An additional constraint on local realism with mixture of ten-particle Greenberger-Horne-Zeilinger state implied by rotational invariance

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Rotational invariance of physical laws is an accepted principle in Newton's theory. We show that it leads to an additional constraint on local realistic theories with mixture of ten-particle Greenberger-Horne-Zeilinger state. This new constraint rules out such theories even in some situations in which standard Bell inequalities allow for explicit construction of such theories.

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I. INTRODUCTION

The results of the argumentation, dealing with consequences for the assumption of rotational invariance for tests of non-locality, have been published [1–3]. Our intention is to engage with and expand upon these same results with mixture of ten-particle Greenberger-Horne-Zeilinger (GHZ) state [4]. In this argumentation we present violation of the Bell inequality by mixture of ten-particle GHZ state in different bases and white noise. The inequality was derived by us in the argumentation cited as [5]. The same state, but for six spins, was also considered there.

Non-locality in quantum physics means the possibility of distributing correlations that cannot be due to previously shared randomness, without signaling [6]. Some quantum predictions violate Bell inequalities [7], which form necessary conditions for local realistic theories for the results of measurements. Thus, some quantum predictions do not accept local realistic theories.

Leggett-type nonlocal realistic theory [8] is experimentally investigated [9–11]. The experiments report that the quantum theory does not accept Leggett-type nonlocal realistic theory. These experiments are performed by using entangled states (two spins $\frac{1}{2}$).

Rotational invariance of physical laws is a generally accepted principle in Newton's theory. It states that the value of a correlation function does not depend on the coordinate systems used by the observers. The measurement setup classifies realistic theories [10–12].

Many of the recent advances in quantum information theory suggest that the highly-non-local properties of quantum states that lead to violations of Bell inequalities can be used as a resource to achieve success in some tasks, which are locally impossible. Examples can serve quantum cryptography and quantum communication complexity [13–15]. Therefore as the impossibility of existence of local realistic theories for some processes leads to various quantum informational applications it is important to learn what the ultimate bounds for such theories are.

We aim to show that the fundamental property of the known laws of Newton's theory, their rotational invariance can be used to find new hypothesis by using disqualification of experimentally accessible local realistic theories with ten particles in a new state.

This argumentation is as follows. Assume that we have some correlation function. This correlation function has a form which is rotationally invariant (cf. Equation (2)). We want to build rotationally invariant local realistic theory for the rotationally invariant correlation function. We see that the demand that the resulting correlation function must be rotationally invariant leads to a generalized Bell inequality [5], which restricts additionally possible local realistic theories. Further, even if "standard" two-setting Bell inequalities [16–22] allow local realistic theories for the given set of data (i.e., a set of correlation function values obtained in a Bell type experiment), the new restriction, derived from rotational invariance, can invalidate such theories, for some range of parameters.

This paper is organized as follows.

In Sec. II, we discuss generalized Bell inequality.

In Sec. III, we discuss mixture of ten-qubit GHZ state.

In Sec. IV, we discuss violation of rotational invariance of local realistic models with mixture of GHZ state.

Section V summarizes this paper.

II. GENERALIZED BELL INEQUALITY

Assume that we have a set of N spins $\frac{1}{2}$. Each of them is in a separate laboratory. As is well known the measurements (observables) for such spins are parameterized by a unit vector $\vec{n}_j$ (its direction along which the spin component is measured). The results of measurements are ± 1. We can introduce the "Bell" correlation function, which is the average of the product of the local results:

$$E(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \langle r_1(\vec{n}_1) r_2(\vec{n}_2) \cdots r_N(\vec{n}_N) \rangle_{\rm avg}, \quad (1)$$

where $r_j(\vec{n}_j)$ is the local result, ± 1, which is obtained if the measurement direction is set at $\vec{n}_j$.

