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A computational proof of locality in entanglement.

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ABSTRACT: In this paper the design and coding of a local hidden variables model is presented that violates the Clauser, Horne, Shimony and Holt, $|\mathrm{CHSH}| \leq 2$ inequality. Numerically we find with our local computer program, $\mathrm{CHSH} \approx 1 + \sqrt{2}$.

Contents

1 Introduction.

In the debate of the foundation of quantum theory, Bell's theorem [2] is considered an important milestone.In order to study Einsteins incompleteness criticism [1], Bell formulated an expression for the correlation between distant spin measurements. With this formulation it was possible to answer Einstein's question of completeness with an experiment. It is important to note here the following. The experimenters using Bell's correlation form did not "look under the hood" for extra parameters. They mainly employed statistics in spin measurement experiments without much physics theory about hidden variables. Moreover, Einsteins criticism initially did not include the spin. The reformulation of Einsteins criticism [1] into the entanglement between spins was provided by David Bohm [3] and [4]. Bells formulation of the problem looked like a big step from philosophy to physics.

For the ease of the argument, let us say that Einstein argued for extra hidden parameters to explain spin correlation. Einstein insisted that the A wing of the experiment is independent of what is done in the B wing and vice versa [6].

Without loss of generality we may write the quantum correlation as $E(a,b) = a \cdot b = \sum_{k=1}^{3} a_k b_k$. Here, the $a \in \mathbb{R}^3$ and $b \in \mathbb{R}^3$ are unit-length parameter vectors. The a and b vectors direct two paradigmatic Stern Gerlach magnets for spin measurement. In practical experiments other means are employed to measure spin. That doesn't affect our computer model.

According to Einstein, additional local hidden extra parameters "somewhere" in the experimental system explain the quantum correlation. The restriction of locality was introduced because the entanglement correlation is independent of the distance between the sites of measurement. The Einsteinian locality concept can be tested with the use of the Clauser, Horne, Shimony and Holt (CHSH) inequality. The inequality is derived [5] from Bells formula for the correlation [2], $E(a, b)$. Bells formula reads

$$E(a,b) = \int d\lambda \rho_\lambda A_\lambda(a) B_\lambda(b) \tag{1.1}$$

In equation (1.1) the probability density of the hidden variables, λ, is $\rho_\lambda \geq 0$. In addition, $\int d\lambda \rho_\lambda = 1$. Physically, think e.g. of λ as a hidden, but locally confined, field. The local effect of the λ, e.g. an array (λ_1, λ_2), can be accomplished if e.g. λ_1 is assigned and confined to the A wing and λ_2 to the B wing of the experiment. Furthermore, the measurement functions $A_\lambda(a)$ and $B_\lambda(b)$ both project in $\{-1, 1\}$ to represent binairy spin variables (e.g. up=1, down=-1). The a and b represent the already introduced unit parameter vectors.

Given (1.1) we can study the following four term;

$$S = E(1,1) - E(1,2) - E(2,1) - E(2,2) \tag{1.2}$$

The CHSH inequality $|S| \leq 2$ can be derived from (1.2). See [5] and e.g. [6]. So for an $E(a, b)$ in the form (1.1) we have by necessity $|S| \leq 2$. However, note that the CHSH can be violated with $E(a, b) = a \cdot b$ for certain proper (a, b) combinations of setting parameters. To be sure, the labels 1 and 2 in (1.2) refer to a and b vectors that can be set in the experiment. E.g. 1 on the A side, operated by Alice, is $a^1 = (a^1_1, a^1_2, a^1_3)$ etc, with $||a^1||^2 = a^1 \cdot a^1 = 1$. The $|| \cdot ||$ is the Euclidean norm. Similarly the 2 is associated to a^2 on the A side. Moreover, for B we have a similar assignments, b^1 and b^2. Below a numerical example of $|S| \leq 2$ violating setting combinations will be given.

In the present paper we first will show a somewhat restricted design and proof of concept. In the second place the design is extended such as to meet e.g. Weihs's experiment [7].

1.1 Correlation in experiment

Here we answer the question how to obtain in experiment the E values to be used in (1.2). It is technically still impossible to measure directly the $E(a, b)$ for a single pair. The correlation is therefore derived from counting measurement results. The results enter the raw product moment correlation [12] to approximate the correlation $E(a, b)$.

