Primes obtained concatenating p-1 with q^2 where p and q are primes or Poulet numbers

Abstract. In this paper I make the following eight conjectures: (Ia) for any p prime, p > 3, there exist an infinity of primes q such that the number n obtained concatenating p – 1 to the right with q^2 is prime; (Ib) there exist an infinity of terms in any of the sequences above (for any p) such that r = (p – 1)*q^2 + 1 is prime; (IIa) for any q prime, q > 3, there exist an infinity of primes p such that the number n obtained concatenating q^2 to the left with p – 1 is prime; (IIb) there exist an infinity of terms in any of the sequences above (for any q) such that r = (p – 1)*q^2 + 1 is prime; (IIIa) for any Poulet number P, not divisible by 3, there exist an infinity of primes q such that the number n obtained concatenating P – 1 to the right with q^2 is prime; (IIIb) there exist an infinity of terms in any of the sequences above (for any P) such that r = (P – 1)*q^2 + 1 is prime; (IVa) for any Poulet number Q, not divisible by 3 or 5, there exist an infinity of primes p such that the number n obtained concatenating Q^2 to the left with p – 1 is prime; (IVb) there exist an infinity of terms in any of the sequences above (for any Q) such that r = (p – 1)*Q^2 + 1 is prime.

Conjecture 1a:

For any p prime, p > 3, there exist an infinity of primes q such that the number n obtained concatenating p – 1 to the right with q^2 is prime (example: for p = 5, the number n = 449 obtained for q = 7 is prime).

The sequence of primes n for p = 5:

: 449, 4289, 41681, 41849, 42209, 43481, 43721, 45329, 46889, 49409 (...) obtained for q = 7, 17, 41, 59, 61, 79, 83, 97 (...)

The sequence of primes n for p = 7:

: 6121, 6361, 6529, 6841, 6961, 61681, 66889 (...) obtained for q = 7, 19, 23, 29, 31, 41, 83 (...) [note the chain of four primes n (6361, 6529, 6841, 6961) obtained for four consecutive primes q (19, 23, 29, 31]

The sequence of primes n for p = 11:

: 1049, 10169, 10289, 10529, 101681 (...) obtained for q = 7, 13, 17, 23, 41 (...
The sequence of primes \( n \) for \( p = 13 \):

: 1249, 12289, 12841, 121369, 122209, 124489, 125329, 126241, 127921 (\( \ldots \)) obtained for \( q = 7, 17, 29, 37, 47, 67, 73, 79, 89 \) (\( \ldots \))

The sequence of primes \( n \) for \( p = 17 \):

: 16361, 16529, 162209, 163481, 165041, 169409 (\( \ldots \)) obtained for \( q = 19, 23, 47, 59, 71, 97 \) (\( \ldots \))

**Conjecture 1b:**

There exist an infinity of terms in any of the sequences above (for any \( p \)) such that \( r = (p - 1)q^2 + 1 \) is prime.

The sequence of primes \( r \) for \( p = 5 \):

: 197 (\( = 4\times49 + 1 \)), 8837 (\( = 4\times2209 + 1 \)), 21317 (\( = 4\times5329 + 1 \))\( \ldots \)

The sequence of primes \( r \) for \( p = 7 \):

: 727 (\( = 6\times121 + 1 \)), 41047 (\( = 6\times841 + 1 \))\( \ldots \)

The sequence of primes \( r \) for \( p = 11 \):

: 491 (\( = 10\times49 + 1 \)), 16811 (\( = 10\times1681 + 1 \))\( \ldots \)

The sequence of primes \( r \) for \( p = 13 \):

: 3469 (\( = 12\times289 + 1 \)), 10093 (\( = 12\times841 + 1 \)), 63949 (\( = 12\times5329 + 1 \))\( \ldots \)

The sequence of primes \( r \) for \( p = 17 \):

: 55697 (\( = 16\times3481 + 1 \)), 80657 (\( = 16\times5041 + 1 \))\( \ldots \)

**Conjecture 2a:**

For any \( q \) prime, \( q > 3 \), there exist an infinity of primes \( p \) such that the number \( n \) obtained concatenating \( q^2 \) to the left with \( p - 1 \) is prime;

The sequence of primes \( n \) for \( q^2 = 7^2 = 49 \):

: 449, 1049, 1249, 3049, 4049, 4649, 5849, 8849, 9649 (\( \ldots \)), obtained for \( p = 5, 11, 13, 31, 41, 47, 59, 89, 97 \) (\( \ldots \))
The sequence of primes \( n \) for \( q^2 = 11^2 = 121 \):

: \( 6121, 18121, 52121, 70121, 78121 \) (\( \ldots \)), obtained for \( p = 7, 19, 53, 71, 79 \) (\( \ldots \))

The sequence of primes \( n \) for \( q^2 = 13^2 = 169 \):

: \( 10169, 18169, 30169, 40169, 42169, 58169, 60169, 66169, 72169, 88169 \) (\( \ldots \)), obtained for \( p = 11, 19, 31, 41, 43, 59, 61, 67, 73, 89 \) (\( \ldots \))

The sequence of primes \( n \) for \( q^2 = 104729^2 = 10968163441 \):

: \( 1810968163441, 4010968163441, 5210968163441, 7810968163441, 8810968163441 \) (\( \ldots \)), obtained for \( p = 19, 41, 53, 79, 89 \) (\( \ldots \))

**Conjecture 2b:**

There exist an infinity of terms in any of the sequences above (for any \( q \)) such that \( r = (p - 1) \cdot q^2 + 1 \) is prime.