If an experimental correlation function admits rotationally invariant tensor structure familiar from Newton's theory, we can introduce the following form:

$$E(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \hat{T} \cdot (\vec{n}_1 \otimes \vec{n}_2 \otimes \cdots \otimes \vec{n}_N), \qquad (2)$$

where $\otimes$ denotes the tensor product, $\cdot$ the scalar product in $\mathrm{R}^{3\mathrm{N}}$, and $\hat{T}$ the correlation tensor given by

$$E(\vec{x}_1^{(i_1)}, \vec{x}_2^{(i_2)}, \ldots, \vec{x}_N^{(i_N)}) = T_{i_1 \ldots i_N}, \qquad (3)$$

where $\vec{x}_j^{(i_j)}$ is a unit directional vector of the local coordinate system of the jth observer; $i_j = 1, 2, 3$ gives the full set of orthogonal vectors defining the local Cartesian coordinates. Obviously the assumed form of (2) implies rotational invariance, because the correlation function does not depend on the coordinate systems used by the observers. Rotational invariance simply states that the value of $E(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N)$ cannot depend on the local coordinate systems used by the N observers. There is an important, although obvious, implication of rotational invariance.

Assume that one knows the values of all 3^N components of the correlation tensor, $T_{i_1 \ldots i_N}$, which are obtainable by performing specific 3^N measurements of the correlation function, (cf. Eq. (3)). Then, with the use of formula (2) we can reproduce the value of the correlation functions for all other possible sets of local settings. Using this rotationally invariant structure of the correlation function, we shall derive a necessary condition for the existence of rotationally invariant local realistic theory of the experimental correlation function given in (2). If the correlation function is described by rotationally invariant local realistic theory, then the correlation function must be simulated by the following structure

$$E_{\mathrm{LR}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \int d\lambda \rho(\lambda) I^{(1)}(\vec{n}_1, \lambda) I^{(2)}(\vec{n}_2, \lambda) \cdots I^{(N)}(\vec{n}_N, \lambda), \qquad (4)$$

where λ is some local hidden variable, $\rho(\lambda)$ is a probabilistic distribution, and $I^{(j)}(\vec{n}_j, \lambda)$ is the predetermined "hidden" result of the measurement of all the dichotomic observable $\vec{n} \cdot \sigma$ with values ± 1. The dependence on the choice of the correlation function is unit vector $\vec{n}_j$.

Those assumptions are now demystified by Hess and Phillip [27, 28], as they show that Bell's inequalities may be violated even for objective local random variables.

Let us parametrize the arbitrary unit vector in a spherical coordinate system defined by $\vec{x}_j^{(1)}$, $\vec{x}_j^{(2)}$, and $\vec{x}_j^{(3)}$ in the following way:

$$\vec{n}_j(\theta_j, \phi_j) = \sin\theta_j \cos\phi_j \vec{x}_j^{(1)} + \sin\theta_j \sin\phi_j \vec{x}_j^{(2)} + \cos\theta_j \vec{x}_j^{(3)}, \qquad (5)$$

where $\vec{x}_j^{(1)}$, $\vec{x}_j^{(2)}$, and $\vec{x}_j^{(3)}$ are the Cartesian axes relative to which spherical angles are measured.

We shall show that the scalar product of rotationally invariant local realistic correlation function

$$E_{\mathrm{LR}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \int d\lambda \rho(\lambda) I^{(1)}(\vec{n}_1, \lambda) I^{(2)}(\vec{n}_2, \lambda) \cdots I^{(N)}(\vec{n}_N, \lambda), \qquad (6)$$

with the rotationally invariant experimental correlation function, that is

$$E(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) = \hat{T} \cdot (\vec{n}_1 \otimes \vec{n}_2 \otimes \cdots \otimes \vec{n}_N), \qquad (7)$$

is bounded by a specific number dependent on $\hat{T}$, namely:

$$(E_{\mathrm{LR}}, E) = \int d\Omega_1 \int d\Omega_2 \cdots \int d\Omega_N \, E_{\mathrm{LR}}(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) E(\vec{n}_1, \vec{n}_2, \ldots, \vec{n}_N) \leq (2\pi)^N T_{\max}, \qquad (8)$$

where $T_{\max}$ is the maximal possible value of the correlation tensor component, i.e.,

$$T_{\max} = \max_{\theta_1, \phi_1, \ldots, \theta_N, \phi_N} E(\theta_1, \phi_1, \ldots, \theta_N, \phi_N). \qquad (9)$$