Suppose we measure $4N$ spin pairs. After the last measurement in the series, the correlation $E(a, b)$ is computed approximately. We count the number of times $S_{A(a),n} = S_{B(b),n}$, is found i.e., $N(a, b \,|\, S_{A(a),n} = S_{B(b),n})$. In addition we count the number of times $S_{A(a),n} = -S_{B(b),n}$, i.e. $N(a, b \,|\, S_{A(a),n} = -S_{B(b),n})$. It is noted that ideally, $N(a, b \,|\, S_{A(a),n} = S_{B(b),n}) + N(a, b \,|\, S_{A(a),n} = -S_{B(b),n}) = N(a, b) = N$. Hence, we obtain the expression

$$E(a,b) = \frac{N(a,b \,|\, S_{A(a),n} = S_{B(b),n}) - N(a,b \,|\, S_{A(a),n} = -S_{B(b),n})}{N(a,b \,|\, S_{A(a),n} = S_{B(b),n}) + N(a,b \,|\, S_{A(a),n} = -S_{B(b),n})} \tag{1.3}$$

This type of computation of E is also employed in the algorithm and its presented proof of concept.

2 Preliminaries in the computer design

Commonly it is believed that a computer violation of the CHSH inequality $|S| \leq 2$, see (1.2), with a local model is not possible. Peres [6] formulates it thus: "......, a hidden variable theory which would predict individual events must violate the canons of special relativity....". Furthermore, the program must mimic an important experiment in the test of locality performed by Weihs [7]. Note that Weihs's experiment is related to but also differs from important work of Aspect [8]. In Weihs's experiment strict locality conditions were closely approximated and a violation $|S| > 2$ was observed for violating setting combinations of a and b with a quantum correlation $a \cdot b$.

In [9], however, the present author already showed that there is a nonzero probability that a local hidden variables model may violate the CHSH. Objections to the probability loophole claim in [9] were raised in [11] but were answered in [13]. The present paper completes the rejection of what has been claimed in [11] and observes the metaphor requirements of [12].

2.1 Settings and information hiding as a warrant of locality

In the present paper, a local model is presented that can be implemented in a simple computer program and leads to $S \approx 1 + \sqrt{2}$ for the following violating settings. On the A side Alice has $1 = \frac{1}{\sqrt{2}}(1, 0, 1)$ and $2 = (\frac{-1}{2}, \frac{1}{\sqrt{2}}, \frac{1}{2})$ at her disposal. On the B side, Bob has $1 = (1, 0, 0)$ and $2 = (0, 0, -1)$. For the ease of the argument we inspect, $E(a, b) = a \cdot b$. A simple computation then shows that for a quantum outcome we would see $E(1, 1) = 1/\sqrt{2}$, $E(1, 2) = -1/\sqrt{2}$ while $E(2, 1) = -1/2$ and $E(2, 2) = -1/2$. Hence, looking at (1.2), $S = 1 + \sqrt{2} > 2$ is expected in an experiment. The setting parameters a and b are given a value when the A- and B-wing particles leave the source. In flight we allow B (Bob) to change his setting.

Needless to say that infromation hiding between Alice and Bob is the algorithmic realization of strict locality. Furthermore, in the computer simulation A doesn't know anything about B and vice versa. All computations are "encapsulated" i.e. local, despite the fact that in the proof of concept, they occur in a single loop (Appendix A).

3 Design of the algorithm based on a local model

3.1 Random sources

In the first place let us assume random sources to represent random selection of setting. We look at the randomness from the point of view of creating an algorithm. If there are N trials, i.e particle pairs, in the experiment then e.g. two independent random sources can be seen as two arrays with index running from 1 to N. If $\mathcal{N}_N = (1, 2, 3, ..., N)$, then we define three random source arrays

$$\begin{aligned} \underline{\mathcal{R}}_{AS} &= \text{sample}(\mathcal{N}_N) \\ \underline{\mathcal{R}}_{B} &= \text{sample}(\mathcal{N}_N) \\ \underline{\mathcal{R}}_{C} &= \text{sample}(\mathcal{N}_N) \end{aligned} \tag{3.1}$$

