# A list of 33 sequences of primes obtained by the method of concatenation

Abstract. In this paper I list a number of 33 sequences of primes obtained by the method of concatenation; some of these sequences are presented and analyzed in more detail in my previous papers, gathered together in five books of collected papers: "Two hundred conjectures and hundred and fifty open problems one on Fermat pseudoprimes", "Two hundred and thirteen conjectures on primes", "Conjectures on primes and Fermat pseudoprimes, many based on Smarandache function", "Sequences of integers, conjectures and new arithmetical tools", "Formulas and polynomials which generate primes and Fermat pseudoprimes".

## Sequence 1

Conjecture: there exist an infinity of primes p obtained concatenating the square of a prime q, to the right, with the group of digits 0001.

The sequence of primes p:

: 490001, 2890001, 8410001, 18490001, 22090001, 50410001, 114490001, 171610001, 193210001, 327610001, 364810001 (...), obtained for q = 7, 17, 29, 43, 47, 71, 107, 131, 139, 181, 191 (...)

# Sequence 2

Conjecture: there exist an infinity of primes p = n/q, where n is the number obtained concatenating the square of a prime q, to the right, with q (example: p = 497/7 = 71):

The sequence of primes p: : 31, 71, 1301, 1901, 3701, 6101, 6701, 7901, 103001, 109001, 181001 (...), obtained for q = 3, 7, 13, 19, 37, 61, 67, 79, 103, 109, 181 (...)

# Sequence 3

Conjecture: there exist an infinity of primes p = n/q, where n is the number obtained concatenating the square of a prime q, to the left, with q (example: p = 749/7 = 107):

The sequence of primes p:

: 13, 107, 1013, 1019, 10037, 10061, 10067, 10079, 100103, 100109 (...), obtained for q = 3, 7, 13, 19, 37, 61, 67, 79, 103, 109 (...)

Conjecture: there exist an infinity of pairs of primes (p1, p2) defined as follows: p1 = m/q, where m is the number obtained concatenating the square of a prime q, to the left, with q, and p2 = n/q, where n is the number obtained concatenating the square of the prime q, to the right, with q.

The sequence of pairs of primes (p, q):

: (13, 31), (107, 71), (1013, 1301), (1019, 1901), (10037, 3701), (10061, 6101), (10067, 6701), (10079, 7901), (100103, 103001), (100109, 109001) (...), obtained for q = 3, 7, 13, 19, 37, 61, 67, 79, 103, 109 (...)

# Sequence 5

Conjecture: there exist, for any k positive integer, k > 1, an infinity of primes p = n/q, where n is the number obtained concatenating a prime q raised to power k, to the left, with q (example: for k = 3, p = 7343/7 = 1049):

The sequence of primes p for k = 3:
 109, 1049, 10169, 101681 (...), obtained for q = 3,
 7, 13, 41 (...)

The sequence of primes p for k = 4: : 127, 10343, 102197 (...), obtained for q = 3, 7, 13 (...)

#### Sequence 6

Conjecture: there exist, for any k positive integer, k > 1, an infinity of primes p = n/q, where n is the number obtained concatenating a prime q raised to power k, to the right, with q (example: for k = 3, p = 3437/7 = 491):

The sequence of primes p for k = 4: : 271 (...), obtained for q = 3 (...)

#### Sequence 7

Conjecture: there exist, for any k positive integer, k > 1, an infinity of pairs of primes (p1, p2) defined as follows: p1 = m/q, where m is the number obtained concatenating the square of a prime q raised to power k, to the left, with q, and p2 = n/q, where n is the number obtained concatenating the square of the prime q raised to power k, to the right, with q.

The sequence of pairs of primes (p, q) for k = 3: : (1049, 491), (10169, 16901) (...), obtained for q = 7, 13 (...)

The sequence of pairs of primes (p, q) for k = 4: : (127, 271) (...), obtained for q = 3 (...)

# Sequence 8

Primes p obtained through successive concatenation of the numbers q, R(q), q, R(q) and q, where q is an emirp (prime whose reversal is a different prime) and R(q) its reversal:

The sequence of primes q: : 1331133113, 9779977997, 769967769967769, 15111151151111511511 (...), obtained for q = 13, 97, 769, 1511 (...)