We use decomposition (5). We introduce the usual measure $d\Omega_j = \sin\theta_j d\theta_j d\phi_j$ for the system of the jth observer. Note that, due to the integrations in (8), we are looking for rotationally invariant theory described by the entire range of settings. A necessary condition for the existence of rotationally invariant local realistic theory E_{LR} of the rotationally invariant experimental correlation function

$$E(\theta_1, \phi_1, \ldots, \theta_N, \phi_N) = E(\vec{n}_1(\theta_1, \phi_1), \ldots, \vec{n}_N(\theta_N, \phi_N)), \qquad (10)$$

that is for E_{LR} equal to E, is that we have $(E_{\mathrm{LR}}, E) = (E, E)$. If we have, e.g., $(E_{\mathrm{LR}}, E) < (E, E)$, then the rotationally invariant experimental correlation function cannot be explainable by rotationally invariant local realistic theory.

In what follows, we derive the upper bound (8). Since the rotationally invariant local realistic theory is an average over λ, it is enough to find the bound of the following expression:

$$\int d\Omega_1 \cdots \int d\Omega_N I^{(1)}(\vec{n}_1, \lambda) \cdots I^{(N)}(\vec{n}_N, \lambda) \times \sum_{i_1, i_2, \ldots, i_N = 1,2,3} T_{i_1 i_2 \ldots i_N} c_1^{i_1} c_2^{i_2} \cdots c_N^{i_N}, \qquad (11)$$

where

$$\vec{c}_j = (c_j^1, c_j^2, c_j^3) = (\sin\theta_j \cos\phi_j, \sin\theta_j \sin\phi_j, \cos\theta_j), \qquad (12)$$

and

$$T_{i_1 i_2 \ldots i_N} = \hat{T} \cdot (\vec{x}_1^{(i_1)} \otimes \vec{x}_2^{(i_2)} \otimes \cdots \otimes \vec{x}_N^{(i_N)}). \qquad (13)$$

Let us analyze the structure of this integral (11). It is easy to notice that (11) is a sum, with coefficients given by $T_{i_1 i_2 \dots i_N}$, which is a product of the following integrals:

$$\int d\Omega_j I^{(j)}(\theta_j,\phi_j)\sin\theta_j\cos\phi_j,$$
$$\int d\Omega_j I^{(j)}(\theta_j,\phi_j)\sin\theta_j\sin\phi_j, \quad (14)$$

and

$$\int d\Omega_j I^{(j)}(\theta_j,\phi_j)\cos\theta_j. \quad (15)$$

Notice that we deal here with integrals, or rather scalar products of $I^{(j)}(\theta_j,\phi_j)$ with three orthogonal functions. Simply we have

$$\int d\Omega_j c_j^\alpha c_j^\beta = (4\pi/3)\delta_{\alpha,\beta}. \quad (16)$$

The normalized functions $\sqrt{3/4\pi}\sin\theta_j\cos\phi_j$, $\sqrt{3/4\pi}\sin\theta_j\sin\phi_j$, and $\sqrt{3/4\pi}\cos\theta_j$ form a basis of a three-dimensional real functional space, which we shall call $S^{(3)}$. Using these three functions we can write the projection of function $I^{(j)}(\theta_j,\phi_j)$ onto them such as

$$\int d\Omega_j I^{(j)}(\theta_j,\phi_j)\sqrt{3/4\pi}\sin\theta_j\cos\phi_j = \sin\beta_j\cos\gamma_j\|I^{(j)||}\|,$$
$$\int d\Omega_j I^{(j)}(\theta_j,\phi_j)\sqrt{3/4\pi}\sin\theta_j\sin\phi_j = \sin\beta_j\sin\gamma_j\|I^{(j)||}\|,$$
$$\int d\Omega_j I^{(j)}(\theta_j,\phi_j)\sqrt{3/4\pi}\cos\theta_j = \cos\beta_j\|I^{(j)||}\|, \quad (17)$$

where $\|I^{(j)||}\|$ is the length of the projection, and β_j and γ_j are some angles. Going back to expression (11), we have

$$\left(\frac{4\pi}{3}\right)^{N/2}\prod_{j=1}^{N}\|I^{(j)||}\|$$
$$\times \sum_{i_1,i_2,\dots,i_N=1,2,3} T_{i_1 i_2\dots i_N} e_1^{i_1} e_2^{i_2}\cdots e_N^{i_N}, \quad (18)$$

with a normalized vector

$$(e_j^1,e_j^2,e_j^3) = (\sin\beta_j\cos\gamma_j, \sin\beta_j\sin\gamma_j, \cos\beta_j). \quad (19)$$