Technically, the map $\mathcal{N}_N \mapsto \underline{\mathcal{R}}_.$ is 1-1 but randomized. As an example, suppose we have $\mathcal{N}_5 = (2,3,5,1,4)$ and so, $\mathcal{N}_{5,1} = 2$. Then in the first trial $n = 1$, the $\mathcal{N}_{5,n}$ - th element of another array, e.g. $q = (0.1, 0.4, -0.9, 1.2, 1.0)$ is randomly selected, hence, $q(n = 1) = 0.4$. In the second trial, looking at $\mathcal{N}_5$, we see, $\mathcal{N}_{5,2} = 3$ so $q(n = 2) = -0.9$, etcetera. Note that this two array procedure is similar to rolling a five-sided dice. If e.g. $\mathcal{N}_5$ is replaced by $\mathcal{M}_{10}$ and multiples are allowed, such as in e.g. $\mathcal{M}_{10} = (2,3,5,1,4,4,5,1,3,3)$ this q "dice" will in 10 turns show three times the side with -0.9.

In this way a random source $\mathcal{N}$ can be employed in a program and be looked upon as a physical factor giving rise to randomness. The "freely tossing of a coin" is now replaced with "freely randomizing" the $\underline{\mathcal{R}}_X$ by filling it with sample($\mathcal{N}_N$). There can be no fundamental objection to this particular two array form of randomizing.

3.2 Design time settings

Experimentalists may claim the construction of their measuring instruments. Hence, servers in the experiment may be tuned in design time. There is no fundamental reason to reject design time to the designer of a computer experiment. There is also no reason in physics to reject the observers Alice and Bob access to the information in design time.

Let us also note that there can be no fundamental reason to reject our proof design merely because one wants to set the a and b setting parameters at the proper time with the toss of a coin. If e.g. Alice has no access to the complete $\underline{\mathcal{R}}_A$, i.e. is created and implemented in server A at design time, then there is no difference when Alice toss a coin at the n-th particle measurement or employs the $\underline{\mathcal{R}}_{A,n}$ in the selection of a. The question then transforms into the infrastructure of the servers which is a genuine locality issue.

Furthermore, the designer may assume that one random source is shared by A and by S. This is the $\underline{\mathcal{R}}_{AS}$. Because there is a flow of particles between the A and the S this sharing, i.e. $\underline{\mathcal{R}}_A = \underline{\mathcal{R}}_S = \underline{\mathcal{R}}_{AS}$, cannot be prevented at run time in a real experiment. The latter is related to the infrastructure of servers in the numerical experiment.The a_n in the experiment are based on the $\underline{a}$ array. For instance $\underline{a} = (1,2,1,2,1,2,...)$. In design time the designer is allowed to introduce a spin-like variable $\sigma \in \{-1,1\}$. In the sequence of trials, σ_n is selected from $\underline{\sigma} = (-1,1,-1,1,-1,1,...)$.

We may note that, because of $\underline{\mathcal{R}}_A = \underline{\mathcal{R}}_S$ the relation $a_n = 1 + \frac{1}{2}(1+\sigma_n)$ occurs on the A side of the experiment. The setting a_n can be either 1 or 2 and is already presented in terms of selection unit parameter vectors in $\mathbb{R}^3$.

Note that the σ_n can be send to Bob and to Alice without any additional information conveying its meaning. So, Bob cannot derive anything from σ_n even though the designer knows the relation. This is because Bob is only active in run time, not in design time.

Finally, the source may also send a $\zeta \in \{-1,1\}$ to both Alice and Bob. The ζ_n in the experiment is based on the $\underline{\mathcal{R}}_C = \text{sample}(\mathcal{N}_N)$ and derives from a $\underline{\zeta}$ array.

The second random source, $\underline{\mathcal{R}}_B$ is used by B exclusively, the third random source, $\underline{\mathcal{R}}_C$ is used by the source exclusively. There appears to be no physical arguments why this is a violation of locality or cannot be found in nature.

3.3 Random sources $\mathcal{R}$. and particles

The source sends a $\sigma \in \{-1, 1\}$ and a $\zeta \in \{-1, 1\}$. to both A and B. In a formal format,

$$[A(a_n)] \leftarrow (\sigma, \zeta)_n \leftarrow [S] \rightarrow (\sigma, \zeta)_n \rightarrow [B(b_n)]$$

Here, e.g. $[A(a)]$ represents the measuring instrument A where Alice has the a setting. This setting "runs synchronous" with σ in the source because of the "shared" random source. The particle pair source is represented by $[S]$.

The σ and ζ going into the direction of A are equal to the σ and ζ going to B. Each particle is, in the algorithm, a pair (σ, ζ). We note that ζ derives from $\underline{\mathcal{R}}_C$.