## Sequence 9

Primes p obtained through successive concatenation of a prime q with its reversal, not necessarily prime, R(q):

The least prime p obtained for the following primes q: for q = 13, p = 1331133113 is prime; for q = 19, p = 19911991199119 is prime; for q = 29, p = 2992299229 is prime; for q = 31, p = 31133113311331 is prime; for q = 43, p = 4334433443 is prime; for q = 97, p = 9779977997 is prime; for q = 127, p = 127721127721127 is prime.

Note the following patterns: (we will note with "]c[" the operation "concatenate to")

p = q]c[R(q)]c[q]c[R(q)]c[q;

: p = q]c[R(q)]c[q]c[R(q)]c[q]c[R(q)]c[q].

#### Sequence 10

Primes p obtained through successive concatenation of the numbers q1, q2, q1, q2 and q1, where q1 and q2 are primes such that q1 - q2 or q2 - q1 is multiple of 18 (note that in the case of reversible primes n - R(n) or R(n) - n is multiple of 18). I conjecture that there exist an infinity of primes p.

The sequence of pairs of primes (q1, q2):

: for (q1, q2) = (13, 31), p = 1331133113 is prime; for (q1, q2) = (29, 47), p = 2947294729 is prime; for (q1, q2) = (53, 71), p = 5371537153 is prime; for (q1, q2) = (79, 61), p = 7961796179 is prime; for (q1, q2) = (89, 71), p = 8971897189 is prime; for (q1, q2) = (97, 79), p = 9779977997 is prime; (...)

# Sequence 11

Primes p obtained through successive concatenation of the numbers q1, q2, q1, q2, q1, q2 and q1, where q1 and q2 are primes such that q1 - q2 or q2 - q1 is multiple of 18. I conjecture that there exist an infinity of primes p.

The sequence of pairs of primes (q1, q2):

: for (q1, q2) = (13, 31), p = 31133113311331 prime; : for (q1, q2) = (41, 23), p = 41234123412341 prime; (...)

## Sequence 12

Primes p = n/11, where n is obtained concatenating a prime q with its reversal, not necessarily prime, R(q):

The sequence of primes p:

: 181, 283, 647, 727, 9391, 9791, 10301, 12721, 14341, 14851 (...), obtained for q = 19, 31, 71, 79, 103, 109, 113, 139, 157, 163 (...)

#### Sequence 13

Conjecture: there exist an infinity of primes q obtained concatenating to the left with 1 the square of a Poulet number P:

The sequence of primes q: : q = 1116281, 12989441, 17295401, 119105641, 121911761, 171927361, 1163865601, 1188815081, 1195468361, 1907274641 (...), obtained for P = 341, 1729, 2701, 4371, 4681, 8481, 12801, 13741, 13981, 30121 (...)

# Sequence 14

Conjecture: there exist an infinity of primes q obtained concatenating to the left with 9 the square of a Poulet number P:

The sequence of primes q: : q = 9116281, 97958041, 919088161, 921911761, 969239041, 9209989081 (...), obtained for P = 341, 2821, 4369, 4681, 8321, 14491 (...)

#### Sequence 15

Conjecture: there exist an infinity of primes p obtained concatenating a prime q to the left with n where n has the sum of its digits s(n) equal to q.

Primes p that belong to this sequence:

p = 9413, where (q, n) = (13, 94) and s(n) = 13 = q; : p = 9817, where (q, n) = (17, 98) and s(n) = 17 = q; : p = 99119, where (q, n) = (19, 991) and s(n) = 19 =: q; p = 19919, where (q, n) = (19, 199) and s(n) = 19 =: q; p = 95923, where (q, n) = (23, 959) and s(n) = 23 =: q; : p = 999431, where (q, n) = (31, 9994) and s(n) = 31= q; p = 949931, where (q, n) = (31, 9499) and s(n) = 23: = q.

## Sequence 16

Conjecture: there exist an infinity of primes p obtained concatenating a prime q to the left with n where n has the sum of its digits s(n) equal to the sum of the digits s(q) of q.

