A simple model of quantization: an approach from chaos

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Abstract. There is a paradigm in Quantum Mechanics that explains quantization through normal vibration modes called *Eigenstates* that arise from Schrödinger wave equation. In this contribution we propose an alternative methodology of quantization by using basic concepts of mechanics and chaos from which a *Toy Model* is built.

1. Motivation

Let us assume that a pair of particles interact with quantum noise [3][11] such that they are perturbed in the form of *kicks* [1][10] and besides, these particles attract each another due to a central force. The Lagrangian that describes this phenomenon consists of one term associated to the central force acting on the total mass of the system and the *Ansatz* that models the complex interaction between the quantum noise and the particles:

$$\mathcal{L} = \frac{1}{2}\left(m\dot{r}^2 + mr^2\dot{\theta}^2\right) - V(r) - K\theta\sin\left(\Omega + \xi\dot{\theta}\right)\sum_{j=-\infty}^{\infty}\delta(t - jT)\,. \tag{1}$$

Where $j\epsilon\mathbb{Z}$, $-\pi \leq \Omega \leq \pi$ and, K, ξ are parameters that will be defined later on, and T is the perturbation period.

From Euler-Lagrange equations [4] we have

$$\frac{d}{dt}(m\dot{r}) - mr\dot{\theta}^2 + \frac{\partial}{\partial r}V(r) + \epsilon(K, \xi(m,r), T) = 0 \tag{2}$$

$$\frac{d}{dt}\left(mr^2\dot{\theta}\right) + K\sin\left(\Omega + \xi\dot{\theta}\right)\sum_{j=-\infty}^{\infty}\delta(t - jT) = 0 \tag{3}$$

with $\epsilon(K, \xi(m,r), T) \ll \frac{\partial}{\partial r}V(r)$.

Let us assume that r is a constant, thus from equation (3) we have

$$\ddot{\theta} = -\frac{K}{mr^2}\sin\left(\Omega + \xi\dot{\theta}\right)\sum_{j=-\infty}^{\infty}\delta(t - jT) \tag{4}$$

$$\dot{\theta}_{j+1} = \dot{\theta}_j - \frac{K}{mr^2}\sin\left(\Omega + \xi\dot{\theta}_j\right). \tag{5}$$

Taking the fixed points [5] in (5), $\dot{\theta}_{j+1} = \dot{\theta}_j = \dot{\theta}^*$ then

$$\sin\left(\Omega + \xi\dot{\theta}^*\right) = 0.$$

Thus, the fixed points are

$$\dot{\theta}_n^* = \frac{n\pi - \Omega}{\xi} \quad n\epsilon\mathbb{Z}. \tag{6}$$

In order to obtain the stable fixed points we take [5]

$$\left|f'\left(\dot{\theta}^*\right)\right| < 1 \tag{7}$$

where

$$f'\left(\dot{\theta}^*\right) = 1 - \frac{K\xi}{mr^2}\cos\left(\Omega + \xi\dot{\theta}^*\right). \tag{8}$$

Taking the n even stable fixed points in equation (6) we have

$$\dot{\theta}_n^* = \frac{2n\pi - \Omega}{\xi} \quad n\epsilon\mathbb{Z} \tag{9}$$

and from (7), (8) and (9):

$$0 < \frac{K\xi}{mr^2} < 2. \quad (10)$$

Taking $\frac{K\xi}{mr^2} = 2\pi\mu_0$ and $0 < \mu_0 < \frac{1}{\pi}$ we can write equation (9) as:

$$\dot{\theta}_n^* = \frac{(2n\pi - \Omega)\mathrm{K}}{2\pi\mu_0 mr^2} \quad n\epsilon\mathbb{Z}. \quad (11)$$

Now, if $\Omega = 0$ and $K = \mu_0 Ħ$ in equation (11) then

$$L_n = nĦ$$

2. Experimental consequences

An interesting consequence from the stability condition is that, if $K = \mu_0\hbar$ in (10) where $\hbar$ is the Planck's constant, and taking r as in Bohr's model [7][8][9] $r^2 = \frac{n^4\hbar^4}{m^2k^2e^4}$ where k is the Coulomb constant, we obtain

$$\xi < \frac{n^4}{2\pi\mu_0 Rc} \quad (12)$$

where $R = 1.0972 \times 10^7\ m^{-1}$ is the Rydberg's constant and c is the speed of light, wich leads to $Rc \sim 10^{15}\ Hz$.