Note that the sum in (18) over the components of this vector is just $\hat{T}\cdot(\vec{e}_1\otimes\vec{e}_2\otimes\cdots\otimes\vec{e}_N)$, i.e., it is a component of the tensor $\hat{T}$ in the local Cartesian coordinate systems specified by the vectors $\vec{e}_j$. If we know all the values of $T_{i_1 i_2\dots i_N}$, we can always find the maximal possible value of such a component, and it is equal to $T_{\max}$, of Eq. (9). Thus,

$$\sum_{i_1,i_2,\dots,i_N=1,2,3} T_{i_1 i_2\dots i_N} e_1^{i_1} e_2^{i_2}\cdots e_N^{i_N} \le T_{\max}. \quad (20)$$

It remains to show the upper bound on the norm $\|I^{(j)||}\|$. From the definition the norm is given by a maximal possible value of the scalar product between $I^{(j)}(\theta_j,\phi_j)$ and any normalized function belonging to $S^{(3)}$:

$$\|I^{(j)||}\| = \max_{|\vec{d}|=1}\left[\sqrt{\frac{3}{4\pi}}\int d\Omega_j I^{(j)}(\theta_j,\phi_j)\sum_{k=1}^{3} d_k c_j^k\right] \quad (21)$$

where $\vec{d} = (d_1,d_2,d_3)$ and $|\vec{d}| = \sum_{k=1}^3 d_k^2 = 1$. Since $|I^{(j)}(\theta_j,\phi_j)| = 1$, we have, for the integral of the modulus,

$$\|I^{(j)||}\| \le \max_{|\vec{d}|=1}\left[\sqrt{\frac{3}{4\pi}}\int d\Omega_j\left|\vec{d}\cdot\vec{c}_j\right|\right], \quad (22)$$

where the dot between three-dimensional vectors denotes the usual scalar product in R^3. The values of this scalar product are then integrated (summed) over all values of θ_j and ϕ_j, i.e., over vectors $\vec{c}_j$ on the whole sphere. Since the measure is rotationally invariant the integral does not depend on particular $\vec{d}$ and we choose it as a unit vector in direction $\vec{z}$. For this choice

$$\|I^{(j)||}\| \le \int d\Omega_j\left|\sqrt{\frac{3}{4\pi}}\cos\theta_j\right| = 2\pi\sqrt{\frac{3}{4\pi}}. \quad (23)$$

Finally, we have

$$(E_{LR},E) \le (2\pi)^N T_{\max}. \quad (24)$$

The relation (24) is a generalized N-qubit Bell inequality with the entire range of measurement settings. Rotationally invariant local hidden variable theories, $E_{\rm LR}$, which rebuild experimental rotationally invariant correlations, E, satisfy it. Below we show that if we replaces $E_{\rm LR}$ by E we may have a violation of the inequality (24). We have

$$(E,E) = \int d\Omega_1\cdots\int d\Omega_N\left(\sum_{i_1,\dots,i_N=1}^{3} T_{i_1\dots i_N} c_1^{i_1}\cdots c_N^{i_N}\right)^2$$
$$= (4\pi/3)^N\sum_{i_1,\dots,i_N=1}^{3} T_{i_1\dots i_N}^2, \quad (25)$$

where we use the orthogonality relation $\int d\Omega_j\ c_j^\alpha c_j^\beta = (4\pi/3)\delta_{\alpha,\beta}$. The structure of condition (24) and the value (25) suggests that the value of (25) does not have to be smaller than (24). That is there may be such correlation functions E, which have the property that for any $E_{\rm LR}$ we have $(E_{\rm LR},E) < (E,E)$, which implies impossibility of modeling E by rotationally invariant local realistic correlation function $E_{\rm LR}$.