Primes p that belong to this sequence:

p = 211, where (q, n) = (11, 2) and s(q) = s(n) = 2; : p = 2011, where (q, n) = (11, 20) and s(q) = s(n) =: 2; p = 20011, where (q, n) = (11, 200) and s(q) = s(n): = 2; p = 431, where (q, n) = (31, 4) and s(q) = s(n) = 4; : p = 4013, where (q, n) = (13, 40) and s(q) = s(n) =: 4; p = 22013, where (q, n) = (13, 220) and s(q) = s(n): = 4.

#### Sequence 17

Conjecture: there exist an infinity of primes p obtained concatenating a prime q to the left with n where n has the digital root d(n) equal to the digital root d(q) of q.

Primes p that belong to this sequence:

: p = 1019, where (q, n) = (19, 10) and d(q) = d(n) =
1;
: p = 100019, where (q, n) = (19, 1000) and d(q) =
d(n) = 1;
: p = 1129, where (q, n) = (29, 11) and d(q) = d(n) =
2.

## Sequence 18

Conjecture: there exist an infinity of primes p obtained concatenating a prime q to the right with n where n has the sum of its digits s(n) equal to q.

Primes p that belong to this sequence:

: p = 523, where (q, n) = (5, 23) and s(n) = 5 = q; : p = 761, where (q, n) = (7, 61) and s(n) = 7 = q; : p = 1367, where (q, n) = (13, 67) and s(n) = 13 = q; : p = 11821, where (q, n) = (11, 821) and s(n) = 11 = q.

# Sequence 19

Conjecture: there exist an infinity of primes p obtained concatenating a prime q to the right with n where n has the sum of its digits s(n) equal to the sum of the digits s(q) of q.

Primes p that belong to this sequence:

: p = 2341, where (q, n) = (23, 41) and s(q) = s(n) =
5;
: p = 200341, where (q, n) = (2003, 41) and s(q) =
s(n) = 5.

## Sequence 20

Conjecture: there exist an infinity of primes p obtained concatenating a prime q to the right with n where n has the digital root d(n) equal to the digital root d(q) of q.

Primes p that belong to this sequence:

- : p = 29100109, where (q, n) = (29, 100109) and s(q) = s(n) = 2;
- : p = 20000911000009, where (q, n) = (200009, 11000009) and s(q) = s(n) = 2.

Conjecture: for any prime p greater than or equal to 7 there exist n, a power of 2, such that, concatenating to the left p with n, the number resulted is a prime.

The sequence of the primes obtained, for  $p \ge 7$ :

47, 211, 1613, 3217, 419, 223, 229, 431, 1637, 241, 443, 1638447, 853, 859, 461, 467, 271, 6473, 479, 283, 12889, 1697, 8101, 16103, 2048107, 64109, 2113, 4127, 2131 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtaiend:

2, 1, 4, 5, 2, 1, 1, 2, 4, 1, 2, 14, 3, 3, 2, 2, 1, 6, 2, 1, 7, 4, 3, 4, 11, 6, 1, 2, 1 (...)

# Sequence 22

Conjecture: for any odd prime p there exist n, a power of 2, such that, subtracting one from the number resulted concatenating to the right p with n, is obtained a prime.

The sequence of the primes obtained, for odd primes p:

31, 53, 71, 113, 131, 173, 191, 233, 293, 311, 373, 41257, 431, 47262143, 531023, 593, 613, 673, 71257 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtaiend:

1, 2, 1, 2, 1, 2, 1, 2, 2, 1, 2, 8, 1, 18, 10, 2, 2, 2, 8 (...)

## Sequence 23

Conjecture: for any odd prime p there exist n, a power of 2, such that, adding one to the number resulted concatenating to the right p with n, is obtained a prime.

The sequence of the primes obtained, for odd primes p:

317, 53, 73, 113, 139, 173, 193, 233, 293, 313, 373, 419, 479, 5333, 613, 673, 719, 733, 7933, 839, 163, 8933 (...)

The corresponding sequence of the exponents of 2 for which a prime is obtaiend:

4, 1, 1, 1, 3, 1, 1, 1, 1, 1, 1, 3, 5, 1, 1, 3, 1, 5, 3, 1, 5 (...)