Now, taking Lyman's [9] series with $n \geq 2$ we have that

$$\frac{1}{\lambda_n} = R\left(1 - \frac{1}{n^2}\right). \quad (13)$$

Let be $\frac{c}{\lambda_\xi} = \frac{1}{\xi}$ in equations (12) and (13):

$$\mu_0 < \frac{1}{2\pi\lambda_\xi R(1 - \frac{1}{\lambda_n R})^2}. \quad (14)$$

Let be $\lambda_\xi > \lambda_n$ then $\lambda_\xi = A\lambda_n$ with $A > 1$ and when $\mu_0 \approx \frac{1}{\pi}$ in equation (14) we have that

$$A \approx \frac{n^4 - n^2}{2}.$$

When in Lyman's series $n = 8$ we know that $\lambda_8 = 9.26 \times 10^{-8}\ m$ and $\nu_\xi = \frac{1}{\xi}$

$$\nu_\xi = \frac{2c}{\lambda_n(n^4 - n^2)} \approx 1.606 \times 10^{12}\ Hz.$$

The latter shows that $\nu_\xi \approx 1.606 \times 10^{12}\ Hz$, is sufficient but not necessary to keep the system stable.

According to this model, is the correct candidate for perturbations, we must have only seven stable series

$$\nu_\xi = \frac{2c}{\lambda_n(\frac{n^4}{m^2} - n^2)}$$

where $m = 1,2,3,4,5,6,7$ with $n \geq m + 1$ correspond to Lyman, Balmer, Paschen, Brackett, Pfund, Humphreys and 7th respectively, in all cases when $n = 8$ we have $\nu_\xi \approx 1.606 \times 10^{12}\ Hz$. If $\xi(m,r)$ is a constant then $\epsilon(K, \xi(m,r), T)$ must be zero.

3. Fixed points of M period: an interesting observation

We can scale the system to a convenient scale, if in equation (5) we take $\Omega = 0$, $\xi = 2\pi\ [s]$, $mr^2 = 1\ [kg \cdot m^2]$ and $Ħ = 1\ [J \cdot s]$ we have

$$\dot{\theta}_{j+1} = \dot{\theta}_j - K\sin(2\pi\dot{\theta}_j)$$

in this case $K = \mu_0$, and $0 < K < \frac{1}{\pi}$.

If we increase K beyond period one we obtain the following stable period *cascades* [6] (see Figure 1). This diagram is also known as a bifurcation diagram.[1]

[1] *Bifurcation diagrams were first discovered by Robert May.*

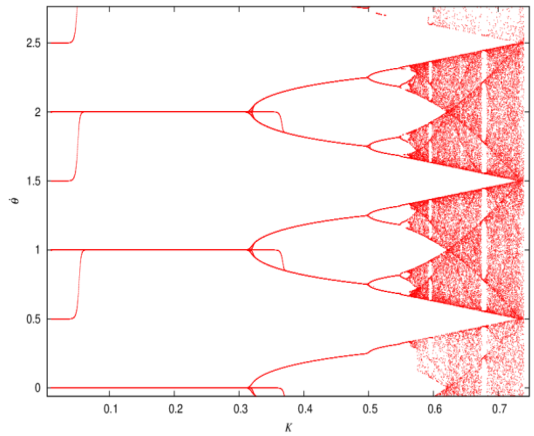

Figure 1 $\dot{\boldsymbol{\theta}}$ $\boldsymbol{vs}$ $\boldsymbol{K}$

These periodic points follow an order established by Sharkovskii's theorem [2]:

$$3 \prec 5 \prec 7 \prec \cdots \prec 2 \cdot 3 \prec 2 \cdot 5 \prec 2 \cdot 7 \prec \cdots \prec 2^N \cdot 3 \prec 2^N \cdot 5 \prec 2^N \cdot 7 \prec \cdots \prec 8 \prec 4 \prec 2 \prec 1 \quad N \epsilon \mathbb{N}$$