3.4 A side processing of the (σ, ζ)

Firstly, let us for the ease of the presentation define a $\sigma_{A,n} = \frac{1+\sigma_n}{2}$. The σ_n at the n-th trial from the source S is a result of the sharing of $\underline{\mathcal{R}}_{AS}$.

The way the information is used remains hidden to B in order to maintain locality in the model. So, secondly, we have the setting $a_n = \sigma_{A,n} + 1$. Furthermore, we define two functions $\varphi^-_{A,n} = \sigma_{A,n}$ and $\varphi^+_{A,n} = 1 - \sigma_{A,n}$. The two functions, together with ζ_n produce, in turn, a function

$$f_{\zeta_n}(a_n) = \zeta_n \varphi^+_{A,n} - \varphi^-_{A,n}$$

Note that $f_{\zeta_n} \in \{-1, 1\}$. Hence, we can store the outcome of the computations on the A side immediately in an N-size array $S_{A,n}$ for trial number n and $n = 1, 2, 3, ...N$.

3.5 B side processing of the (σ, ζ)

In the first place, let us determine with the B associated die the setting b_n. This results from the hypothetical random source $\underline{\mathcal{R}}_B$. Then, secondly and similar such as in the case of A, but of course completely hidden from A, the $(\sigma, \zeta)_n$ information from the source is processed. We have, $\sigma_{B,n} = \frac{1+\sigma_n}{2}$, then $\varphi^-_{B,n} = \sigma_{B,n}$ and $\varphi^+_{B,n} = \sigma_{B,n} + (\delta_{1,b} - \delta_{2,b})(1 - \sigma_{B,n})$. This leads to the function

$$g_\zeta(b) = \zeta \varphi^+_B + \frac{1-\zeta}{\sqrt{2}} \varphi^-_B$$

For $g_{\zeta_n}(b_n)$ we may note that it projects in the real interval $[-\sqrt{2}, \sqrt{2}]$. If $\sigma_{B,n} = 1$ then $g_{\zeta_n}(b_n) = 1$ for $\zeta_n = 1$ and $\sqrt{2} - 1$ for $\zeta_n = -1$. If $\sigma_{B,n} = 0$, then $\varphi^-_{B,n} = 0$ and $g_{\zeta_n}(b_n) = \pm 1$.

Hence, in order to generate a response in $\{-1, 1\}$, a random λ_2 from the real interval $[-\sqrt{2}, \sqrt{2}]$ is uniformly drawn and $S_{B,n} = \text{sgn}(g_\zeta(b) - \lambda_2)$ in the n-th trial. We note that as long as Bob doesn't know the meaning of σ_B, derived from σ and related to the $\underline{\mathcal{R}}_{AS}$, locality is warranted. Bob doesn't have access to the design time information.

3.6 Computer infrastructure

In computer infrastructure terms one can imagine three cables from the source server running to the A server and three running from S to the B server. One cable, $\mathcal{C}_{SA}(\sigma)$, carries the σ from S to A and the other cable, $\mathcal{C}_{SB}(\sigma)$, carries σ from S to B. The σ's will also contain synchronous running timing mechanisms that are set upon "creation" of each pulse running through the cable. Secondly, a cable, $\mathcal{C}_{SA}(\zeta)$, carries the ζ from S to A and a cable $\mathcal{C}_{SB}(\zeta)$ carries the ζ from S to B. The third cable, $\mathcal{C}_{AS}(\underline{\mathcal{R}})$ is only used by A to share the (information of) $\underline{\mathcal{R}}_A$ with S. This cable is open only once and carries only one "pulse" that conveys the random source at A.

4 Conclusion & discussion

In the paper a simple design is given that is able to violate the CHSH inequality with numerical values close to the expected quantum mechanics. Please note that no violation of locality is employed. B doesn't know the meaning of the A-S shared information send to B. So the information from S to Alice is inaccessible to Bob. In fact, A server (Alice) and B server (Bob) process their common input (σ, ζ) differently without knowing of each other's existence.

The reader kindly notes that the construction is designed to explain the outcome of the A-S-B experiment such as in Weihs's [7] and should not be confused with experimental configurations unequal to $A(a) \leftarrow S \rightarrow B(b)$.

In the appendix, the essential loop in the R program over $n = 1, 2, 3...N$ is presented. This loop represents the course of events in the computer infrastructre described previously.