Conjecture: there exist an infinity of primes q of the form  $p^2 + 4320$ , where p is prime.

The sequence of the primes q:

: 4339, 4441, 5281, 5689, 6529, 7129, 9649, 12241, 13729, 14929, 21481 (...)

obtained for p = 11, 19, 31, 37, 53, 73, 89, 97, 103, 131 (...)

## Sequence 25

Conjecture: there exist an infinity of primes p obtained concatenating an odd power of 2, i.e.  $2^k$ , where k odd number, both to the left and to the right with 1.

The sequence of the primes p:

: 181, 1321, 15121, 1335544321, 121474836481, 1351843720888321, 194447329657392904273921, 1405648192073033408478945025720321, 125961484292674138142652481646100481, 1425352958651173079329218259289710264321, 16805647338418769269267492148635364229121 (...)

obtained for k = 3, 5, 9, 25, 31, 45, 73, 105, 111, 125, 129 (...)

#### Sequence 26

Conjecture: there exist an infinity of primes p obtained concatenating to the right a number n of the form 6\*k + 1 with the group of digits 081.

The sequence of the pairs [n, p]:

[19, 19081]; [31, 31081]; [97, 97081]; [49, 49081]; [85, 85081]; [91, 91081]; [121, 121081]; [127, [157, 157081]; [175, 175081]; [187, 187081]; [199, 199081]; 127081]; 175081]; [181, 181081]; [205, 205081]; [217, 217081]; [229, 229081]; [241, 241081 = 491<sup>2</sup>; [253, 253081]; [259, 259081 = 509<sup>2</sup>]; [295, 295081]; [313, 313081]; [325, 325081]; [331, [343, [337, 343081]; 331081]; 337081]; [349, 385081]; 349081]; [379, 379081]; [385, [409, 409081]; [421, 421081]; [427, 427081]; [439, 439081]; [475, 475081]; [517, 517081]; [559, 559081]; [577, 577081]; [563, 563081]; [569, 569081]; [595, 595081]; [607, 607081]...

Conjecture: there exist an infinity of primes p obtained concatenating to the right a multiple of 30, m, with a power of prime, n.

Triplets [m, n, p] that belong to this sequence:

: [30, 49, 3049]; [30, 169, 30169]; [30, 529, 30529]; [30, 841, 30841]; [30, 1681, 301681]; [30, 4489, 304489]; [30, 5329, 305329]; [60, 169, 60169]; [60, 289, 60289]; [60, 961, 60961]; [60, 1849, 601849]; [60, 5329, 605329]; [60, 6241, 606241]; [60, 7921, 607921]; [90, 49, 9049]; [90, 121, 90121]; [90, 289, 90289]; [90, 529, 90529]; [90, 841, 90841]; [90, 4489, 904489]; [90, 5329, 905329]; [90, 9409, 909409]; [120, 49, 12049]; [120, 121, 120121]; [150, 169, 150169]; [180, 49, 18049]; [180, 289, 180289]; [210, 361, 210361]; [240, 49, 24049]; [270, 121, 270121]; [300, 961, 300961]; [330, 49, 33049]...

## Sequence 28

Conjecture: there exist an infinity of primes p obtained concatenating to the right the numbers  $q^2 - 1$ , where q are primes of the form 6\*k - 1, with the digit 1.

The sequence of the primes p:

: 241, 1201, 5281, 28081, 68881, 79201, 102001, 127681, 278881, 299281, 320401, 364801, 388081 (...) (...)

obtained for q = 5, 11, 23, 53, 83, 89, 101, 113, 167, 173, 179, 191, 197 (...)

# Sequence 29

Observation: taking a number having just even digits and concatenating it three times with itself and then to the right with the digit 1 seems that are great chances to obtain a number with very few prime factors and (I conjecture that an infinite sequence of) primes p.

The sequence of primes p:

: 2221, 4441, 6661, 2424241, 2828281, 4040401, 4242421, 6262621, 6868681, 8282821, 2002002001, 2242242241, 2422422421, 2482482481, 2602602601, 2622622621, 2642642641, 4044044041, 4424424421, 4824824821, 6226226221, 6266266261, 6486486481, 6646646641, 6666666661, 6846846841, 8448448441, 8648648641, 200420041, 2024202420241, 2042204220421 (...)