The sequence of primes \( r \) for \( q^2 = 7^2 = 49 \):

: \( 197, 491, 1471 \) (= \( 30 \cdot 49 + 1 \)), \( 2843 \) (= \( 58 \cdot 49 + 1 \)) (\( \ldots \))

The sequence of primes \( r \) for \( q^2 = 11^2 = 121 \):

: \( 727, 2179 \) (= \( 18 \cdot 121 + 1 \)), \( 9439 \) (= \( 70 \cdot 121 + 1 \)) (\( \ldots \))

The sequence of primes \( r \) for \( q^2 = 13^2 = 169 \):

: \( 6761 \) (= \( 40 \cdot 169 + 1 \)), \( 9803 \) (= \( 58 \cdot 169 + 1 \)), \( 10141 \) (= \( 60 \cdot 169 + 1 \)) (\( \ldots \))

**Conjecture 3a:**

For any Poulet number \( P \), not divisible by 3, there exist an infinity of primes \( q \) such that the number \( n \) obtained concatenating \( P - 1 \) to the right with \( q^2 \) is prime.

The sequence of primes \( n \) for \( P = 341 \):

: \( 34049, 340169, 3409409 \) (\( \ldots \)), obtained for \( q = 11, 13, 97 \) (\( \ldots \))

The sequence of primes \( n \) for \( P = 1105 \):

: \( 1104289, 11041369, 11042209, 11043481, 11044489, 11046241, 11047921 \) (\( \ldots \)), obtained for \( q = 17, 37, 47, 59, 67, 79, 89 \) (\( \ldots \))
The sequence of primes \( n \) for \( P = 1387 \):

\[
1386361, \ 13861369, \ 13862809, \ 13867921 \ (\ldots),
\]

obtained for \( q = 19, 37, 53, 89 \ (\ldots) \)

The sequence of primes \( n \) for \( P = 1729 \):

\[
172849, \ 1728121, \ 1728361, \ 17281681, \ 17283481, \ 17286889, \ 17289409 \ (\ldots),
\]

obtained for \( q = 7, 11, 19, 41, 59, 83, 97 \ (\ldots) \)

**Conjecture 3b:**

There exist an infinity of terms in any of the sequences above (for any \( P \)) such that \( r = (P - 1)q^2 + 1 \) is prime.

The sequence of primes \( r \) for \( P = 341 \):

\[
16661 \ (= 340\times49 + 1), \ 3199061 \ (= 340\times9409 + 1) \ldots
\]

The sequence of primes \( r \) for \( P = 1105 \):

\[
319057 \ (= 1104\times289 + 1) \ldots
\]

The sequence of primes \( r \) for \( P = 1387 \):

\[
14871781 \ (= 1104\times7921 + 1) \ldots
\]

The sequence of primes \( r \) for \( P = 1729 \):

\[
84673 \ (= 1728\times49 + 1), \ 209089 \ (= 1728\times121 + 1), \ 6015169 \ (= 1728\times3481 + 1) \ldots
\]

**Conjecture 4a:**

For any Poulet number \( Q \), not divisible by 3 or 5, there exist an infinity of primes \( p \) such that the number \( n \) obtained concatenating \( Q^2 \) to the left with \( p - 1 \) is prime.

The sequence of primes \( n \) for \( Q^2 = 341^2 = 116281 \):

\[
6116281, \ 18116281, \ 40116281, \ 42116281, \ 58116281, \ 60116281, \ 72116281, \ 78116281 \ (\ldots),
\]

obtained for \( p = 7, 19, 41, 43, 59, 61, 73, 79 \ (\ldots) \)

The sequence of primes \( n \) for \( Q^2 = 1387^2 = 1923769 \):

\[
181923769, \ 281923769, \ 881923769 \ (\ldots),
\]

obtained for \( p = 19, 23, 89 \ (\ldots) \)
The sequence of primes \( n \) for \( Q^2 = 1729^2 = 2989441 \): 

\[
62989441, 162989441, 222989441, 722989441, 822989441 (\ldots), \text{obtained for } p = 7, 17, 23, 72, 82 (\ldots)
\]

**Conjecture 4b:**

There exist an infinity of terms in any of the sequences above (for any \( Q \)) such that \( r = (p - 1)Q^2 + 1 \) is prime.

The sequence of primes \( r \) for \( Q^2 = 341^2 = 116281 \):

\[
697687 (= 6\times116281 + 1), 6744299 (= 58\times116281 + 1), 6976861 (= 6\times116281 + 1), 8372233 (= 72\times116281 + 1)\ldots
\]

The sequence of primes \( r \) for \( Q^2 = 1387^2 = 1923769 \):

\[
169291673 (= 88\times1923769 + 1)\ldots
\]

The sequence of primes \( r \) for \( Q^2 = 1729^2 = 2989441 \):

\[
65767703 (= 22\times2989441 + 1)\ldots
\]