

Special cases of Goldbach conjecture: For every even integer $2n$, there exists infinite integers “d” greater than one, such that the product $2nd$, may be expressed as sum of two primes

Abstract: In this paper, we show that for all even integers “ $2n$ ”, there exists infinite positive integers “d” greater than one, such that their product “ $2nd$ ” is a sum of two primes. Any two odd primes add to give even integers. However this general method does not allow us to understand the property or relationship among even numbers numbers derived in this manner. On the other hand, our results suggests existence of even integers of the specific form “ $2nd$ ” that can be written as a sum of two primes.

Results:

Consider an even integer $2n$, where $n=1,2,3,\dots$

Then for each $2n$, there exists integers $2n-1$ and $2n+1$ which are both co-prime to $2n$.

Therefore by Dirichlet’s theorem of arithmetic progressions, infinite integers a and b exist such that

$$(2n-1)+a(2n)=p, \text{ where } p \text{ is prime } \dots\dots\dots(1)$$

and

$$(2n+1)+b(2n)=q, \text{ where } q \text{ is prime } \dots\dots\dots(2)$$

Adding the two equations (1) and (2)

$$4n+2n(a+b)=p+q \dots\dots\dots \text{ (sum of two primes)}$$

$$2n(2+a+b)=p+q$$

Replacing $(2+a+b)$ by integer d , we get

$$2nd = p+q$$

This suggests that for every even integer $2n$, there exists infinite number of suitable integers “d” where d is greater than one, such the product $2nd$ can be expressed as the sum of two primes.

For each such even integer “ $2nd$ ”, the even Goldbach conjecture is true.

In a special case when a and b are both zero, and $2n-1$ and $2n+1$ are twin primes, d takes the minimum value of 2.