#### Sequence 30

Conjecture: there exist an infinity of primes p obtained concatenating a number of the form 6\*k + 1 with its reversal then with 1.

The sequence of primes p:

: 23321, 29921, 35531, 41141, 47741, 59951, 71171, 1255211, 1311311, 1855811, 1911911, 2033021, 2099021, 2155121, 2277221, 2333321, 2511521, 2699621, 2755721, 2999921 (...)

Note the chain of five consecutive primes (23321, 29921, 35531, 41141, 47741) obtained for five consecutive numbers of the form 6\*k + 1 (23, 29, 35, 41, 47).

Few larger primes p:

: 1046399364011, 1046811864011, 1047233274011.

#### Sequence 31

Conjecture: there exist an infinity of primes p obtained concatenating a square of a prime  $q^2$ , q greater than 5, to the left, with a number of the form 6\*k.

The sequence of primes p for  $q^2 = 49$ :

: 1849, 3049, 5449, 9049, 9649 (...), obtained for k = 3, 5, 9, 15, 16 (...)

The sequence of primes p for  $q^2 = 121$ :

: 6121, 18121, 24121, 48121, 54121, 78121, 84121, 90121 (...), obtained for k = 1, 3, 4, 8, 9, 13, 14, 15 (...)

The sequence of primes p for  $q^2 = 169$ :

: 18169, 24169, 30169, 42169, 60169, 66169, 72169 (...), obtained for k = 3, 4, 5, 7, 10, 11, 12 (...)

The sequence of primes p for  $q^2 = 289$ :

: 12289, 18289, 60289, 90289, 96289 (...), obtained for k = 2, 3, 10, 15, 16 (...)

Few larger primes p:

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: 12010963555849, for q<sup>2</sup> = 104707<sup>2</sup> and k = 20;
: 121096433521, for q<sup>2</sup> = 104711<sup>2</sup> and k = 2;
: 1810965650089, for q<sup>2</sup> = 104717<sup>2</sup> and k = 3;
: 6610966906729, for q<sup>2</sup> = 104723<sup>2</sup> and k = 11;
: 1810968163441, for q<sup>2</sup> = 104729<sup>2</sup> and k = 3.
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## Sequence 32

Conjecture: there exist an infinity of primes p obtained concatenating a square of a number n of the form 6\*k + 1 with the square of the number 2\*n - 1.

The sequence of primes p:

:	49169 (49 = $7^2$ and 169 = $13^2$ where 13 = $2*7 - 1$ );
:	$3611369$ ( $361 = 19^2$ and $1369 = 37^2$ where $37 = 2*19$
:	- 1); 6252401 (625 = 25^2 and 2401 = $49^2$ where $49 = 2*25$
	- 1);
:	$13695329$ (1369 = $37^2$ and $5329$ = $73^2$ where $73$ =
	2*37 - 1);
:	$372114641$ ( $3721 = 61^2$ and $14641 = 121^2$ where $121 =$
	2*61 - 1);
:	$624124649$ ( $6241 = 79^2$ and $24649 = 157^2$ where $157 = 6472$
	2*79 - 1);
	$(\ldots)$

#### Sequence 33

Conjecture: for any n of the form 3\*k + 1 there exist an infinity of primes q such that the numbers  $p1 = n \setminus (n + 1) \setminus p$  and  $p2 = (n + 1) \setminus n \setminus p$  are both primes. The operator "\\" is used with the meaning "concatenated to".

- : for n = 1, there exist the following pairs of (P1, P2): (1213, 2113), (1229, 2129), (1231, 2131), (1237, 2137), (1279, 2179), (12101, 21101), (12107, 21107), (12149, 21149)...
- : for n = 4, there exist the following pairs of (P1, P2): (457, 547), (4513, 5413), (4517, 5417), (4519, 5419), (4583, 5483), (45139, 54139), (45181, 54181)...
- : for n = 7, there exist the following pairs of (P1, P2): (787, 877), (7841, 8741), (7853, 8753), (7879, 8779), (7883, 8787), (78179, 87179)...