_B - x'_A)^2 + (y'_B - y'_A)^2 + (z'_B - z'_A)^2}}{\sqrt{u'^2_{x'} + u'^2_{y'} + u'^2_{z'}}} \ . \tag{12}$$

From (11) and (12), we get the following consequence: $\Delta t > 0$ relative to $S \Rightarrow \Delta t' > 0$ relative to S', which is, of course, the necessary and sufficient conditions for cause-effect (causal) relationship. Remark that until now we have not specified if u and u' are subluminal, luminal or superluminal relative velocities. Furthermore, we have according to LT (4a):

$$\Delta t' = \gamma\left(1 - \frac{u_x v}{c^2}\right)\Delta t \ . \tag{13}$$

Recalling u_x is only the first component of $\mathbf{u}$ relative to S and u_x may be negative, null or positive, which according to (5b) may be written as

$$u_x = \frac{u'_{x'} + v}{1 + \frac{u'_{x'} v}{c^2}} \ . \tag{14}$$

From (13) and (14), we obtain after substitution

$$\Delta t' = \gamma\left[1 - \left(\frac{u'_{x'} v + v^2}{u'_{x'} v + c^2}\right)\right]\Delta t \ . \tag{15}$$

Now, let us consider the very important case, that is, when the hypothetical readable signal propagating along the x–axis, the magnitude of the signal (relative) velocity with respect to S and S' becomes according to (8): $u' = |u'_{x'}|$ and $u = |u_x|$, and the relations (12) and (15) turn out to be, respectively:

$$\Delta t' = \left|\frac{x'_B - x'_A}{u'_{x'}}\right| = \frac{|x'_B - x'_A|}{u'} \ , \tag{16}$$

$$\Delta t' = \gamma\left[1 - \left(\frac{u' v + v^2}{u' v + c^2}\right)\right]\Delta t \ . \tag{17}$$

Mathematically and Physically, relation (16) by itself implies that the elapsing time between A and B cannot be negative and to confirm this result it suffices to note that the quantity $\left(\frac{u' v + v^2}{u' v + c^2}\right)$ included in relation (17) is identical to (vi), therefore, *whatever* the values of u' the elapsing time $\Delta t'$ remains always positive when $u' < c$, $u' = c$ and/or $u' > c$. Consequently, the causality principle and the relativity principle are universal in the framework of SRT.

4. Conclusion

In this paper we have confirmed the universality of causality principle in the framework of SRT, that is, the causality is absolutely valid for subluminal, luminal and superluminal signals under any natural and/or artificial circumstances, also the so-called causality paradoxes are proven to be in reality a pure mental construction resulted from some too-common misconceptions which mainly spring from the confusion between the concept of (relative) velocity and the concept of (relative) speed.

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