III. MIXTURE OF TEN-QUBIT GHZ STATE

We shall present an important quantum state. We assume $N = 10$. Consider the following ten-qubit Greenberger-Horne-Zeilinger (GHZ) state [4]

$$|\psi_3\rangle = \frac{1}{\sqrt{2}}\Big(|z+\rangle_1\cdots|z+\rangle_9|z-\rangle_{10} + |z-\rangle_1\cdots|z-\rangle_9|z+\rangle_{10}\Big), \quad (26)$$

where $|z\pm\rangle_j$ is the eigenstate of the local σ_z operator of the jth observer. Note that the states of the last party are flipped with respect to the states of the other parties. We rotate the states of all individual qubits by the angle $\alpha = 2\pi/3$ around the axis $\vec{m} = \frac{1}{\sqrt{3}}(1,1,1)$ on the Bloch sphere. This rotation cyclically permutes the directions of the Cartesian coordinate system. The unitary realizing this rotation is given by:

$$U = e^{-i\frac{\alpha}{2}\vec{m}\cdot\vec{\sigma}} = \frac{1}{2}\begin{pmatrix} 1-i & -1-i \\ 1-i & 1+i \end{pmatrix}, \tag{27}$$

with $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$ being a vector of local Pauli operators. Applying U to all the qubits gives a new state $|\psi_1\rangle \equiv U^{\otimes 10}|\psi_3\rangle$. With the double application we get $|\psi_2\rangle \equiv U^{\otimes 10}|\psi_1\rangle$. The states $|\psi_1\rangle$ and $|\psi_2\rangle$ are, up to a global phase which does not contribute to correlations, of the same form as $|\psi_3\rangle$, but are written in the local bases of σ_x and σ_y operators, respectively. Finally, we introduce a mixture of Greenberger-Horne-Zeilinger correlations and white noise:

$$\rho = \frac{V}{3}\sum_{k=1}^{3}|\psi_k\rangle\langle\psi_k| + (1-V)\rho_{\text{noise}}, \tag{28}$$

where $|\psi_k\rangle$ is the GHZ state and $\rho_{\text{noise}} = \frac{1}{2^{10}}I$ is the random noise admixture. The value of V can be interpreted as the reduction factor of the interferometric contrast observed in the ten-particle correlation experiment.

IV. VIOLATION OF ROTATIONAL INVARIANCE OF LOCAL REALISTIC MODELS WITH MIXTURE OF GHZ STATE

We present here a simple, but important example of violation (24). Imagine 10 observers who can choose between three orthogonal directions of spin measurement, $\vec{x}_j^{(1)}$, $\vec{x}_j^{(2)}$, and $\vec{x}_j^{(3)}$ for the jth one.

Let us assume that the source of 10 entangled spin-carrying particles emits them in a state, which can be described as a mixture of Greenberger-Horne-Zeilinger correlations, given in (28). We can show that if the observers limit their settings to $\vec{x}_j^{(1)} = \hat{x}_j$, $\vec{x}_j^{(2)} = \hat{y}_j$, and $\vec{x}_j^{(3)} = \hat{z}_j$, there are

$$3\left({}_{10}C_2 +_{10} C_4 + \cdots +_{10} C_8\right) + 3 = 3(2^{10-1} - 1) \tag{29}$$

components of $\hat{T}$ of the value $\pm V/3$. Other x-y-z components vanish.

It is easy to see that

$$\begin{aligned} &T_{\max} = V/3, \\ &\sum_{i_1,i_2,\ldots,i_{10}=1,2} T^2_{i_1 i_2 \ldots i_{10}} = 2^{10-1}\left(\frac{V}{3}\right)^2, \\ &\sum_{i_1,i_2,\ldots,i_{10}=2,3} T^2_{i_1 i_2 \ldots i_{10}} = 2^{10-1}\left(\frac{V}{3}\right)^2, \\ &\sum_{i_1,i_2,\ldots,i_{10}=3,1} T^2_{i_1 i_2 \ldots i_{10}} = 2^{10-1}\left(\frac{V}{3}\right)^2, \\ &\sum_{i_1,i_2,\ldots,i_{10}=1,2,3} T^2_{i_1 i_2 \ldots i_{10}} = 3(2^{10-1}-1)\left(\frac{V}{3}\right)^2. \end{aligned} \tag{30}$$