Let us consider the following subset

$$\ldots \prec 2 \cdot 3 \prec 2 \cdot 5 \prec 2 \cdot 7 \prec \cdots \prec 2 \prec 1 = \cdots \prec 6 \prec 10 \prec 14 \prec \cdots \prec 2 \prec 1$$

wich coincidently emerges in the range of energies of the different chemical elements (see Figures 2 and 3). Thus, to analytically calculate these periods is necessary to estimate the compositions for each corresponding period [5], $f^M = x$, i.e.:

$$f^6 = f \circ f \circ f \circ f \circ f \circ f$$

$$f^{10} = f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f$$

$$f^{14} = f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f \circ f$$

$$\vdots$$

$$f^2 = f \circ f$$

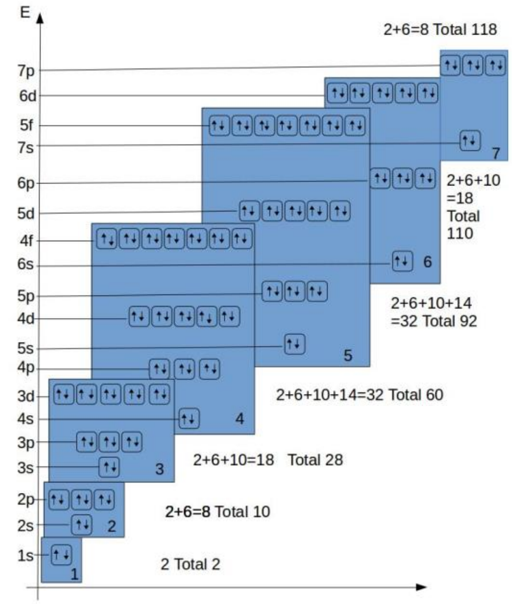

Figure 2 Energy levels

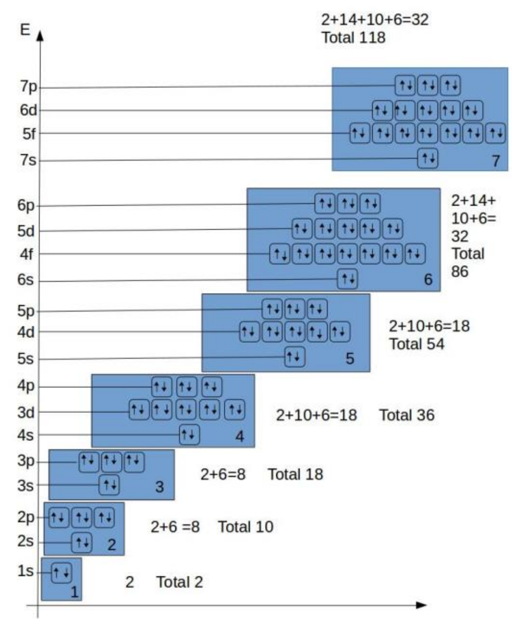

Figure 3 Sharkovskii's order

for wich would exist $K_{s \ldots fdp}$ or $\mu_{s \ldots fdp}$ to be determined.

These fixed points of higher periods may, for example, propagate through the energy levels according to the following modified equation from Bohr's model

$$E = \frac{mk^2Z^2e^4}{2\hbar^2(n \pm \Omega_{s\ldots fdp} \pm \zeta_{s\ldots fdp})^2}$$

where $\Omega_{s\ldots fdp}$ and $\zeta_{s\ldots fdp}$ would be related to the fixed points of periods $2, \ldots, 14, 10, 6$ possibly to the sub-level $s, \ldots, f, d, p$.

4. Discussion and Conclusions

Even so, it would be necessary to take into account relativistic effects, and extra dimension factors as well as the effects of the spin of the electron as is currently carried out by modern quantum mechanics to describe the fine-structure; however this represents a simplified model.

In this work we present a methodology based on basic concepts of stability to quantize the angular moment. It was also shown that, in order for this model to be stable, it is sufficient condition that the model is immersed in noise with the same frequency and, that the order of the energy levels agree with the order given by Sharkovskii's theorem. Finally, we can apply this methodology to other scales.

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