In the explanation of entanglement with locality, three random sources $\underline{\mathcal{R}}_{AS}$, $\underline{\mathcal{R}}_C$ and $\underline{\mathcal{R}}_B$ are employed. We note that nobody knows whether or not in the experiment the measuring instrument, A, and the particle source, S, share a random source yes or no. Moreover information from design time is not accessible in run time and there is a flow of particles between S and A. From S to A the flow is "forced" by the experimenter. In this design, flow of information from A to S is enforced by nature on the experimenter. It is perhaps like 'tHooft once claimed: ".... every no-go theorem comes with small print" [10].

The initial conceptual weakness of the computer simulation presented here lies in the fact that, in real experiment, both Alice and Bob may change their settings when the two particles $(\sigma, \zeta)_{A\,wing}$ and $(\sigma, \zeta)_{B\,wing}$ are created and are in flight heading to their targets A and B. In our computer model, only Bob may change his setting "in flight".

Changing "in flight" settings at Bob's together with no access to design time is a very strong form of information hiding between Alice and Bob. Moreover, "shared random sources" together with "meaning-hidden information transport" via the particles and "synchronized random clocks" cannot be rejected in nature beforehand. We think at minimum we have provided another way to look at the criticism raised by Einstein [1].

As required by the author of [11] a computer simulation, be it initially a somewhat restricted in some details, rejects the criticism raised in [11]. We may claim this because our "freezing the setting of a at particle creation" is a valid CHSH type of experiment. It would be strange to say that locality and causality cannot occur in an experiment where "in flight" changes in both wings are allowed whereas one must admit that locality and causality occurs when only B wing "in flight" changes of setting may occur.

Moreover, the σ in the computational model may act like a kind of clock $\sigma(t)$. The synchronization of the A wing and B wing $\sigma(t)$ clocks starts at creation of the particle in the source. Then the separate $\sigma(t)$ may synchronously change "in flight" until $(\sigma(t),\zeta)_n$ hits the measuring instrument. Hence Alice can have "in flight" changes of a too.

Because the metaphor requirements of [12] are met, a local hidden variables explanation of the correlation in a "one wing freeze setting at particle creation & other wing freely in flight change of setting" type of experiment entails the following. Such a violation of the CHSH criterion would not have been possible without a probability loophole in the CHSH [9]. The freely selected settings e.g. of a can be accomplished by hiding the $\underline{\mathcal{R}}_A$ for Alice, i.e. create and implement at desgin time. Similarly, $\underline{\mathcal{R}}_B$ is hidden for Bob. In this way the selection of (a,b) is similar to the one where a coin is tossed. It is hard to see how a particle pair in a distant source would behave differently when Alice employs an unknown sequence $\underline{\mathcal{R}}_A$ for her setting selection, compared to the case where Alice employs a coin.

Moreover, using the clock-type synchronization for both $\sigma_{A\,wing}(t)$ and $\sigma_{B\,wing}(t)$ in the complete design, allows both Alice and Bob setting changes during "time of flight" of the particles. The latter comes the closest to a local explanation of the results from Weihs's experiment [7]. This feature cannot be demonstrated in a proof of concept but one can see that the implementation of it in the server infrastructure will provide the solution. Hence, we are allowed to claim that the present paper corrects Peres' statement [6], that violations of the CHSH inequality "violate the canons of special relativity".

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Appendix A: Here the nucleus of the algorithm is shown.

```
for (n in 1:N){
#Source section
  zetah<-zeta[RC[n]]
  sygma<-sigma[RAS[n]]
#A section
  aSet<-a[RAS[n]]
  aKeep[n]<-aSet
  phiAmin<-((sygma+1)/2)
  phiAplus<-1-((sygma+1)/2)
  f<-zetah*phiAplus-phiAmin
  scoreA[aSet,n]<-f
#B section
  phiBmin<-((sygma+1)/2)
  bSet<-b[RB[n]]
  bKeep[n]<-bSet
  if(((sygma+1)/2)==1){
    phiBplus<-1
  }else{
    if(bSet==1){
      phiBplus<-1
    }
    if(bSet==2){
      phiBplus<-(-1)
    }
  }
  g<-zetah*phiBplus
  g<-g+((1-zetah)*phiBmin/sqrt(2))
  lambda_2<-runif(1)*sqrt(2)
  lambda_2<-sign(0.5 - runif(1))*lambda_2
  scoreB[bSet,n]<-sign(g-lambda_2)
}
```