Then, we have $(E_{\text{LR}}, E) \leq (2\pi)^{10}V/3$ and $(E,E) = 3(2^{10-1}-1)(4\pi/3)^{10}\left(\frac{V}{3}\right)^2$. It is clear that if we have ten spins the rotational invariance puts an additional, non trivial, constraint on a local realistic theory. For V given by

$$0.112847 \simeq \frac{1}{2^{10-1}-1}\left(\frac{3}{2}\right)^{10} < V \leq \frac{3}{\sqrt{2^{10-1}}} \simeq 0.132583 \tag{31}$$

despite the fact that there exists a local realistic theory for the actually measured values of the correlation function, the rotational invariance principle disqualifies this theory. As it is shown in [22] if the correlation tensor satisfies the following conditions

$$\begin{aligned} &\sum_{i_1,i_2,\ldots,i_{10}=1,2} T^2_{i_1 i_2 \ldots i_{10}} \leq 1, \\ &\sum_{i_1,i_2,\ldots,i_{10}=2,3} T^2_{i_1 i_2 \ldots i_{10}} \leq 1, \\ &\sum_{i_1,i_2,\ldots,i_{10}=3,1} T^2_{i_1 i_2 \ldots i_{10}} \leq 1 \end{aligned} \tag{32}$$

then there always exists an *explicit* local realistic theory for the set of correlation function values $E(\vec{x}_1^{(i_1)}, \vec{x}_2^{(i_2)}, \ldots, \vec{x}_{10}^{(i_{10})})$, $i_1, i_2, \ldots, i_{10} = 1,2,3$. For our example the condition (32) is met whenever $V \leq \frac{3}{\sqrt{2^{10-1}}}$. Nevertheless the rotational invariance principle excludes local realistic theories for $V > \frac{1}{2^{10-1}-1}\left(\frac{3}{2}\right)^{10}$. Thus the situation is such that for $V \leq \frac{3}{\sqrt{2^{10-1}}}$ for all two settings per observer experiments we can construct a local realistic theory for the values of the correlation function for the settings chosen in the experiment. But these theories must be consistent with each other, if we want to extend their validity beyond the 2^{10} settings to which each of them pertains. Our result clearly indicates that this is impossible for $V > \frac{1}{2^{10-1}-1}\left(\frac{3}{2}\right)^{10}$ That is, theories built to reconstruct the 2^{10} data points, when compared with each other, must be inconsistent - therefore they are invalidated. The theories must contradict each other. In other words the explicit theories, given in [22], work only for the specific set of settings in the given experiment, but cannot be extended to all settings. We utilize rotational invariance to show this.

Please note that all information needed to get this conclusion can be obtained in a three-orthogonal-settings-per-observer experiments with ten particles. Simply to get both the value of (25) ($N = 10$) and of $T_{\max}$ it is enough to measure all values of $T_{i_1 i_2 \dots i_{10}}$, $i_1, i_2, \dots, i_{10} = 1, 2, 3$.

V. CONCLUSIONS

In conclusions, rotational invariance of physical laws has been an accepted principle in Newton's theory. We have shown that it leads to an additional constraint on local realistic theories with mixture of ten-particle Greenberger-Horne-Zeilinger state. This new constraint has ruled out such theories even in some situations in which standard Bell inequalities allow for explicit construction of local realistic theories.

The interesting feature is that Bell's theorem rules out realistic interpretation of some quantum mechanical predictions, and therefore of quantum mechanics in general, provided one assumes locality. Locality is a consequence of the general symmetries of the Poincaré group of the Special Relativity Theory. However it is a direct consequence of the Lorentz transformations (boosts), as they define the light-cone. As our discussion shows a subgroup of the Poincaré group, rotations of the Cartesian coordinates, introduces an additional constraint on the local realistic models.

The results presented are the result of the definition of E_{LR}, equation (4), this type of hidden variable has been widely studied, see [23–28] and it is known that for this type of hidden variable is not compatible with the experimental results. The great debate is that we can not describe the action of the hidden variable as in (4) for most cases, this has been necessarily the case when we have a stochastic force acting on the system among others, see [24–28].

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